Wavelet Toolbox™

Reference

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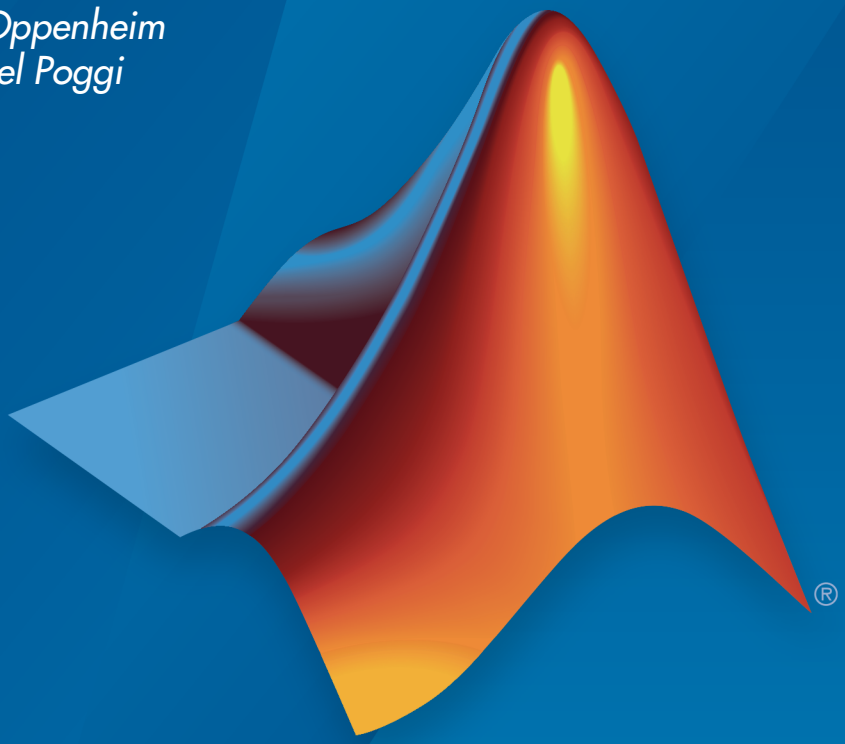

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Wavelet Toolbox™ Reference

Revision History

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addlift

Add lifting steps to lifting scheme

Syntax

```
LSN = addlift(LS,ELS)
LSN = addlift(LS,ELS,'begin')
LSN = addlift(LS,ELS,'end')
addfilt(LS,ELS)
```

Description

`LSN = addlift(LS,ELS)` returns the new lifting scheme *LSN* obtained by appending the elementary lifting step *ELS* to the lifting scheme *LS*.

`LSN = addlift(LS,ELS,'begin')` prepends the specified elementary lifting step.

ELS is either a cell array (see `lsinfo`)

```
{TYPEVAL, COEFS, MAX_DEG}
```

or a structure (see `liftfilt`)

```
struct('type',TYPEVAL,'value',LPVAL)
```

with

```
LPVAL = laurpoly(COEFS, MAX_DEG)
```

`LSN = addlift(LS,ELS,'end')` is equivalent to `addfilt(LS,ELS)`.

If *ELS* is a sequence of elementary lifting steps, stored in a cell array or an array of structures, then each of the elementary lifting steps is added to *LS*.

For more information about lifting schemes, see `lsinfo`.

Add Primal Lifting Step

This example shows how to start with the Haar lifting scheme and add a primal lifting step.

```
LSbegin = liftwave('haar');
```

Display the lifting scheme.

```
displs(LSbegin);
```

```
LSbegin = {...
'd'              [ -1.00000000]  [0]
'p'              [  0.50000000]  [0]
[  1.41421356]   [  0.70710678]  []
};
```

Create a primal lifting step.

```
pstep = { 'p', [-1 2 -1]/4 , 1 };
```

Add the primal lifting step.

```
LSend = addlift(LSbegin,pstep);
```

Display the final lifting scheme.

```
displs(LSend);
```

```
LSend = {...
'd'              [ -1.00000000]                            [0]
'p'              [  0.50000000]                            [0]
'p'              [ -0.25000000  0.50000000 -0.25000000]    [1]
[  1.41421356]   [  0.70710678]                            []
};
```

See Also

`liftfilt`

Introduced before R2006a

allnodes

Tree nodes

Syntax

```
N = allnodes(T)
N = allnodes(T,'deppos')
```

Description

`allnodes` is a tree management utility that returns one of two node descriptions: either indices, or depths and positions.

The nodes are numbered from left to right and from top to bottom. The root index is 0.

`N = allnodes(T)` returns the indices of all the nodes of the tree *T* in column vector *N*.

`N = allnodes(T,'deppos')` returns the depths and positions of all the nodes in matrix *N*.

`N(i,1)` is the depth and `N(i,2)` the position of the node `i`.

Return Nodes of Wavelet Packet Tree

This example shows how to obtain the depth-position and linear indices of a wavelet packet tree.

Load the noisy Doppler signal and obtain the wavelet packet decomposition down to the level 4 using the `'db2'` wavelet.

```
load noisdopp;
T = wpdec(noisdopp,4,'db2');
```

Obtain the depth-position indices.

```
DepthPosition = allnodes(T,'deppos');
```

Obtain the corresponding linear indices.

```
LinearIndices = allnodes(T);
```

Display the correspondance in a table.

```
table(DepthPosition,LinearIndices)
```

```
ans =

    DepthPosition    LinearIndices
    _____________    _____________

    0      0          0
    1      0          1
    1      1          2
    2      0          3
    2      1          4
    2      2          5
    2      3          6
    3      0          7
    3      1          8
    3      2          9
    3      3         10
    3      4         11
    3      5         12
    3      6         13
    3      7         14
    4      0         15
    4      1         16
    4      2         17
    4      3         18
    4      4         19
    4      5         20
    4      6         21
    4      7         22
    4      8         23
    4      9         24
    4     10         25
    4     11         26
    4     12         27
    4     13         28
    4     14         29
    4     15         30
```

Introduced before R2006a

appcoef

1-D approximation coefficients

Syntax

```
A = appcoef(C,L,'wname',N)
A = appcoef(C,L,'wname')
A = appcoef(C,L,Lo_R,Hi_R)
A = appcoef(C,L,Lo_R,Hi_R,N)
```

Description

`appcoef` is a one-dimensional wavelet analysis function.

`appcoef` computes the approximation coefficients of a one-dimensional signal.

`A = appcoef(C,L,'wname',N)` computes the approximation coefficients at level *N* using the wavelet decomposition structure `[C,L]` (see `wavedec` for more information).

`'wname'` is a string containing the wavelet name. Level *N* must be an integer such that `0 ≤ N ≤ length(L)-2`.

`A = appcoef(C,L,'wname')` extracts the approximation coefficients at the last level: length(*L*)-2.

Instead of giving the wavelet name, you can give the filters.

For `A = appcoef(C,L,Lo_R,Hi_R)` or `A = appcoef(C,L,Lo_R,Hi_R,N)`, *Lo_R* is the reconstruction low-pass filter and *Hi_R* is the reconstruction high-pass filter (see `wfilters` for more information).

Level 3 Approximation Coefficients

This example shows how to extract the level 3 approximation coefficients.

Load the signal consisting of electricity usage data.

```
load leleccum;
```

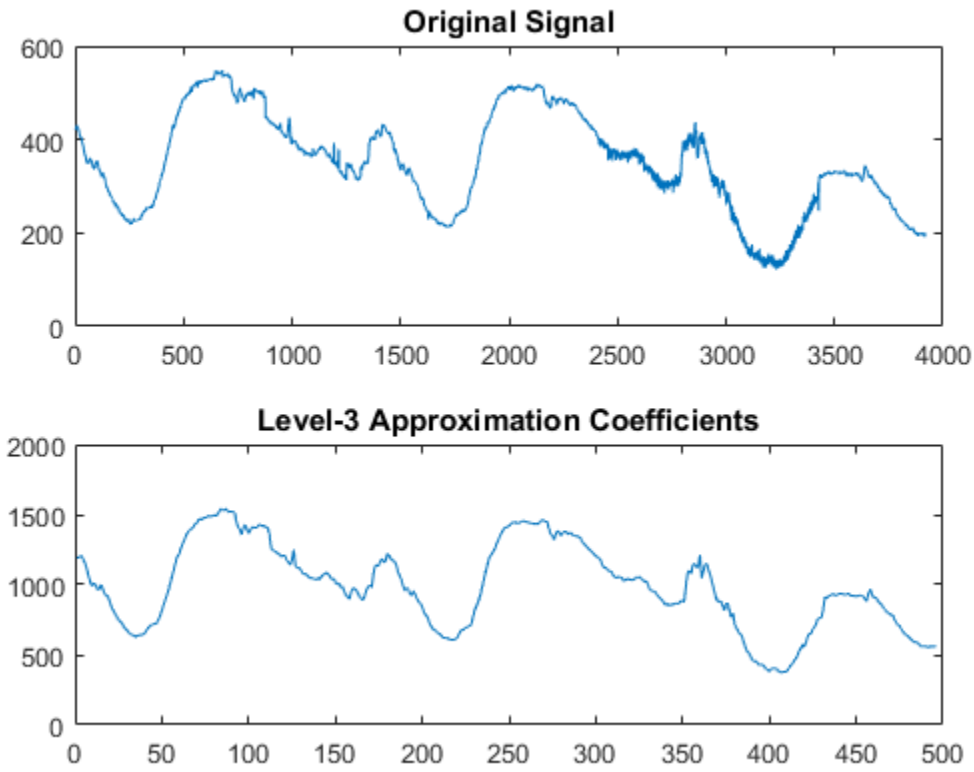
```
sig = leleccum(1:3920);
```

Obtain the DWT down to level 5 with the `'sym4'` wavelet.

```
[C,L] = wavedec(sig,5,'sym4');
```

Extract the level-3 approximation coefficients. Plot the original signal and the approximation coefficients.

```
Lev = 3;
a3 = appcoef(C,L,'sym4',Lev);
subplot(2,1,1)
plot(sig); title('Original Signal');
subplot(2,1,2)
plot(a3); title('Level-3 Approximation Coefficients');
```



You can substitute any value from 1 to 5 for `Lev` to obtain the approximation coefficients for the corresponding level.

More About

Algorithms

The input vectors *C* and *L* contain all the information about the signal decomposition.

Let `NMAX = length(L)-2`; then `C = [A(NMAX) D(NMAX) ... D(1)]` where *A* and the *D* are vectors.

If `N = NMAX`, then a simple extraction is done; otherwise, `appcoef` computes iteratively the approximation coefficients using the inverse wavelet transform.

See Also

`detcoef` | `wavedec`

Introduced before R2006a

appcoef2

2-D approximation coefficients

Syntax

```
A = appcoef2(C,S,'wname',N)
A = appcoef2(C,S,'wname')
A = appcoef2(C,S,Lo_R,Hi_R)
A = appcoef2(C,S,Lo_R,Hi_R,N)
```

Description

`appcoef2` is a two-dimensional wavelet analysis function. It computes the approximation coefficients of a two-dimensional signal. The syntaxes allow you to give the wavelet name or the filters as inputs.

`A = appcoef2(C,S,'wname',N)` computes the approximation coefficients at level *N* using the wavelet decomposition structure [*C*,*S*] (see `wavedec2` for more information).

`'wname'` is a string containing the wavelet name. Level *N* must be an integer such that `0 ≤ N ≤ size(S,1)-2`.

`A = appcoef2(C,S,'wname')` extracts the approximation coefficients at the last level: `size(S,1)-2`.

`A = appcoef2(C,S,Lo_R,Hi_R)` or `A = appcoef2(C,S,Lo_R,Hi_R,N)`, *Lo_R* is the reconstruction low-pass filter and *Hi_R* is the reconstruction high-pass filter (see `wfilters` for more information).

Examples

```
% The current extension mode is zero-padding (see dwtmode).

% Load original image.
load woman;
```

```
% X contains the loaded image.

% Perform decomposition at level 2
% of X using db1.
[c,s] = wavedec2(X,2,'db1');
sizex = size(X)
sizex =
    256 256

sizec = size(c)

sizec =
     1     65536
    val_s = s

val_s =
    64     64
    64     64
   128    128
   256    256

% Extract approximation coefficients
% at level 2.
ca2 = appcoef2(c,s,'db1',2);
sizeca2 = size(ca2)

sizeca2 =
    64     64

% Compute approximation coefficients
% at level 1.
ca1 = appcoef2(c,s,'db1',1);
sizeca1 = size(ca1)

sizeca1 =
   128    128
```

More About

Tips

If C and S are obtained from an indexed image analysis or a truecolor image analysis, A is an m-by-n matrix or an m-by-n-by-3 array, respectively.

For more information on image formats, see the `image` and `imfinfo` reference pages.

Algorithms

The algorithm is built on the same principle as `appcoef`.

See Also

`detcoef2` | `wavedec2`

Introduced before R2006a

bestlevt

Best level tree wavelet packet analysis

Syntax

```
T = bestlevt(T)
[T,E] = bestlevt(T)
```

Description

`bestlevt` is a one- or two-dimensional wavelet packet analysis function.

`bestlevt` computes the optimal complete subtree of an initial tree with respect to an entropy type criterion. The resulting complete tree may be of smaller depth than the initial one.

`T = bestlevt(T)` computes the modified wavelet packet tree *T* corresponding to the best level tree decomposition.

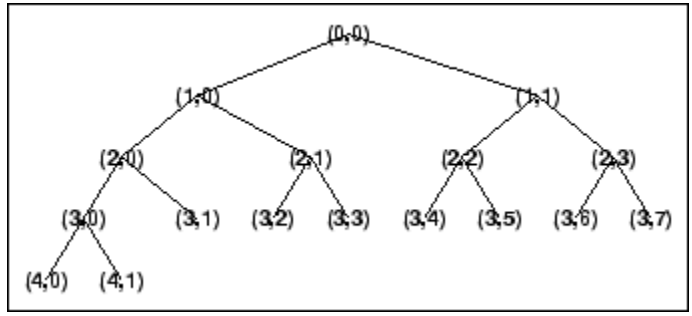
`[T,E] = bestlevt(T)` computes the best level tree *T*, and in addition, the best entropy value *E*.

The optimal entropy of the node, whose index is `j-1`, is `E(j)`.

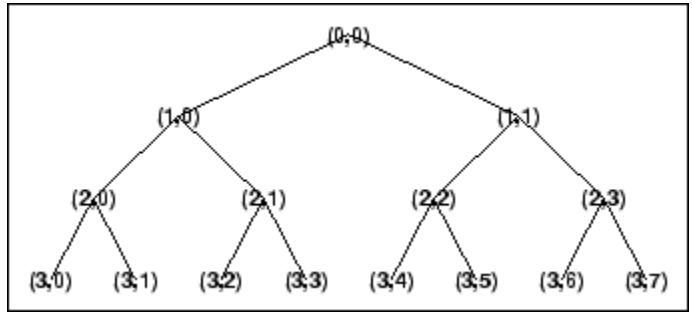
Examples

```
% The current extension mode is zero-padding (see dwtmode).

% Load signal.
load noisdopp;
x = noisdopp;

% Decompose x at depth 3 with db1 wavelet, using default
% entropy (shannon).
wpt = wpdec(x,3,'db1');
```

```
% Decompose the packet [3 0].
wpt = wpsplt(wpt,[3 0]);

% Plot wavelet packet tree wpt.
plot(wpt)
```




```
% Compute best level tree.
blt = bestlevt(wpt);

% Plot best level tree blt.
plot(blt)
```



More About

Algorithms

See `besttree` algorithm section. The only difference is that the optimal tree is searched among the complete subtrees of the initial tree, instead of among all the binary subtrees.

See Also

`besttree` | `wpdec` | `wenergy` | `wpdec2`

Introduced before R2006a

besttree

Best tree wavelet packet analysis

Syntax

```
T = besttree(T)
[T,E] = besttree(T)
[T,E,N] = besttree(T)
```

Description

`besttree` is a one- or two-dimensional wavelet packet analysis function that computes the optimal subtree of an initial tree with respect to an entropy type criterion. The resulting tree may be much smaller than the initial one.

Following the organization of the wavelet packets library, it is natural to count the decompositions issued from a given orthogonal wavelet.

A signal of length $N = 2^L$ can be expanded in α different ways, where α is the number of binary subtrees of a complete binary tree of depth L.

As a result, we can conclude that $\alpha \geq 2^{N/2}$ (for more information, see the Mallat's book given in References at page 323).

This number may be very large, and since explicit enumeration is generally intractable, it is interesting to find an optimal decomposition with respect to a convenient criterion, computable by an efficient algorithm. We are looking for a minimum of the criterion.

`T = besttree(T)` computes the best tree *T* corresponding to the best entropy value.

`[T,E] = besttree(T)` computes the best tree *T* and, in addition, the best entropy value *E*.

The optimal entropy of the node, whose index is `j-1`, is `E(j)`.

`[T,E,N] = besttree(T)` computes the best tree *T*, the best entropy value *E* and, in addition, the vector *N* containing the indices of the merged nodes.

Best Wavelet Packet Tree

This example shows to obtain the optimal wavelet packet tree based on an entropy criterion.

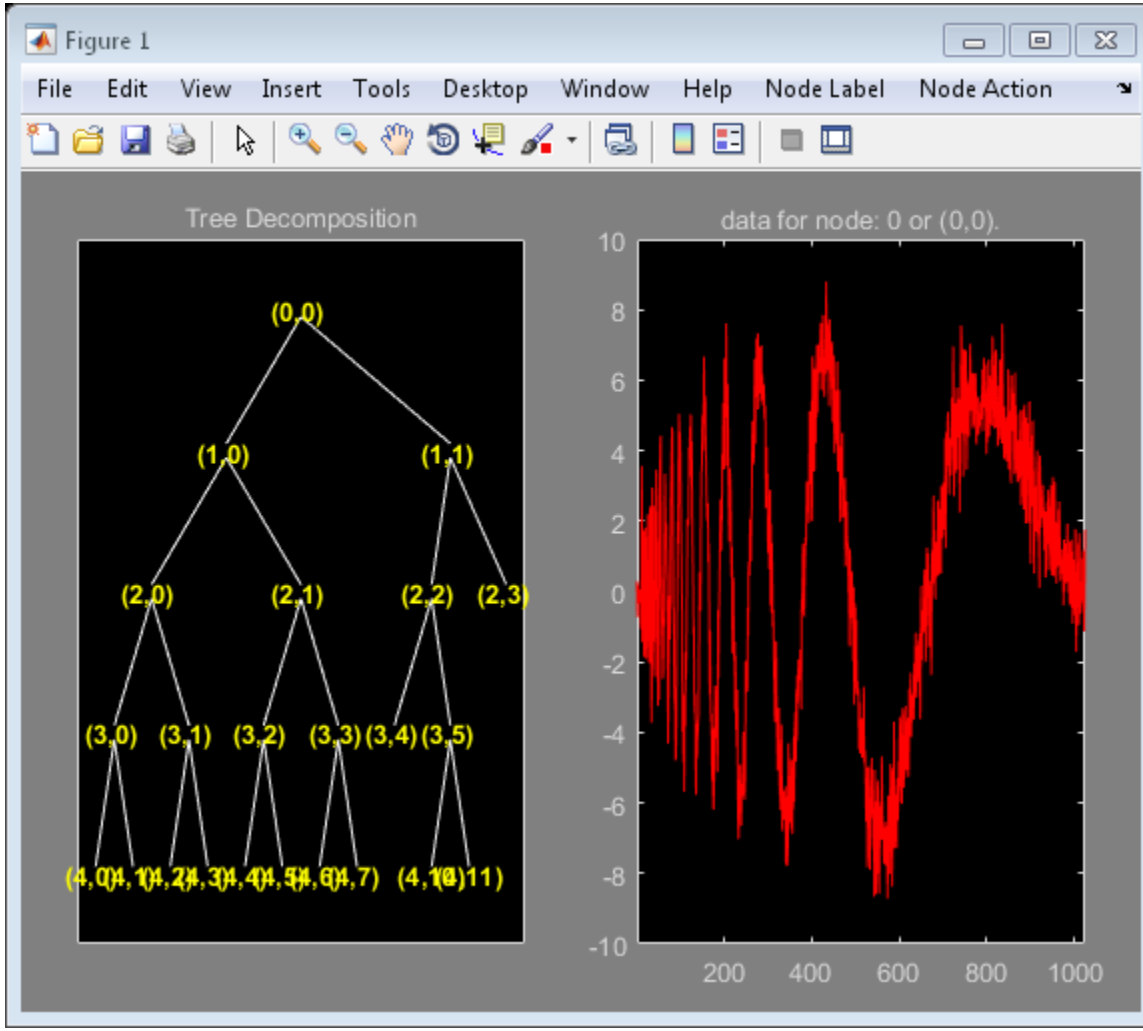
Load the noisy Doppler signal. Obtain the wavelet packet tree down to level 4 with the `'sym4'` wavelet. Use the periodic extension mode.

```
dwtmode('per');
load noisdopp;
T = wpdec(noisdopp,4,'sym4');
```

```
!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!
!  WARNING: Change DWT Extension Mode  !
!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!

**************************************
**  DWT Extension Mode: Periodization  **
**************************************
```

Obtain the best wavelet packet tree and plot the result.

```
BstTree = besttree(T);
plot(BstTree)
```



Return the DWT extension mode to the default value.

```
dwtmode('sym');
```

```
!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!
!  WARNING: Change DWT Extension Mode  !
!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!
```

```
*****************************************************
**  DWT Extension Mode: Symmetrization (half-point)  **
*****************************************************
```

More About

Algorithms

Consider the one-dimensional case. Starting with the root node, the best tree is calculated using the following scheme. A node N is split into two nodes N1 and N2 if and only if the sum of the entropy of N1 and N2 is lower than the entropy of N. This is a local criterion based only on the information available at the node N.

Several entropy type criteria can be used (see `wenergy` for more information). If the entropy function is an additive function along the wavelet packet coefficients, this algorithm leads to the best tree.

Starting from an initial tree T and using the merging side of this algorithm, we obtain the best tree among all the binary subtrees of T.

- "Reconstructing a Signal Approximation from a Node"

References

Coifman, R.R.; M.V. Wickerhauser (1992), "Entropy-based algorithms for best basis selection," *IEEE Trans. on Inf. Theory*, vol. 38, 2, pp. 713–718.

Mallat, S. (1998), *A wavelet tour of signal processing*, Academic Press.

See Also

`bestlevt` | `wenergy` | `wpcoef` | `wpdec` | `wpdec2` | `wprcoef`

Introduced before R2006a

biorfilt

Biorthogonal wavelet filter set

Syntax

```
[Lo_D,Hi_D,Lo_R,Hi_R] = biorfilt(DF,RF)
[Lo_D1,Hi_D1,Lo_R1,Hi_R1,Lo_D2,Hi_D2,Lo_R2,Hi_R2] =
biorfilt(DF,RF,'8')
```

Description

The `biorfilt` command returns either four or eight filters associated with biorthogonal wavelets.

`[Lo_D,Hi_D,Lo_R,Hi_R] = biorfilt(DF,RF)` computes four filters associated with the biorthogonal wavelet specified by decomposition filter *DF* and reconstruction filter *RF*. These filters are

`Lo_D`	Decomposition low-pass filter
`Hi_D`	Decomposition high-pass filter
`Lo_R`	Reconstruction low-pass filter
`Hi_R`	Reconstruction high-pass filter

`[Lo_D1,Hi_D1,Lo_R1,Hi_R1,Lo_D2,Hi_D2,Lo_R2,Hi_R2] = biorfilt(DF,RF,'8')` returns eight filters, the first four associated with the decomposition wavelet, and the last four associated with the reconstruction wavelet.

It is well known in the subband filtering community that if the same FIR filters are used for reconstruction and decomposition, then symmetry and exact reconstruction are incompatible (except with the Haar wavelet). Therefore, with biorthogonal filters, two wavelets are introduced instead of just one:

One wavelet, $\tilde{\psi}$, is used in the analysis, and the coefficients of a signal s are

$$\tilde{c}_{j,k} = \int s(x)\tilde{\psi}_{j,k}(x)dx$$

The other wavelet, ψ, is used in the synthesis:

$$s = \sum_{j,k} \tilde{c}_{j,k}\psi_{j,k}$$

Furthermore, the two wavelets are related by duality in the following sense:
$\int \tilde{\psi}_{j,k}(x)\psi_{j',k'}(x)dx = 0$ as soon as $j \neq j'$ or $k \neq k'$ and
$\int \tilde{\phi}_{0,k}(x)\phi_{0,k'}(x)dx = 0$ as soon as $k \neq k'$.

It becomes apparent, as A. Cohen pointed out in his thesis (p. 110), that "the useful properties for analysis (e.g., oscillations, null moments) can be concentrated in the $\tilde{\psi}$ function; whereas, the interesting properties for synthesis (regularity) are assigned to the ψ function. The separation of these two tasks proves very useful."

$\tilde{\psi}$ and ψ can have very different regularity properties, ψ being more regular than $\tilde{\psi}$.

The $\tilde{\psi}$, ψ, $\tilde{\phi}$ and ϕ functions are zero outside a segment.

Biorthogonal Filters and Transfer Functions

This example shows how to obtain the decompositition (analysis) and reconstruction (synthesis) filters for the `'bior3.5'` wavelet.

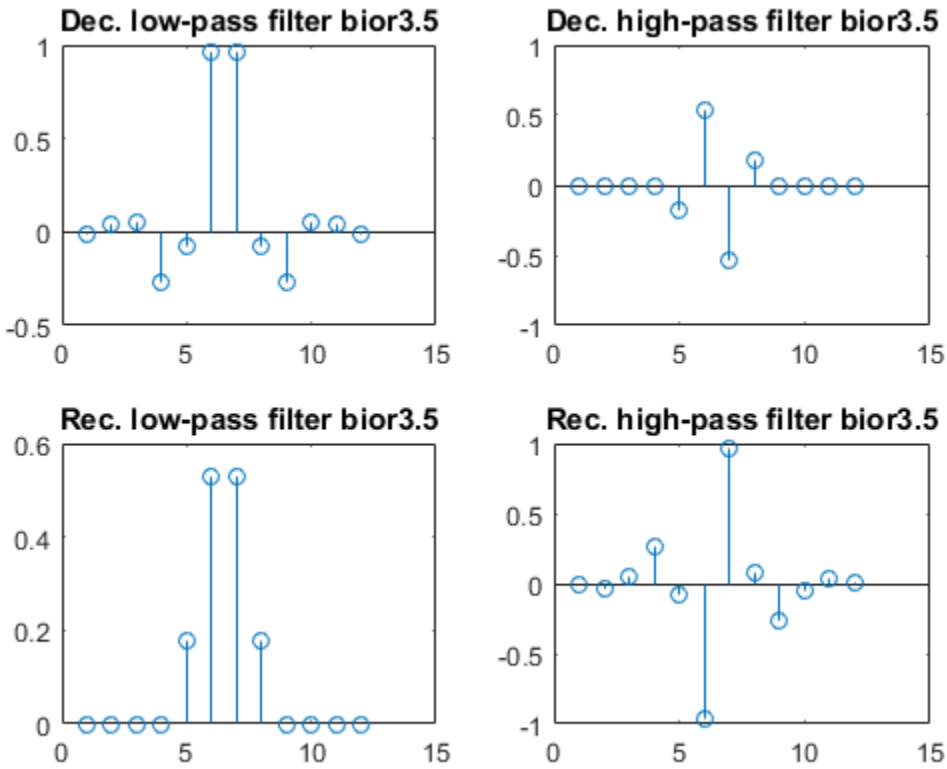
Determine the two scaling and wavelet filters associated with the `'bior3.5'` wavelet.

```
[Rf,Df] = biorwavf('bior3.5');
[LoD,HiD,LoR,HiR] = biorfilt(Df,Rf);
```

Plot the filter impulse responses.

```
subplot(221); stem(LoD);
```

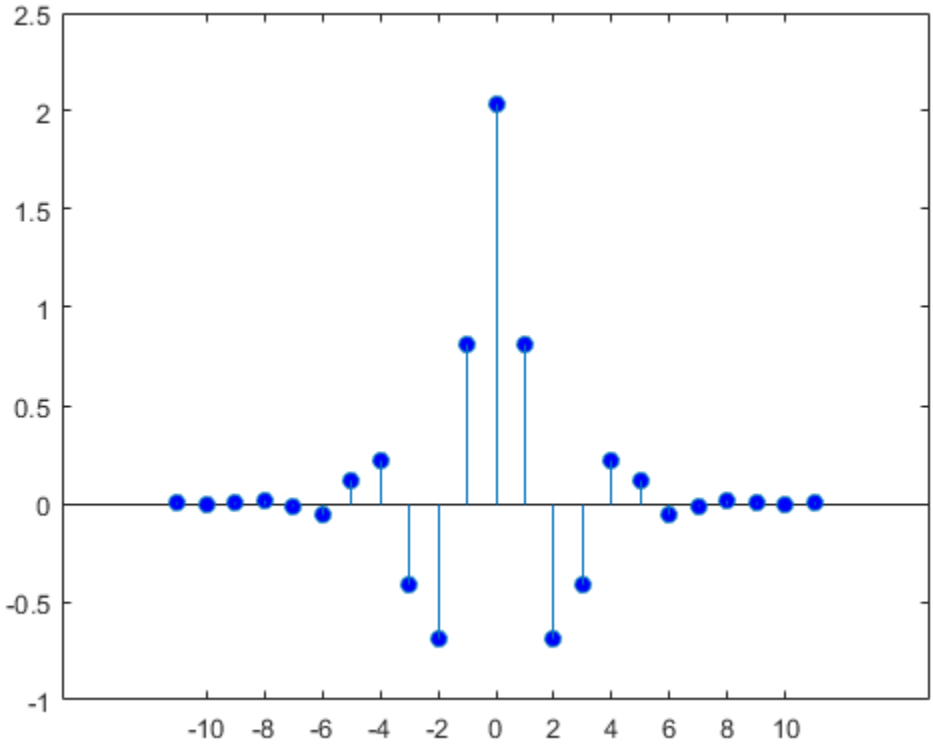
```
title('Dec. low-pass filter bior3.5');
subplot(222); stem(HiD);
title('Dec. high-pass filter bior3.5');
subplot(223); stem(LoR);
title('Rec. low-pass filter bior3.5');
subplot(224); stem(HiR);
title('Rec. high-pass filter bior3.5');
```



Demonstrate that autocorrelations at even lags are only zero for dual pairs of filters. Examine the autocorrelation sequence for the lowpass decomposition filter.

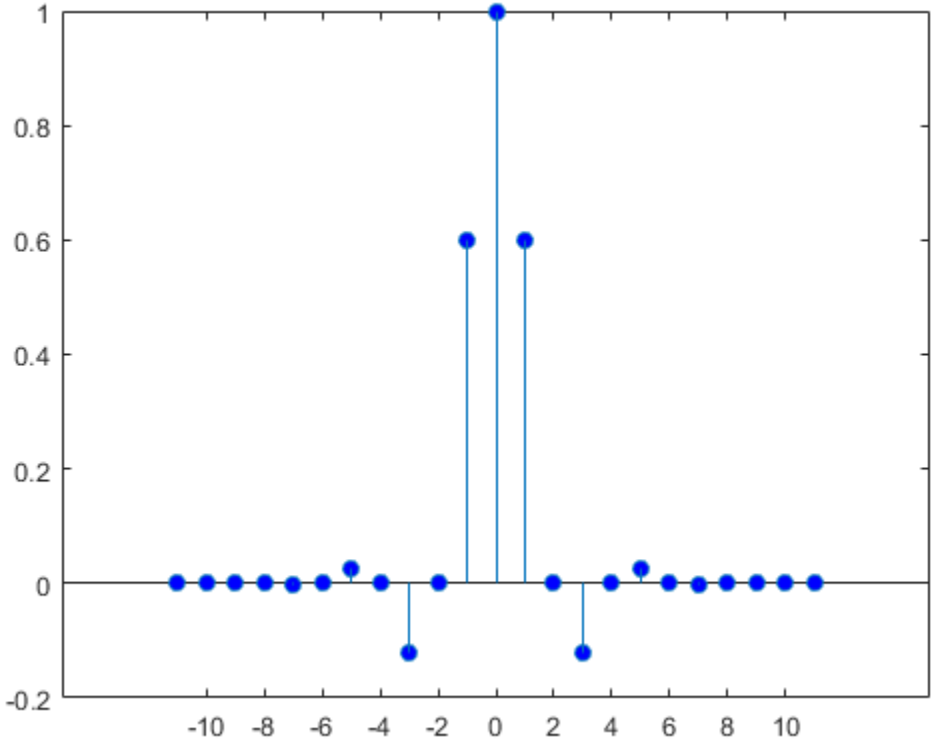
```
npad = 2*length(LoD)-1;
LoDxcr = fftshift(ifft(abs(fft(LoD,npad)).^2));
```

```
lags = -floor(npad/2):floor(npad/2);
figure;
stem(lags,LoDxcr,'markerfacecolor',[0 0 1])
set(gca,'xtick',-10:2:10)
```



Examine the cross correlation sequence for the lowpass decomposition and synthesis filters. Compare the result with the preceding figure.

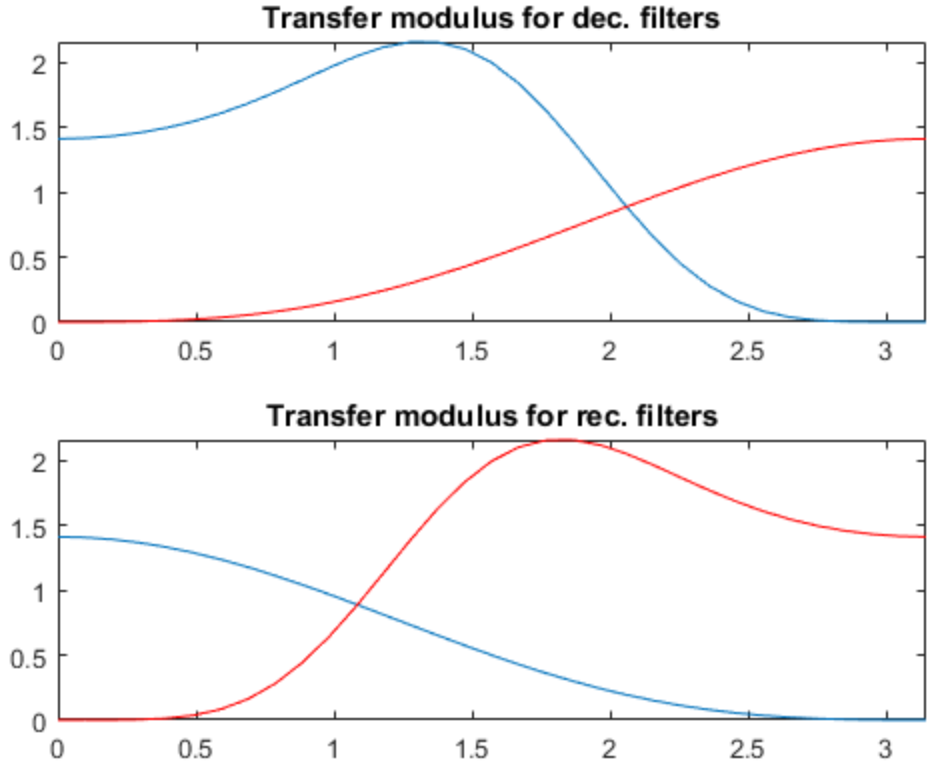
```
npad = 2*length(LoD)-1;
xcr = fftshift(ifft(fft(LoD,npad).*conj(fft(LoR,npad))));
lags = -floor(npad/2):floor(npad/2);
stem(lags,xcr,'markerfacecolor',[0 0 1])
set(gca,'xtick',-10:2:10)
```



Compare the transfer functions of the analysis and synthesis scaling and wavelet filters

```
dftLoD = fft(LoD,64);
dftLoD = dftLoD(1:length(dftLoD)/2+1);
dftHiD= fft(HiD,64);
dftHiD = dftHiD(1:length(dftHiD)/2+1);
dftLoR = fft(LoR,64);
dftLoR = dftLoR(1:length(dftLoR)/2+1);
dftHiR = fft(HiR,64);
dftHiR = dftHiR(1:length(dftHiR)/2+1);
df = (2*pi)/64;
freqvec = 0:df:pi;

subplot(211); plot(freqvec,abs(dftLoD),freqvec,abs(dftHiD),'r');
```

```
axis tight;
title('Transfer modulus for dec. filters')
subplot(212); plot(freqvec,abs(dftLoR),freqvec,abs(dftHiR),'r');
axis tight;
title('Transfer modulus for rec. filters')
```



References

Cohen, A. (1992), “Ondelettes, analyses multirésolution et traitement numérique du signal,” *Ph. D. Thesis*, University of Paris IX, DAUPHINE.

Daubechies, I. (1992), *Ten lectures on wavelets*, CBMS-NSF conference series in applied mathematics. SIAM Ed.

See Also

`biorwavf` | `orthfilt`

Introduced before R2006a

biorwavf

Biorthogonal spline wavelet filter

Syntax

```
[RF,DF] = biorwavf(W)
```

Description

`[RF,DF] = biorwavf(W)` returns the reconstruction (synthesis) and decomposition (analysis) filters associated with the biorthogonal wavelet specified by the string `W`.

`W = 'biorNr.Nd'` where possible values for *Nr* and *Nd* are

Nr = 1	Nd = 1 , 3 or 5
Nr = 2	Nd = 2 , 4 , 6 or 8
Nr = 3	Nd = 1 , 3 , 5 , 7 or 9
Nr = 4	Nd = 4
Nr = 5	Nd = 5
Nr = 6	Nd = 8

The output arguments are filters.

- `RF` is the reconstruction filter.
- `DF` is the decomposition filter.

Examples

Return the biorthogonal spline wavelet scaling filters with 2 vanishing moments.

```
wname = 'bior2.2';
[RF,DF] = biorwavf(wname);
```

See Also

`biorfilt` | `waveinfo`

Introduced before R2006a

bswfun

Biorthogonal scaling and wavelet functions

Syntax

```
[PHIS,PSIS,PHIA,PSIA,XVAL] = bswfun(LoD,HiD,LoR,HiR)
bswfun(LoD,HiD,LoR,HiR,ITER)
bswfun(LoD,HiD,LoR,HiR,'plot')
bswfun(LoD,HiD,LoR,HiR,ITER,'plot')
bswfun(LoD,HiD,LoR,HiR,'plot',ITER)
```

Description

`[PHIS,PSIS,PHIA,PSIA,XVAL] = bswfun(LoD,HiD,LoR,HiR)` returns approximations on the grid `XVAL` of the two pairs of biorthogonal scaling and wavelet functions. `PHIS` and `PSIS` are the scaling and wavelet functions constructed from the decomposition filters, `LoD` and `HiD`. `PHIA` and `PSIA` are the scaling and wavelet functions constructed from the reconstruction filters, `LoR` and `HiR`.

`bswfun(LoD,HiD,LoR,HiR,ITER)` computes the two pairs of scaling and wavelet functions using ITER iterations.

`bswfun(LoD,HiD,LoR,HiR,'plot')` or `bswfun(LoD,HiD,LoR,HiR,ITER,'plot')` or `bswfun(LoD,HiD,LoR,HiR,'plot',ITER)` computes and plots the functions.

Examples

Biorthogonal Scaling and Wavelet from Lifting Scheme

This example shows how to obtain the biorthogonal scaling and wavelet functions corresponding to a lifting scheme. Obtain the lifting scheme for the CDF 3/1 wavelet.

```
lscdf = liftwave('cdf3.1');
```

Display the lifting scheme, which consists of two primal and one dual step.

```
Sc = displs(lscdf);
Sc
```

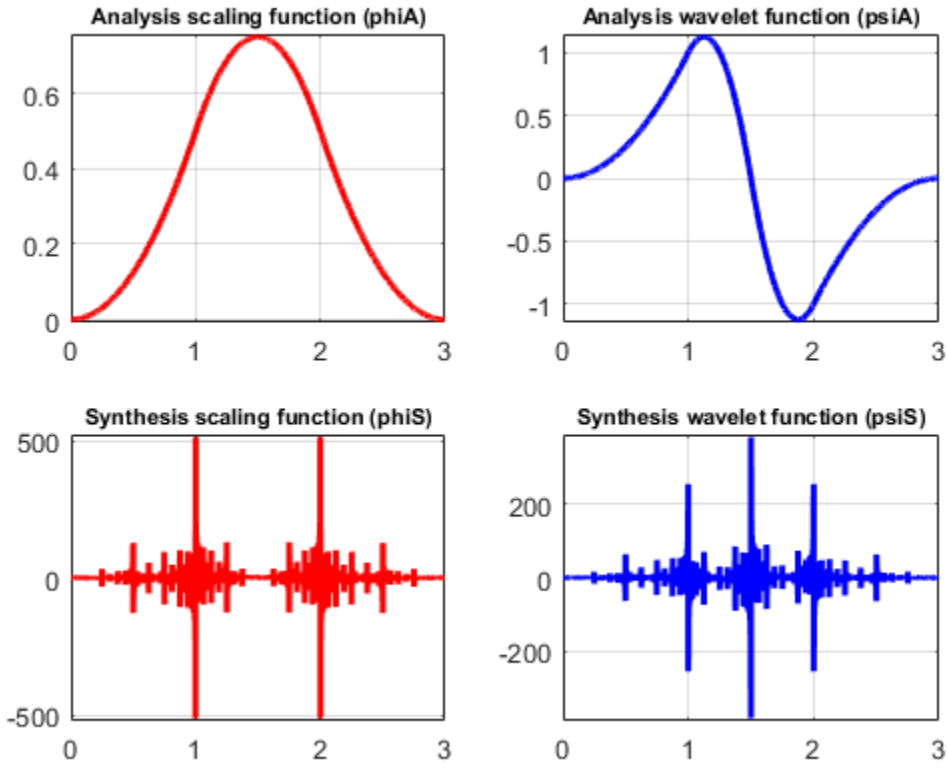
```
Sc =

lscdf = {...
'p'              [ -0.33333333]              [-1]
'd'              [ -0.37500000 -1.12500000]  [1]
'p'              [  0.44444444]              [0]
[  2.12132034]   [  0.47140452]              []
};
```

Obtain the decomposition and reconstruction filters from the lifting scheme.

```
[LoD,HiD,LoR,HiR] = ls2filt(lscdf);
```

Visualize the scaling and wavelet function and their duals.

```
bswfun(LoD,HiD,LoR,HiR,'plot');
```



More About

Algorithms

This function uses the cascade algorithm.

See Also

`wavefun`

Introduced before R2006a

centfrq

Wavelet center frequency

Syntax

```
FREQ = centfrq('wname')
FREQ = centfrq('wname',ITER)
[FREQ,XVAL,RECFREQ] = centfrq('wname',ITER,'plot')
```

Description

`FREQ = centfrq('wname')` returns the center frequency in herz of the wavelet function, `'wname'` (see `wavefun` for more information).

For `FREQ = centfrq('wname',ITER)`, `ITER` is the number of iterations performed by the function `wavefun`, which is used to compute the wavelet.

`[FREQ,XVAL,RECFREQ] = centfrq('wname',ITER,'plot')` returns, in addition, the associated center frequency based approximation `RECFREQ` on the 2^{ITER} points grid `XVAL` and plots the wavelet function and `RECFREQ`.

Examples

Determine Center Frequency

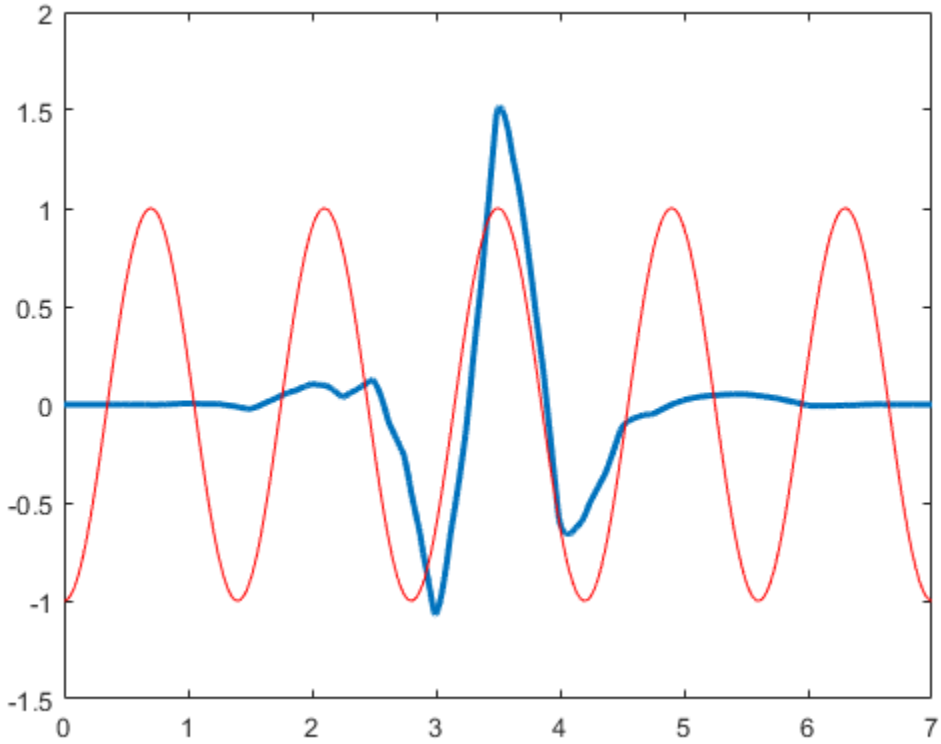
This example shows how to determine the center frequency in hertz for Daubechies' least-asymmetric wavelet with 4 vanishing moments.

```
cfreq = centfrq('sym4');
```

Obtain the wavelet and create a sine wave with a frequency equal to the center frequency, `cfreq`, of the wavelet. Use a starting phase of $-\pi$ for the sine wave to visualize how the oscillation in the sine wave matches the oscillation in the wavelet.

```
[~,psi,xval] = wavefun('sym4');
y = cos(2*pi*cfreq*xval-pi);
```

```
plot(xval,psi,'linewidth',2);
hold on;
plot(xval,y,'r');
```



Convert Scales to Frequencies

This example shows to convert scales to frequencies for the Morlet wavelet. There is an approximate inverse relationship between scale and frequency. Specifically, scale is inversely proportional to frequency with the constant of proportionality being the center frequency of the wavelet.

Construct a vector of scales with 32 voices per octave over 5 octaves for data sampled at 1 kHz.

```
Fs = 1000;
numvoices = 32;
a0 = 2^(1/numvoices);
numoctaves = 5;
scales = a0.^(numvoices:1/numvoices:numvoices*numoctaves).*1/Fs;
```

Convert the scales to approximate frequencies in hertz for the Morlet wavelet.

```
Frq = centfrq('morl')./scales;
```

You can also use `scal2frq` to convert scales to approximate frequencies in hertz.

See Also

`scal2frq` | `wavefun`

Introduced before R2006a

cfs2wpt

Wavelet packet tree construction from coefficients

Syntax

Description

`CFS2WPT` builds a wavelet packet tree (T) and the related analyzed signal or image (X) using the following input information:

`WNAME`: name of the wavelet used for the analysis

`SIZE_OF_DATA`: size of the analyzed signal or image

`TN_OF_TREE`: vector containing the terminal node indices of the tree

`ORDER`: 2 for a signal or 4 for an image

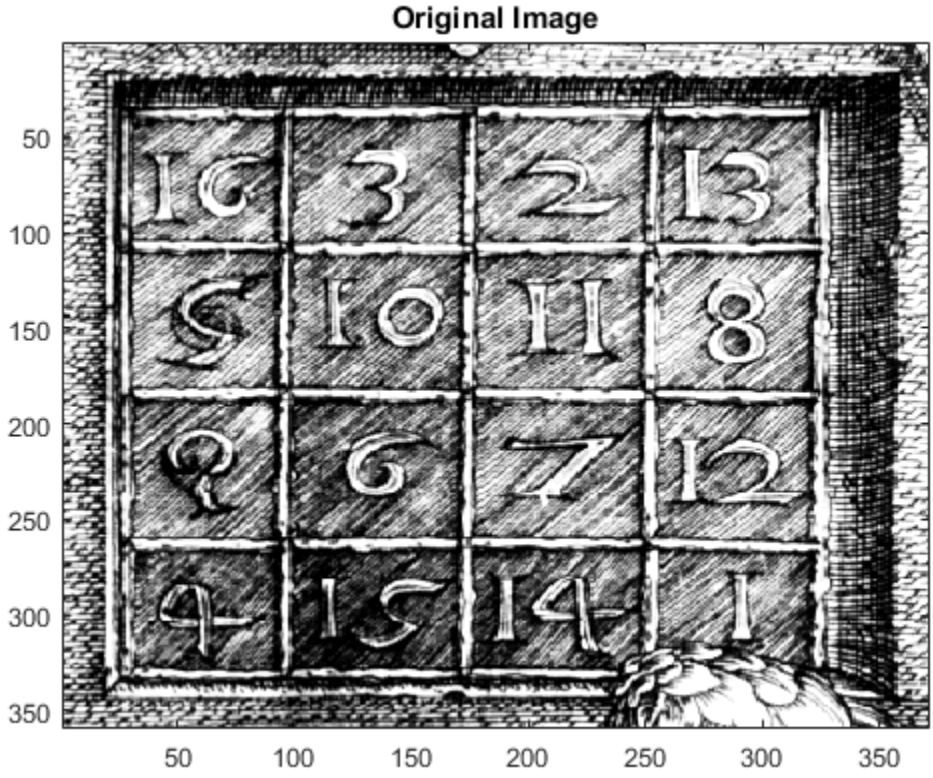
`CFS`: coefficients used to reconstruct the original signal or image. *`CFS`* is optional. When `CFS2WPT` is used without the *`CFS`* input parameter, the wavelet packet tree structure (T) is generated, but all the tree coefficients are null (including X).

Build Wavelet Packet Tree

This example shows how to build a wavelet packet tree in two ways: 1.) By filling the wavelet packet tree with coefficients, and 2.) By creating the wavelet packet tree and using `write`

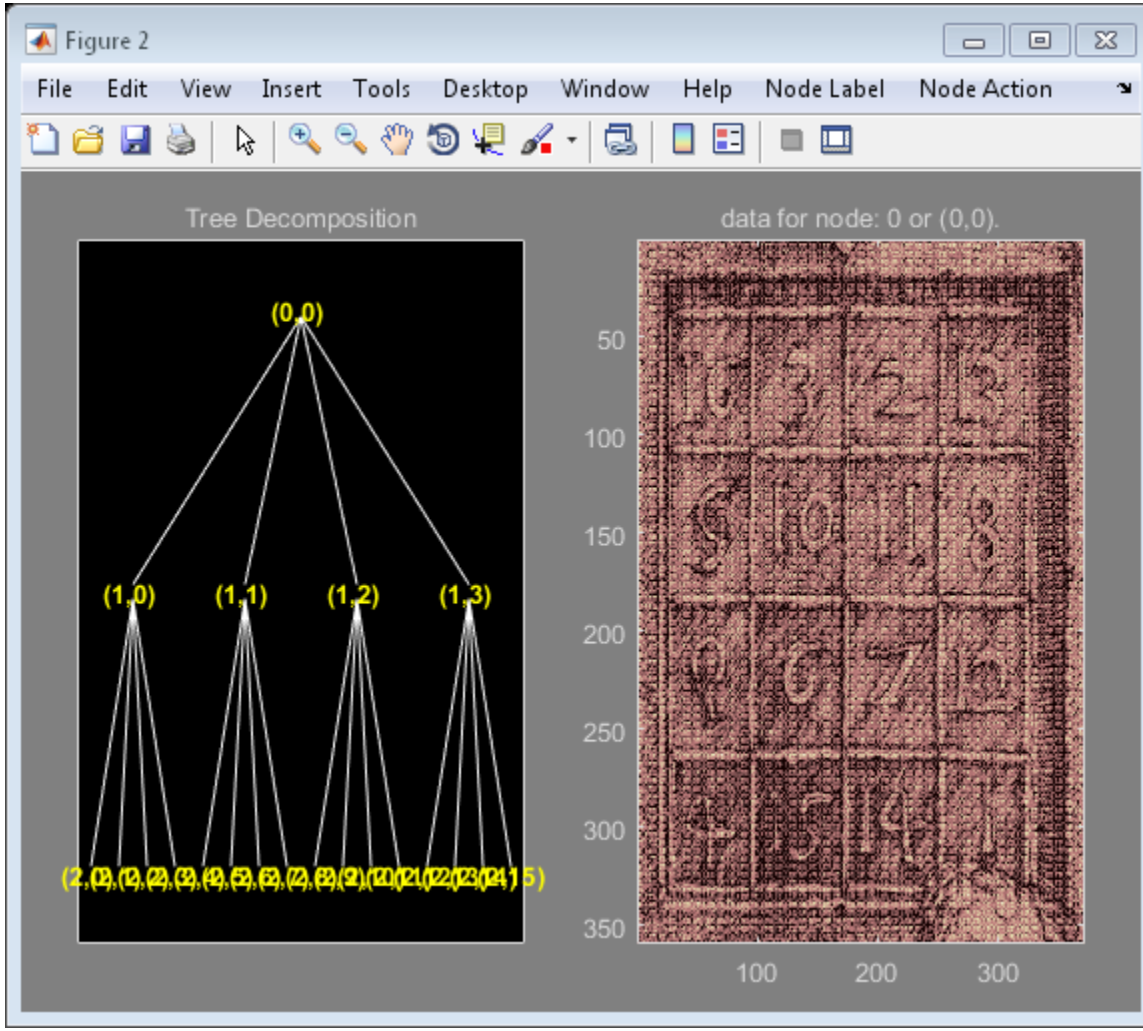
Load an image and obtain the wavelet packet decomposition down to level 2 with the `'sym4'` wavelet.

```
load detail;
imagesc(X); colormap gray; title('Original Image');
Tr = wpdec2(X,2,'sym4');
```



Read the coefficients from the wavelet packet tree. Add $N(0, 40^2)$ noise to the coefficients and plot the new wavelet packet tree.

```
cfs = read(Tr,'allcfs');
noisyCfs = cfs + 40*rand(size(cfs));
noisyT = cfs2wpt('sym4',size(X),tnodes(Tr),4,noisyCfs);
plot(noisyT)
```



To illustrate building a wavelet packet tree using `write`, construct an admissible binary wavelet packet tree with terminal nodes `[2 3 9 10]`. The analyzing wavelet is `'sym4'` and the signal length is 1024.

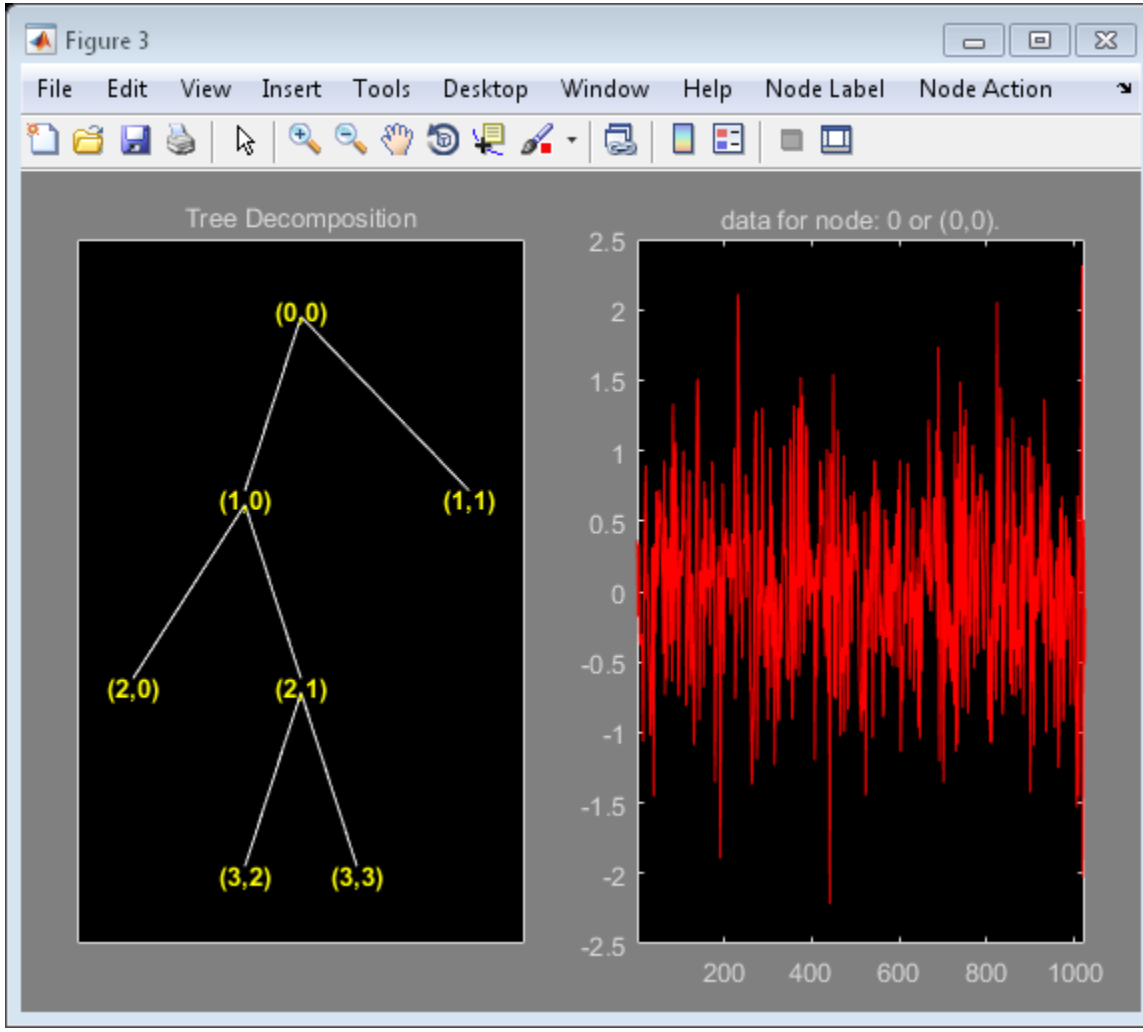
```
tr = cfs2wpt('sym4',[1 1024],[2 3 9 10]',2);
```

Fill terminal nodes `[3 9]` with $N(0,1)$ coefficients.

```
sN = read(tr,'sizes',[3,9]);
sN3 = sN(1,:); sN9 = sN(2,:);
cfsN3 = randn(sN3);
cfsN9 = randn(sN9);
tr = write(tr,'cfs',3,cfsN3,'cfs',9,cfsN9);
```

Plot the resulting wavelet packet tree and synthesized signal.

```
plot(tr)
```



Introduced before R2006a

cgauwavf

Complex Gaussian wavelet

Syntax

```
[PSI,X] = cgauwavf(LB,UB,N)
[PSI,X] = gauswavf(LB,UB,N,P)
[PSI,X] = gauswavf(LB,UB,N,WAVNAME)
```

Description

`[PSI,X] = cgauwavf(LB,UB,N)` returns the 1st order derivative of the complex-valued Gaussian wavelet, `PSI`, on an `N`-point regular grid, `X`, for the interval `[LB,UB]`. The effective support of the complex-valued Gaussian wavelets is `[-5 5]`.

`[PSI,X] = gauswavf(LB,UB,N,P)` returns the `P`th derivative. Valid values of `P` are integers from 1 to 8.

The complex Gaussian function is defined as $C_p e^{-ix} e^{-x^2}$. C_p is such that the 2-norm of the `P`th derivative of `PSI` is equal to 1.

`[PSI,X] = gauswavf(LB,UB,N,WAVNAME)` uses the valid wavelet family short name `WAVNAME` plus the order of the derivative in a string, such as `'cgau4'`. To see valid strings for complex-valued Gaussian wavelets, use `waveinfo('cgau')` or use `wavemngr('read',1)` and refer to the Complex Gaussian section.

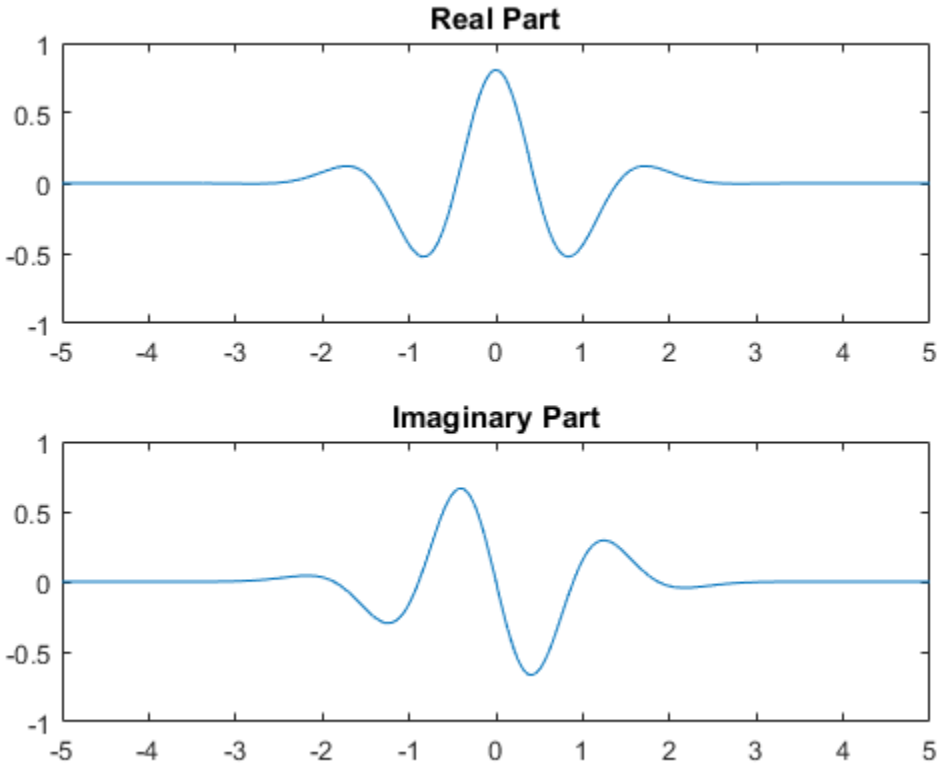
Examples

Create Complex Gaussian Wavelet

This example shows how to create a complex-valued Gaussian wavelet of order 4. The wavelet has an effective support of [-5,5] and is constructed using 1,000 samples.

```
lb = -5;
```

```
ub = 5;
n = 1000;
order = 4;
[psi,x] = cgauwavf(lb,ub,n,order);
subplot(211)
plot(x,real(psi))
title('Real Part');
subplot(212)
plot(x,imag(psi))
title('Imaginary Part');
```



See Also

waveinfo

Introduced before R2006a

chgwdeccfs

Change multisignal 1-D decomposition coefficients

Syntax

```
DEC = chgwdeccfs(DEC,'ca',COEFS)
DEC = chgwdeccfs(DEC,'cd',COEFS,LEV)
DEC = chgwdeccfs(DEC,'all',CA,CD)
DEC = chgwdeccfs(DEC,'all',V)
DEC = chgwdeccfs(...,IDXSIG)
```

Description

`DEC = chgwdeccfs(DEC,'ca',COEFS)` replaces the approximation coefficients at level `DEC.level` with those contained in the matrix `COEFS`. If `COEFS` is a single value *`V`*, all coefficients are replaced by *`V`*.

`DEC = chgwdeccfs(DEC,'cd',COEFS,LEV)` replaces the detail coefficients at level `LEV` with those contained in the matrix `COEFS`. If `COEFS` is a single value *`V`*, then `LEV` can be a vector of levels and all the coefficients that belong to these levels are replaced by *`V`*. `LEV` must be such that $1 \le$ `LEV` $\le$ `DEC.level`

`DEC = chgwdeccfs(DEC,'all',CA,CD)` replaces all the approximation and detail coefficients. *`CA`* must be a matrix and *`CD`* must be a cell array of length `DEC.level`.

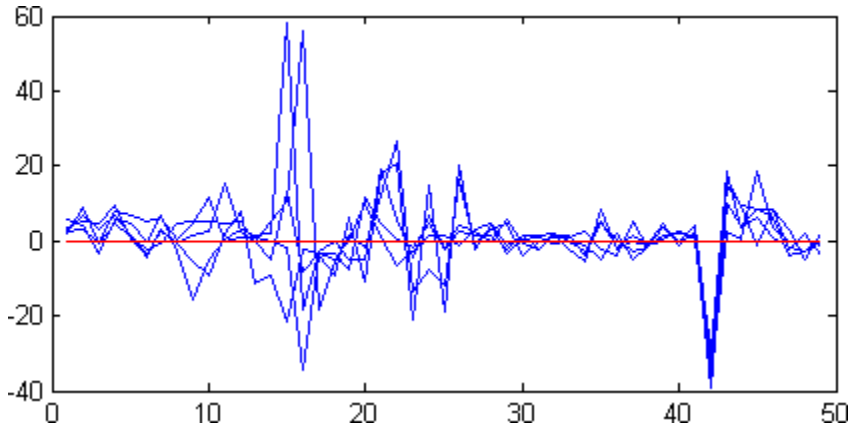
If `COEFS` (or *`CA`* or *`CD`*) is a single number, then it replaces all the related coefficients. Otherwise, `COEFS` (or *`CA`*, or *`CD`*) must be a matrix of appropriate size.

For a real value *`V`*, `DEC = chgwdeccfs(DEC,'all',V)` replaces all the coefficients by *`V`*.

`DEC = chgwdeccfs(...,IDXSIG)` replaces the coefficients for the signals whose indices are given by the vector `IDXSIG`. If the initial data are stored row-wise or column-wise in a matrix `X`, then `IDXSIG` contains the row or column indices, respectively, of the data.

Examples

```
% Load original 1D-multisignal
load thinker

% Perform a decomposition at level 2 using wavelet db2
dec = mdwtdec('r',X,2,'db2');

% Change the coefficients of details at level 1.
% Replace all values by 0.
decBIS = chgwdeccfs(dec,'cd',0,1);

% Change the coefficients of details at level 1 and
% level 2 for signals 31 to 35. Replace all values by 0.
decTER = chgwdeccfs(dec,'cd',0,1:2,31:35);

% Compare original and new coefficients for details
% at level 1 for signals 31 to 35.
plot(dec.cd{1}(31:35,:)','b'); hold on;
plot(decTER.cd{1}(31:35,:)','r')
```



See Also

mdwtdec | mdwtrec

Introduced in R2007a

cmddenoise

Interval-dependent denoising

Syntax

```
sigden = cmddenoise(sig,wname,level)
sigden = cmddenoise(sig,wname,level,sorh)
sigden = cmddenoise(sig,wname,level,sorh,nb_inter)
sigden = cmddenoise(sig,wname,level,sorh,nb_inter,thrParamsIn)

[sigden,coefs] = cmddenoise(___)
[sigden,coefs,thrParamsOut] = cmddenoise(___)

[sigden,coefs,thrParamsOut,int_DepThr_Cell] = cmddenoise(sig,wname,
level,sorh,nb_inter)
[sigden,coefs,thrParamsOut,int_DepThr_Cell,BestNbofInt] =
cmddenoise(sig,wname,level,sorh,nb_inter)
```

Description

`sigden = cmddenoise(sig,wname,level)` returns the denoised signal, `sigden`, obtained from an interval-dependent denoising of the signal, `sig`, using the orthogonal or biorthogonal wavelet and scaling filters, `wname`. `cmddenoise` thresholds the wavelet (detail) coefficients down to level, `level`, and reconstructs a signal approximation using the modified detail coefficients. `cmddenoise` partitions the signal into intervals based on variance change points in the first level detail coefficients and thresholds each interval separately. The location and number of variance change points are automatically selected using a penalized contrast function [2]. The minimum delay between change points is 10 samples. Thresholds are obtained using a minimax threshold rule and soft thresholding is used to modify the wavelet coefficients [1] .

`sigden = cmddenoise(sig,wname,level,sorh)` returns the denoised signal, `sigden`, using the thresholding method, `sorh`, to modify the wavelet coefficients. Valid choices for `sorh` are `'s'` for soft thresholding or `'h'` for hard thresholding.

`sigden = cmddenoise(sig,wname,level,sorh,nb_inter)` returns the denoised signal, `sigden`, with the number of denoising intervals as a positive integer between

1 and 6: 1≤ nb_inter ≤6. For `nb_inter` ≥ 2, `cmddenoise` estimates the location of the change points with a contrast function [2].

`sigden = cmddenoise(sig,wname,level,sorh,nb_inter,thrParamsIn)` returns the denoised signal, `sigden`, with the denoising intervals and corresponding thresholds specified as a cell array of matrices with length equal to `level`. Each element of the cell array contains the interval and threshold information for the corresponding level of the wavelet transform. The elements of `thrParamsIn` are N-by-3 matrices with N equal to the number of intervals. The 1st and 2nd columns contain the beginning and ending indices of the intervals and the 3rd column contains the corresponding threshold value. If you specify `thrParamsIn`, `cmddenoise` ignores the value of `nb_inter`.

`[sigden,coefs] = cmddenoise( ___ )` returns the approximation (scaling) and detail (wavelet) coefficients, `coefs`. The organization of `coefs` is identical to the structure returned by `wavedec`. This syntax can include any of the input arguments used in previous syntaxes.

`[sigden,coefs,thrParamsOut] = cmddenoise( ___ )` returns a cell array, `thrParamsOut`, with length equal to `level`. Each element of `thrParamsOut` is an N-by-3 matrix. The row dimension of the matrix elements is the number of intervals and is determined by the value of the input arguments. Each row of the matrix contains the beginning and end points (indices) of the thresholded interval and the corresponding threshold value.

`[sigden,coefs,thrParamsOut,int_DepThr_Cell] = cmddenoise(sig,wname, level,sorh,nb_inter)` returns a cell array, `int_DepThr_Cell`, with length equal to 6. `int_DepThr_Cell` contains interval and threshold information assuming the number of change points ranges from 0 to 5. The N-th element of `int_DepThr_Cell` is a N-by-3 matrix containing the interval information assuming N-1 change points. Each row of the matrix contains the beginning and end points (indices) of the thresholded interval and the corresponding threshold value. Attempting to output `int_DepThr_Cell` if you use the input argument, `thrParamsIn`, results in an error.

`[sigden,coefs,thrParamsOut,int_DepThr_Cell,BestNbofInt] = cmddenoise(sig,wname,level,sorh,nb_inter)` returns the optimal number of signal intervals based on the estimated variance change points in the level-1 detail coefficients. To estimate the number of change points, `cmddenoise` assumes the total number is less than or equal to 6 and uses a penalized contrast [2]. Attempting to output `BestNbofInt` if you use the input argument, `thrParamsIn`, results in an error.

Examples

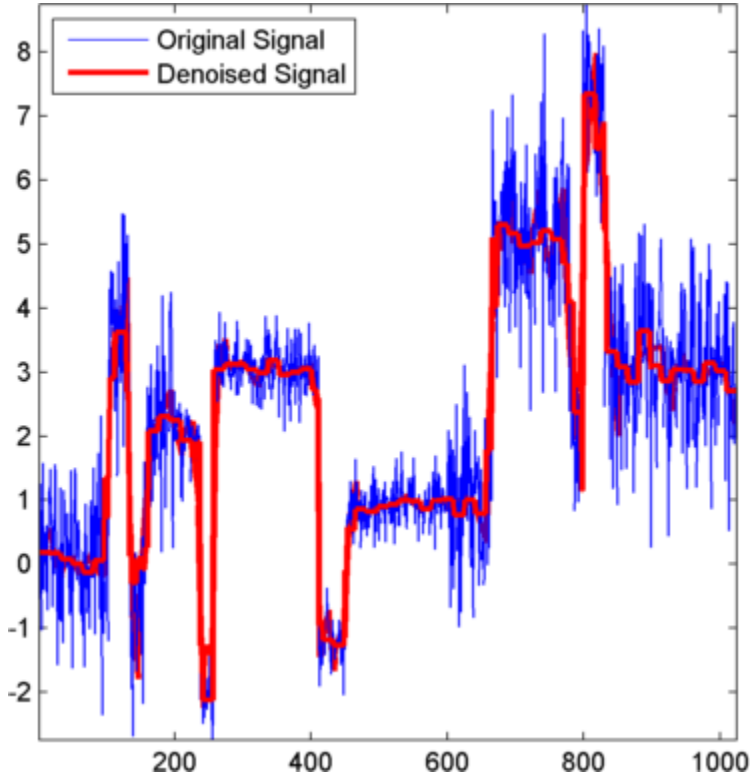
Denoising Blocks Signal with Haar Wavelet

Load the noisy blocks signal, `nblocr1.mat`. The signal consists of a piecewise constant signal in addtive white Gaussian noise. The variance of the additive noise differs in three disjoint intervals.

```
load nblocr1;
```

Apply interval-dependent denoising down to level 4 using the Haar wavelet. `cmddenoise` automatically determines the optimal number and locations of the variance change points. Plot the denoised and original signal for comparison.

```
sigden = cmddenoise(nblocr1,'db1',4);
plot(nblocr1);
hold on;
plot(sigden,'r','linewidth',2);
axis tight;
legend('Original Signal','Denoised Signal','Location','NorthWest');
```



Denoising Blocks Signal with Hard Thresholding

Load the noisy blocks signal, `nblocr1.mat`. The signal consists of a piecewise constant signal in additive white Gaussian noise. The variance of the additive noise differs in three disjoint intervals.

```
load nblocr1;
```

Apply interval-dependent denoising down to level 4 using the Haar wavelet and a hard thresholding rule. `cmddenoise` automatically determines the optimal number and locations of the intervals. Plot the original and denoised signals.

```
sorh = 'h';
sigden = cmddenoise(nblocr1,'db1',4,sorh);
plot(nblocr1);
hold on;
```

```
plot(sigden,'r','linewidth',2);
axis tight;
legend('Original Signal','Denoised Signal','Location','NorthWest');
```

Specify the Number of Intervals

Create a signal sampled at 1 kHz. The signal consists of a series of bumps of various widths.

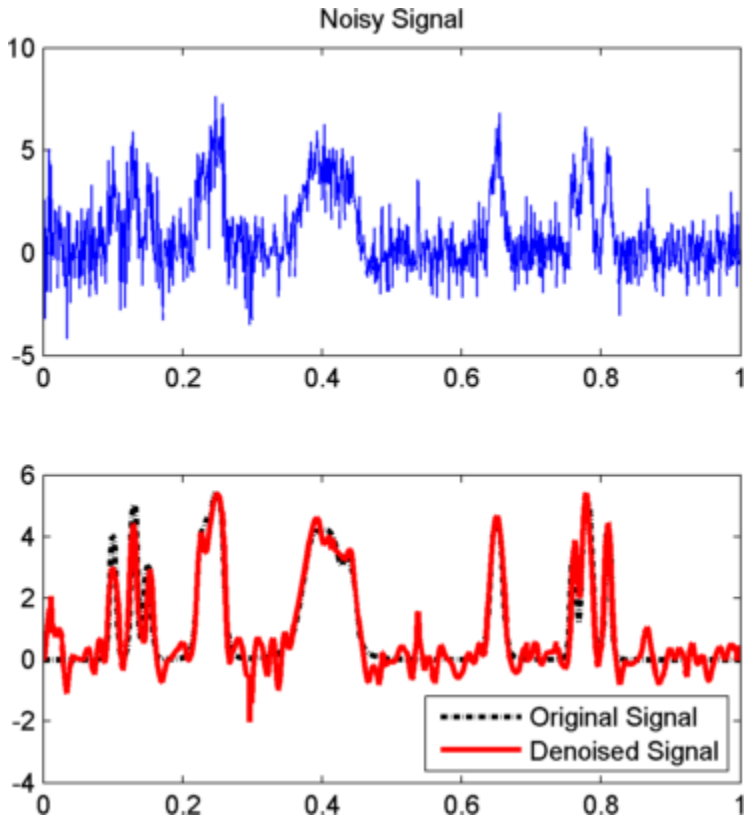
```
t = [0.1 0.13 0.15 0.23 0.25 0.40 0.44 0.65 0.76 0.78 0.81];
h = [4  -5    3   -4    5   -4.2  2.1  4.3 -3.1  5.1 -4.2];
h  = abs(h);
len = 1000;
w  = 0.01*[0.5 0.5 0.6 1 1 3 1 1 0.5 0.8 0.5];
tt = linspace(0,1,len);  x = zeros(1,len);
for j=1:11
  x = x + ( h(j) ./ (1+ ((tt-t(j))/w(j)).^4));
end
plot(tt,x); title('Original Signal');
```

Add white Gaussian noise with different variances to two disjoint segments of the signal. Add zero-mean white Gaussian noise with variance equal to 2 to the signal segment from 0 to 0.3 seconds. Add zero-mean white Gaussian noise with unit variance to the signal segment from 0.3 seconds to 1 second. Set the random number generator to the default settings for reproducible results.

```
rng default;
nv1 = sqrt(2).*randn(size(tt)).*(tt<=0.3);
nv2 = randn(size(tt)).*(tt>0.3);
xx = x+nv1+nv2;
```

Apply interval-dependent denoising using the Daubechies' least-asymmetric wavelet with 5 vanishing moments down to level 3. Set the number of intervals to 2. Plot the noisy signal, original signal, and denoised signal for comparison.

```
sigden = cmddenoise(xx,'sym5',3,'s',2);
subplot(211)
plot(tt,xx); title('Noisy Signal');
subplot(212)
plot(tt,x,'k-.','linewidth',2);
hold on;
plot(tt,sigden,'r','linewidth',2);
legend('Original Signal','Denoised Signal','Location','SouthEast');
```



Specify Intervals and Thresholds

Load the example signal `nbumpr1.mat`. The variance of the additive noise differs in three disjoint intervals.

```
load nbumpr1.mat;
```

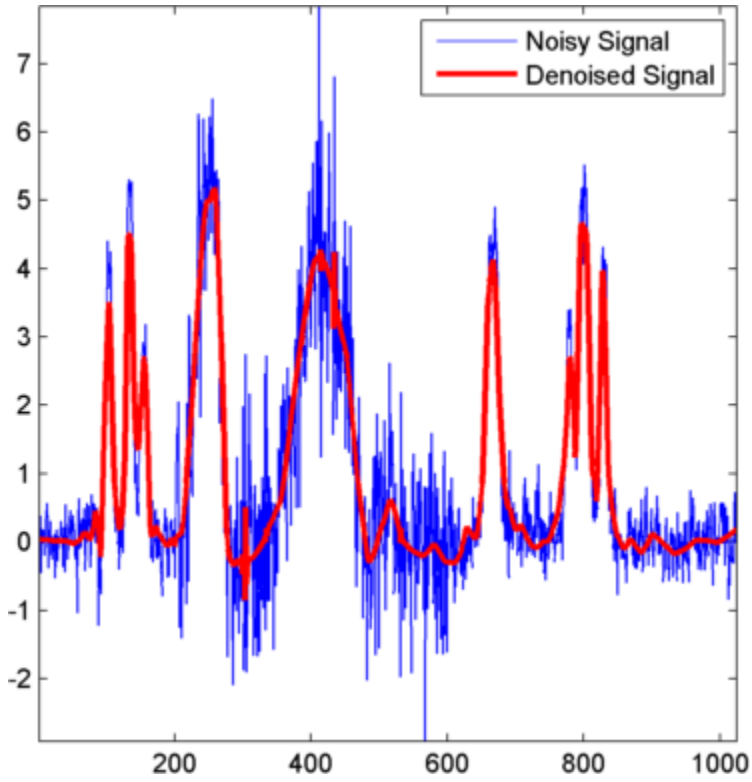
Use a level-5 multiresolution analysis. Create a cell array of length 5 consisting of 3-by-3 matrices. The first two elements of each row contain the beginning and ending indices of the interval and the last element of each row is the corresponding threshold.

```
thrParamsIn =  {...
    [...
    1     207      1.0482; ...
    207   613      2.5110; ...
```

```
613   1024     1.0031; ...
]; ...
[...
1    207      1.04824; ...
207  613      3.8718; ...
613  1024     1.04824; ...
]; ...
[...
1    207      1.04824; ...
207  613      1.99710; ...
613  1024     1.65613; ...
]; ...
[...
1    207      1.04824; ...
207  613      2.09117; ...
613  1024     1.04824; ...
]; ...
[...
1    207      1.04824; ...
207  613      1.78620; ...
613  102      1.04824; ...
]; ...
};
```

Denoise the signal using the threshold settings and the Daubechies' least-asymmetric wavelet with 4 vanishing moments. Use a soft thresholding rule. Plot the noisy and denoised signals for comparison.

```
wname = 'sym4';
level = 5;
sorh = 's';
sigden = cmddenoise(nbumpr1,wname,level,sorh,NaN,...
   thrParamsIn);
plot(nbumpr1); hold on;
plot(sigden,'r','linewidth',2); axis tight;
legend('Noisy Signal','Denoised Signal','Location','NorthEast');
```



Return Denoised Wavelet Coefficients

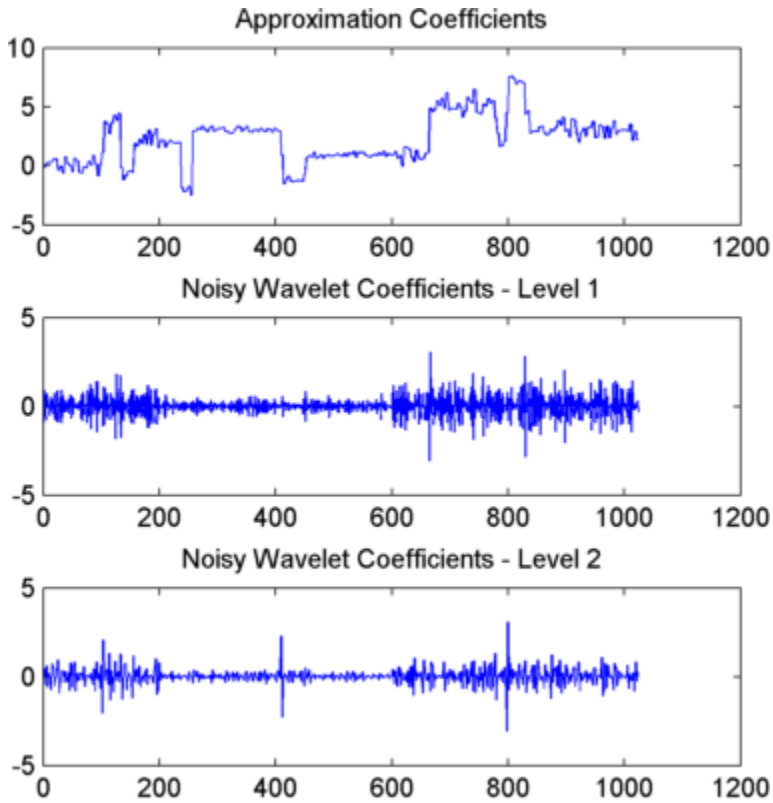
Load the example signal `nblocr1.mat`. Use the Haar wavelet and decompose the signal down to level 2. Obtain the discrete wavelet transform and denoise the signal. Return the wavelet coefficients of the noisy and denoised signals.

```
load nblocr1.mat;
[sigden,coefs] = cmddenoise(nblocr1,'db1',2);
[C,L] = wavedec(nblocr1,2,'db1');
```

Plot reconstructions based on the level-2 approximation and level-2 and level-1 detail coefficients for the noisy signal.

```
app = wrcoef('a',C,L,'db1',2);
subplot(3,1,1);
plot(app); title('Approximation Coefficients');
```

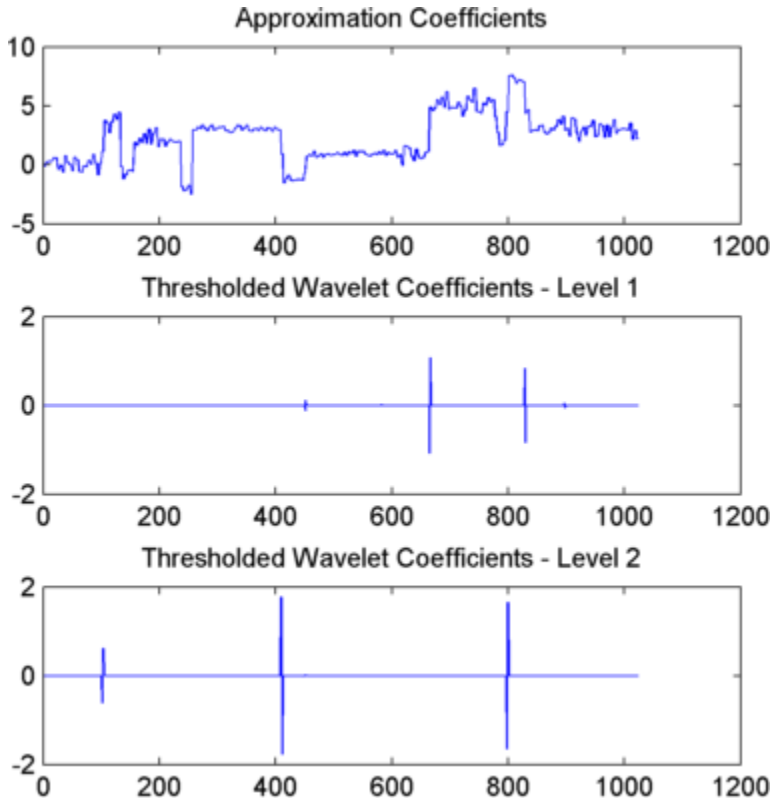
```
for nn = 1:2
    det = wrcoef('d',C,L,'db1',nn);
    subplot(3,1,nn+1)
    plot(det);
    title(['Noisy Wavelet Coefficients - Level '...
       num2str(nn)]);
end
```



Plot reconstructions based on the approximation and detail coefficients for the denoised signal at the same levels.

```
figure;
app = wrcoef('a',coefs,L,'db1',2);
subplot(3,1,1);
plot(app); title('Approximation Coefficients');
for nn = 1:2
```

```
    det = wrcoef('d',coefs,L,'db1',nn);
    subplot(3,1,nn+1)
    plot(det);
    title(['Thresholded Wavelet Coefficients - Level '...
       num2str(nn)]);
end
```



The approximation coefficients are identical in the noisy and denoised signal, but most of the detail coefficients in the denoised signal are close to zero.

Output Intervals and Thresholds

Create a signal sampled at 1 kHz. The signal consists of a series of bumps of various widths.

```
t = [0.1 0.13 0.15 0.23 0.25 0.40 0.44 0.65 0.76 0.78 0.81];
```

```
h = [4  -5   3   -4   5   -4.2  2.1  4.3 -3.1  5.1 -4.2];
h  = abs(h);
len = 1000;
w  = 0.01*[0.5 0.5 0.6 1 1 3 1 1 0.5 0.8 0.5];
tt = linspace(0,1,len);  x = zeros(1,len);
for j=1:11
  x = x + ( h(j) ./ (1+ ((tt-t(j))/w(j)).^4));
end
plot(tt,x); title('Original Signal');
```

Add white Gaussian noise with different variances to two disjoint segments of the signal. Add zero-mean white Gaussian noise with variance equal to 2 to the signal segment from 0 to 0.3 seconds. Add zero-mean white Gaussian noise with unit variance to the signal segment from 0.3 seconds to 1 second. Set the random number generator to the default settings for reproducible results.

```
rng default;
nv1 = sqrt(2).*randn(size(tt)).*(tt<=0.3);
nv2 = randn(size(tt)).*(tt>0.3);
xx = x+nv1+nv2;
```

Apply interval-dependent denoising using the Daubechies' least-asymmetric wavelet with 4 vanishing moments down to level 5. Automatically choose the number of intervals and output the result.

```
[sigden,coefs,thrParamsOut] = cmddenoise(xx,'sym4',5);
thrParamsOut{1}
```

`cmddenoise` identifies one variance change point in the 1st level detail coefficients defining two intervals. The first interval contains samples 1 to 293. The second interval contains samples 293 to 1000. This is close to the true variance change point, which occurs at sample 299.

Partition Signal into Increasing Numbers of Intervals with Thresholds

Load the example signal, `nbumpr1.mat`. Partition the signal into 1 to 6 intervals assuming 0 to 5 change points. Compute the thresholds for each interval. Using the Daubechies' least-asymmetric wavelet with 4 vanishing moments return the intervals and corresponding thresholds. Display the results in the command window.

```
load nbumpr1.mat;
[sigden,~,~,int_DepThr_Cell] = cmddenoise(nbumpr1,'sym4',1);
format bank;
```

```
disp('          Begin        End          Threshold ');
cellfun(@disp,int_DepThr_Cell,'UniformOutput',false);
format;
```

Detect Number of Change Points

Load the example signal, `nbumpr1.mat`. The signal has two variance change points, which results in three intervals.

Use `cmddenoise` to detect the number of change points. Print the result.

```
load nbumpr1.mat;
[sigden,~,thrParamsOut,~,bestNbofInt] = ...
   cmddenoise(nbumpr1,'sym4',1);
fprintf('Found %d change points.\n',bestNbofInt-1);
```

Input Arguments

`sig` — Signal for interval-dependent denoising

1-D row or column vector

Input signal, specified as a 1-D row or column vector. `sig` is the real-valued input signal for interval-dependent denoising. The elements of `sig` are assumed to be equally spaced in time or space. If `sig` contains unequally-sampled data, `cmddenoise` is not appropriate. Use a lifting transform instead. See `lwt` for details.

Data Types: `double`

`wname` — Wavelet name

string

Wavelet name, specified as a character array. `wname` is any valid orthogonal or biorthogonal wavelet. You can use the command: `wtype = wavemngr('fields',wname,'type','file');` to determine if the wavelet name is valid to use with `cmddenoise`. Valid wavelet names return a 1 or 2 for `wtype`.

Example: `'bior2.2'`, `'db4'`, `'sym4'`

Data Types: `char`

`level` — Level of the decimated wavelet transform (multiresolution analysis)

positive integer

Wavelet transform (multiresolution analysis) level, specified as a positive integer. `level` gives the level of the multiresolution decomposition of the input signal using the decimated 1-D discrete wavelet transform, `wavedec`.

Data Types: `double`

`sorh` — Threshold rule

`'s'` (default) | `'h'`

Thresholding rule, specified as a character array. `sorh` is the threshold rule used in the modification of the detail coefficients. Valid choices for `sorh` are `'s'` (default) and `'h'` for soft and hard thresholding.

`nb_inter` — Number of intervals

positive integer in the set {1,2,3,4,5,6} | `NaN`

Number of intervals, specified as a positive integer less than 7. `cmddenoise` divides the input signal into `nb_inter` intervals. `cmddenoise` determines the location of the `nb_inter` change points using a contrast function [2]. If you enter `NaN` for `nb_inter`, `cmddenoise` ignores the input. If you use the input argument `thrParamsIn`, `cmddenoise` disregards any value you enter for `nb_inter`.

Data Types: `double`

`thrParamsIn` — Intervals and thresholds by level

cell array of matrices

Intervals and thresholds by level, specified as a cell array of matrices equal in length to `level`. Each element of `thrParamsIn` contains the interval and threshold information for the corresponding level of the multiresolution analysis. The elements of `thrParamsIn` are N-by-3 matrices with N equal to the number of intervals. The 1st and 2nd columns contain the beginning and ending indices of the intervals and the 3rd column contains the corresponding threshold value. If you specify `thrParamsIn`, you cannot specify the output arguments `int_DepThr_Cell` or `BestNbofInt`.

Data Types: `cell`

Output Arguments

`sigden` — Denoised signal

1-D row or column vector

`sigden` is the denoised version of the input `sig`. `sigden` is a 1-D row vector equal in length to `sig`.

`coefs` — Approximation coefficients and thresholded wavelet coefficients

1-D row vector of approximation coefficients and thresholded wavelet coefficients

`coefs` is a row vector of approximation (scaling) and thresholded detail (wavelet) coefficients. The ordering of the approximation and detail coefficients by level in `coefs` is the same as the output of `wavedec`. `cmddenoise` does not apply thresholding to the approximation coefficients.

Data Types: `double`

`thrParamsOut` — Intervals and thresholds by level

cell array of matrices

`thrParamsOut` is a cell array of matrices equal in length to `level`. Each element of the cell array contains the interval and threshold information for the corresponding level of the multiresolution analysis. The elements of `thrParamsOut` are N-by-3 matrices with N equal to the number of intervals. N is determined by the value of the input arguments. The 1st and 2nd columns contain the beginning and ending indices of the intervals and the 3rd column contains the corresponding threshold value.

Data Types: `cell`

`int_DepThr_Cell` — Intervals and thresholds assuming 0 to 5 change points

cell array of matrices

`int_DepThr_Cell` contains interval and threshold information assuming the number of change points ranges from 0 to 5. The N-th element of `int_DepThr_Cell` is a N-by-3 matrix containing the interval information assuming N-1 change points. Each row of the matrix contains the beginning and ending indices of the thresholded interval and the corresponding threshold value. Attempting to output `int_DepThr_Cell` if you input the number of intervals and thresholds, `thrParamsIn`, results in an error. `int_DepThr_Cell{BestNbofInt}` or `int_DepThr_Cell{nb_inter}` is equal to the matrix elements of `thrParamsOut`.

Data Types: `cell`

`BestNbofInt` — Optimal number of intervals

positive integer ≤6

`BestNbofInt` is the optimal number of intervals based on estimated change points in the variance of the level-1 detail coefficients. The number and location of the change

points are estimated using a penalized contrast method [2]. Attempting to output `BestNbofInt` if you input the number of intervals and thresholds, `thrParamsIn`, results in an error.

References

[1] Donoho, D. and Johnstone, I. “Ideal spatial adaptation by wavelet shrinkage”, *Biometrika*, 1994, 81,3, 425–455.

[2] Lavielle, M. “Detection of multiple changes in a sequence of dependent variables”, *Stochastic Processes and their Applications*, 1999, 83, 79–102.

See Also

`thselect` | `wavedec` | `wthresh` | `wvarchg`

Introduced in R2010a

cmorwavf

Complex Morlet wavelet

Syntax

```
[PSI,X] = cmorwavf(LB,UB,N,FB,FC)
```

Description

`[PSI,X] = cmorwavf(LB,UB,N,FB,FC)` returns values of the complex Morlet wavelet defined by a positive bandwidth parameter *FB*, a wavelet center frequency *FC*, and the expression

```
PSI(X) = ((pi*FB)^(-0.5))*exp(2*i*pi*FC*X)*exp(-X^2/FB)
```

on an N point regular grid for the interval [*LB*,*UB*].

Output arguments are the wavelet function PSI computed on the grid X.

Complex Morlet Wavelet

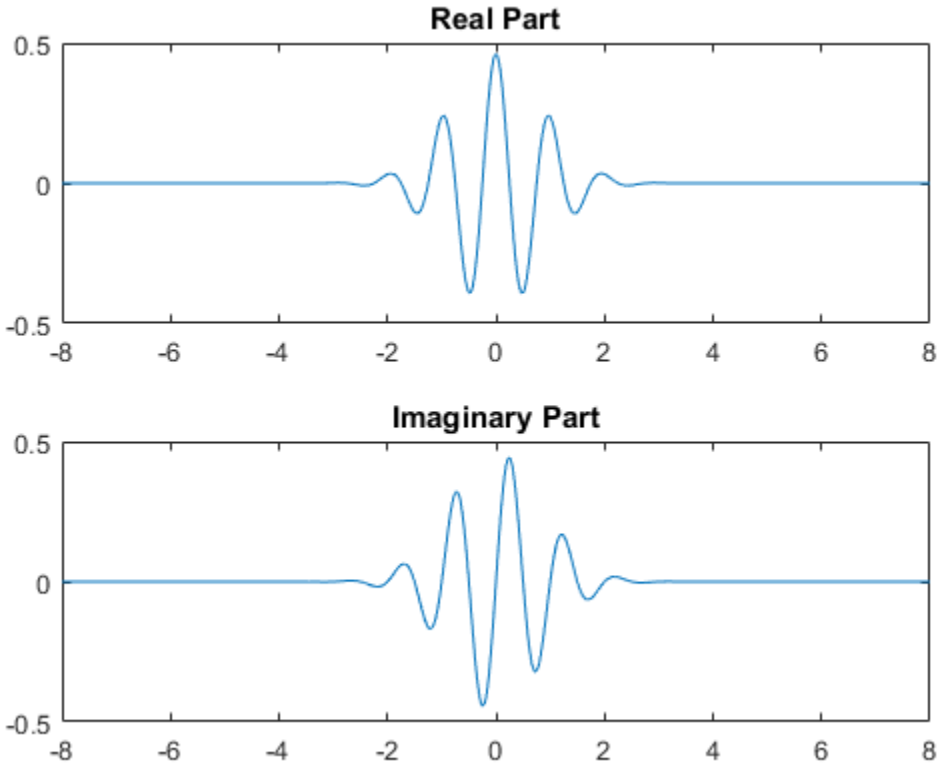
Construct a complex-valued Morlet wavelet with a bandwidth parameter of 1.5 and a center frequency of 1. Set the effective support to $[-8,8]$ and the length of the wavelet to 1000.

```
N = 1000;
Lb = -8;
Ub = 8;
fb = 1.5;
fc = 1;
[psi,x] = cmorwavf(Lb,Ub,N,fb,fc);
```

Plot the real and imaginary parts of the wavelet.

```
subplot(2,1,1)
plot(x,real(psi)); title('Real Part');
subplot(2,1,2)
```

```
plot(x,imag(psi)); title('Imaginary Part');
```



References

Teolis, A. (1998), *Computational signal processing with wavelets*, Birkhauser, p. 65.

See Also

`waveinfo`

Introduced before R2006a

coifwavf

Coiflet wavelet filter

Syntax

```
F = coifwavf(W)
```

Description

`F = coifwavf(W)` returns the scaling filter associated with the Coiflet wavelet specified by the string *W* where `W = 'coifN'`. Possible values for `N` are 1, 2, 3, 4, or 5.

Examples

```
% Set coiflet wavelet name.
wname = 'coif2';

% Compute the corresponding scaling filter.
f = coifwavf(wname)

f =
Columns 1 through 7
0.0116  -0.0293  -0.0476 0.2730 0.5747  0.2949  -0.0541

Columns 8 through 12
-0.0420  0.0167  0.0040 -0.0013 -0.0005
```

See Also

`waveinfo`

Introduced before R2006a

conofinf

Cone of influence

Syntax

```
cone = conofinf(wname,scales,LenSig,SigVal)
[cone,PL,PR] = conofinf(wname,scales,LenSig,SigVal)
[cone,PL,PR,PLmin,PRmax] = conofinf(wname,scales,LenSig,SigVal)
[PLmin,PRmax] = conofinf(wname,scales,LenSig)
[...] = conofinf(...,'plot')
```

Description

`cone = conofinf(wname,scales,LenSig,SigVal)` returns the cone of influence (COI) for the wavelet `wname` at the scales in `scales` and positions in `SigVal`. `LenSig` is the length of the input signal. If `SigVal` is a scalar, `cone` is a matrix with row dimension `length(scales)` and column dimension `LenSig`. If `SigVal` is a vector, `cone` is cell array of matrices.

`[cone,PL,PR] = conofinf(wname,scales,LenSig,SigVal)` returns the left and right boundaries of the cone of influence at scale 1 for the points in `SigVal`. `PL` and `PR` are `length(SigVal)`-by-2 matrices. The left boundaries are `(1-PL(:,2))./PL(:,1)` and the right boundaries are `(1-PR(:,2))./PR(:,1)`.

`[cone,PL,PR,PLmin,PRmax] = conofinf(wname,scales,LenSig,SigVal)` returns the equations of the lines that define the minimal left and maximal right boundaries of the cone of influence. `PLmin` and `PRmax` are 1-by-2 row vectors where `PLmin(1)` and `PRmax(1)` are the slopes of the lines. `PLmin(2)` and `PRmax(2)` are the points where the lines intercept the scale axis.

`[PLmin,PRmax] = conofinf(wname,scales,LenSig)` returns the slope and intercept terms for the first-degree polynomials defining the minimal left and maximal right vertices of the cone of influence.

`[...] = conofinf(...,'plot')` plots the cone of influence.

Input Arguments

`wname`

`wname` is a string corresponding to a valid wavelet. To verify that `wname` is a valid wavelet, `wavemngr('fields',wname)` must return a struct array with a `type` field of 1 or 2, or a nonempty `bound` field.

`scales`

`scales` is a vector of scales over which to compute the cone of influence. Larger scales correspond to stretched versions of the wavelet and larger boundary values for the cone of influence.

`LenSig`

`LenSig` is the signal length and must exceed the maximum of `SigVal`.

`SigVal`

`SigVal` is a vector of signal values at which to compute the cone of influence. The largest value of `SigVal` must be less than the signal length, `LenSig`.If `SigVal` is empty, `conofinf` returns the slope and intercept terms for the minimal left and maximal right vertices of the cone of influence.

Output Arguments

`cone`

`cone` isthe cone of influence. If `SigVal` is a scalar, `cone` is a matrix. The row dimension is equal to the number of `scales` and column dimension equal to the signal length, `LenSig`. If `SigVal` is a vector, `cone` is a cell array of matrices. The elements of each row of the matrix are equal to 1 in the interval around `SigVal` corresponding to the cone of influence.

`PL`

`PL` is the minimum value of the cone of influence on the position (time) axis.

`PR`

`PR` is the maximum value of the cone of influence on the position (time) axis.

PLmin

PLmin is a 1-by-2 row vector containing the slope and scale axis intercept of the line defining the minimal left vertex of the cone of influence. PLmin(1) is the slope and PLmin(2) is the point where the line intercepts the scale axis.

PRmax

PRmax is a 1-by-2 row vector containing the slope and scale axis intercept of the line defining the maximal right vertex of the cone of influence. PRmax(1) is the slope and PRmax(2) is the point where the line intercepts the scale axis.

Examples

Cone of influence for Mexican hat wavelet:

```
load cuspamax
signal = cuspamax;
wname  = 'mexh';
scales = 1:64;
lenSIG = length(signal);
x = 500;
figure;
cwt(signal,scales,wname,'plot');
hold on
[cone,PL,PR,Pmin,Pmax] = conofinf(wname,scales,lenSIG,x,'plot');
set(gca,'Xlim',[1 lenSIG])
```

Left minimal and right maximal vertices for the cone of influence (Morlet wavelet):

```
[PLmin,PRmax] = conofinf('morl',1:32,1024,[],'plot');
% PLmin = -0.1245*u+ 32.0000
% PRmax =  0.1250*u-96.0000
```

More About

Cone of Influence

Let ψ(t) be an admissible wavelet. Assume that the effective support of ψ(t) is [-B,B]. Letting u denote the translation parameter and s denote the scale parameter, the dilated and translated wavelet is:

$$\psi_{u,s}(t) = \frac{1}{\sqrt{s}}\psi(\frac{t-u}{s})$$

andhas effective support [u-sB,u+sB]. The cone of influence (COI) is the set of all t included in the effective support of the wavelet at a given position and scale. This set is equivalent to:

$$|t-u| \leq sB$$

At each scale, the COI determines the set of wavelet coefficients influenced by the value of the signal at a specified position.

- "Continuous and Discrete Wavelet Transforms"

References

Mallat, S. *A Wavelet Tour of Signal Processing*, London:Academic Press, 1999, p. 174.

See Also

`cwt` | `wavsupport`

Introduced in R2010b

cwt

Continuous 1-D wavelet transform

Syntax

```
coefs = cwt(x,scales,'wname')
coefs = cwt(x,scales,'wname','plot')
coefs = cwt(x,scales,'wname','coloration')
coefs = cwt(x,scales,'wname','coloration',xlim)
[coefs,sgram] = cwt(x,scales,'wname','scal')
[coefs,sgram] = cwt(x,scales,'wname','scalCNT')
[coefs,frequencies] = cwt(x,scales,wname, samplingperiod)
[coefs,sgram,frequencies] = cwt(x,scales,wname,
samplingperiod,'scal')
```

Description

`coefs = cwt(x,scales,'wname')` returns the continuous wavelet transform (CWT) of the real-valued signal S. The wavelet transform is computed for the specified scales using the analyzing wavelet *`wname`*. `scales` is a 1-D vector with positive elements. The string *`wname`* denotes a wavelet recognized by `wavemngr`. `coefs` is a matrix with the number of rows equal to the length of `scales` and number of columns equal to the length of the input signal. The k-th row of `coefs` corresponds to the CWT coefficients for the k-th elemens in the scales vector.

`coefs = cwt(x,scales,'wname','plot')` plots the continuous wavelet transform coefficients, using default `coloration` `'absglb'`.

`coefs = cwt(x,scales,'wname','coloration')` uses the specified coloration. See "Definitions" on page 1-75 for coloration options.

`coefs = cwt(x,scales,'wname','coloration',xlim)` colors the coefficients using coloration and `xlim`, where `xlim` is a vector, `[x1 x2]`, with $1 \le$ `x1` $<$ `x2` $\le$ `length(x)`.

`[coefs,sgram] = cwt(x,scales,'wname','scal')` returns and plots the scalogram. `'scal'` produces an image plot of the scalogram.

`[coefs,sgram] = cwt(x,scales,'wname','scalCNT')` displays a contour representation of the scalogram.

`[coefs,frequencies] = cwt(x,scales,wname, samplingperiod)` returns the frequencies in cycles per unit time corresponding to the scales and the analyzing wavelet *`wname`*. `samplingperiod` is a positive real-valued scalar. If the units of `samplingperiod` are seconds, the frequencies are in hertz.

`[coefs,sgram,frequencies] = cwt(x,scales,wname, samplingperiod,'scal')` returns the scalogram and the frequencies corresponding to the scales and the analyzing wavelet. You can also use the flag `'scalCNT'` to output the scalogram if you have at least two elements in `scales`. The `samplingperiod` is only used in the conversion of scales to frequencies. Specifying `samplingperiod` does not affect the appearance of plots generated by `cwt`.

Examples

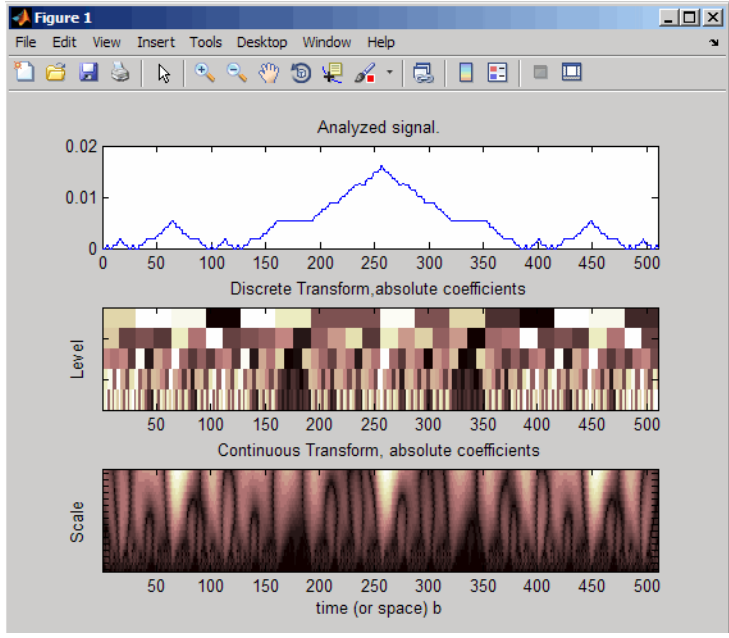
Plot the continuous wavelet transform and scalogram using sym2 wavelet at all integer scales from 1 to 32, using a fractal signal as input:

```
load vonkoch
vonkoch=vonkoch(1:510);
len = length(vonkoch);
cw1 = cwt(vonkoch,1:32,'sym2','plot');
title('Continuous Transform, absolute coefficients.')
ylabel('Scale')
[cw1,sc] = cwt(vonkoch,1:32,'sym2','scal');
title('Scalogram')
ylabel('Scale')
```

Compare discrete and continuous wavelet transforms, using a fractal signal as input:

```
load vonkoch
vonkoch=vonkoch(1:510);
len=length(vonkoch);
[c,l]=wavedec(vonkoch,5,'sym2');
% Compute and reshape DWT to compare with CWT.
cfd=zeros(5,len);
for k=1:5
    d=detcoef(c,l,k);
    d=d(ones(1,2^k),:);
```

```
    cfd(k,:)=wkeep(d(:)',len);
end
cfd=cfd(:);
I=find(abs(cfd) <sqrt(eps));
cfd(I)=zeros(size(I));
cfd=reshape(cfd,5,len);
% Plot DWT.
subplot(311); plot(vonkoch); title('Analyzed signal.');
set(gca,'xlim',[0 510]);
subplot(312);
image(flipud(wcodemat(cfd,255,'row')));
colormap(pink(255));
set(gca,'yticklabel',[]);
title('Discrete Transform,absolute coefficients');
ylabel('Level');
% Compute CWT and compare with DWT
subplot(313);
ccfs=cwt(vonkoch,1:32,'sym2','plot');
title('Continuous Transform, absolute coefficients');
set(gca,'yticklabel',[]);
ylabel('Scale');
```



How To

"Continuous and Discrete Wavelet Transforms"

"1-D Continuous Wavelet Analysis"

"New Wavelet for CWT"

"Time-Frequency Analysis with the Continuous Wavelet Transform"

More About

Scale values

Scale values determine the degree to which the wavelet is compressed or stretched. Low scale values compress the wavelet and correlate better with high frequencies. The low scale CWT coefficients represent the fine-scale features in the input signal vector. High scale values stretch the wavelet and correlate better with the low frequency content of the signal. The high scale CWT coefficients represent the coarse-scale features in the input signal.

Coloration

Coloration is the method used to scale the coefficient values for plotting. Each coefficient is divided by the resulting coloration value.

- `'lvl'` — uses maximum value in each scale
- `'glb'` — uses maximum value in all scales
- `'abslvl'` or `'lvlabs'` — uses maximum absolute value in each scale
- `'absglb'` or `'glbabs'` — uses maximum absolute value in all scales
- `'scal'` — produces a scaled image of the scalogram
- `'scalCNT'` — produces a contour representation of the scalogram

For 3-D plots (surfaces), use the `coloration` parameter preceded by `'3D'`, such as `coefs = cwt(...,'3Dplot')` or `coefs = cwt(...,'3Dlvl') ...`

Scalogram

Scalograms are plots that represent the percentage energy for each coefficient.

References

Daubechies, I. *Ten Lectures on Wavelets*, Philadelphia, PA: Society for Industrial and Applied Mathematics (SIAM), 1992.

Mallat, S. *A Wavelet Tour of Signal Processing*, San Diego, CA: Academic Press, 1998.

See Also

`cwtext` | `dwt` | `wavedec` | `wavefun` | `waveinfo` | `wcodemat`

Introduced before R2006a

cwtft

Continuous wavelet transform using FFT algorithm

Syntax

```
cwtstruct = cwtft(sig)
cwtstruct = cwtft(sig,Name,Value)
cwtstruct = cwtft(...,'plot')
```

Description

`cwtstruct = cwtft(sig)` returns the continuous wavelet transform (CWT) of the 1–D input signal `sig`. `cwtft` uses an FFT algorithm to compute the CWT. `sig` can be a vector, a structure array, or a cell array. If the sampling interval of your signal is not equal to `1`, you must input the sampling period with `sig` in a cell array or a structure array to obtain correct results. If `sig` is a cell array, `sig{1}` is equal to your signal and `sig{2}` is equal to the sampling interval. If `sig` is a structure array, the field `sig.val` contains your signal and `sig.period` contains the sampling interval.

By default, `cwtft` uses the analytic Morlet wavelet. See More About for descriptions of valid analyzing wavelets.

For additional default values, see `scales` in “Name-Value Pair Arguments” on page 1-78.

`cwtstruct = cwtft(sig,Name,Value)` returns the continuous wavelet transform (CWT) of the 1–D input signal `sig` with additional options specified by one or more `Name,Value` pair arguments. See “Name-Value Pair Arguments” on page 1-78 for a comprehensive list.

`cwtstruct = cwtft(...,'plot')` plots the continuous wavelet transform. If the analyzing wavelet is real-valued, the original signal along with the CWT coefficient magnitudes and signed CWT coefficients are plotted. If the analyzing wavelet is complex-valued, the original signal is plotted along with the moduli, real parts, imaginary parts, and angles of the CWT coefficients. You can select the radio button in the bottom left of the plot to superimpose the signal's reconstruction using `icwtft`.

Input Arguments

`sig`

The 1–D input signal. `sig` can be a vector, a structure array, or a cell array. If `sig` is a structure array, `sig` contains two fields: `val` and `period`. `sig.val` is the signal vector and `sig.period` is the sampling period. If `sig` is a cell array, `sig{1}` is the signal vector and `sig{2}` is the sampling period.

If `sig` is a vector, the sampling period defaults to `1`.

Note: If the sampling interval of your input signal is not `1`, you must input the sampling interval with `sig` in a cell array or structure array to obtain correct results. If `sig` is a cell array, `sig{1}` is the 1–D input signal and `sig{2}` is the sampling period. If `sig` is a structure array, the field `sig.val` is the 1–D input signal and `sig.period` is the sampling interval.

Name-Value Pair Arguments

`'scales'`

Scales over which to compute the CWT. The value of `scales` can be a vector, a structure array, or a cell array. If `scales` is a structure array, it contains at most five fields. The first three fields are mandatory. The last two fields are optional.

1. `s0` — The smallest scale. The default `s0` depends on the wavelet. See the More About for descriptions of the default for each wavelet.
2. `ds` — Spacing between scales. The default `ds` depends on the wavelet. See the More About for descriptions of the default for eacj wavelet. You can construct a linear or logarithmic scale vector using `ds`. See type for a description of the type of spacing.
3. `nb` — Number of scales. The default `nb` depends on the wavelet. See the More About for descriptions of the default for each wavelet.
4. `type` — Type of spacing between scales. `type` can be one of `'pow'` or `'lin'`. The default is `'pow'`. If `type` is equal to `'pow'`, the CWT scales are `s0*pow.^((0:nb-1)*ds)`. This results in a constant spacing of `ds` if you take the logarithm to the base `power` of the scales vector. If `type` is equal to `'lin'`, the CWT scales are linearly spaced by `s0 + (0:nb-1)*ds`.

Use the default power of two spacing to ensure an accurate approximation to the original signal based only on select scales. See the second example in "Examples" on page 1-80 for a signal approximation based on select scales.

5 `pow` — The base for `'pow'` spacing. The default is 2. This input is valid only if the `type` argument is `'pow'`.

If `scales` is a cell array, the first three elements of the cell array are identical to the first three elements of the structure array described in the preceding list. The last two elements of the cell array are optional and match the two optional inputs in the structure array described in the preceding list.

`'wavelet'`

Analyzing wavelet. To include a parameter for the wavelet, use a cell array. For example, to specify a fourth order derivative of a Gaussian wavelet, use `'wavelet',{'dog',4}` as in `cwtstruct = (sig,'wavelet',{'dog',4})`.

The supported analyzing wavelets are:

- `'dog'` — m-th order derivative of a Gaussian wavelet where m is a positive even integer. The default value of m is 2.
- `'morl'` — Morlet wavelet. Results in an analytic Morlet wavelet. The Fourier transform of an analytic wavelet is zero for negative frequencies.
- `'morlex'` — non-analytic Morlet wavelet
- `'morl0'` — non-analytic Morlet wavelet with zero mean
- `'mexh'` — Mexican hat wavelet. The Mexican hat wavelet is a special case of the m-th order derivative of a Gaussian wavelet with m=2.
- `'paul'` — Paul wavelet
- `'bump'` — Bump wavelet

See the More About for formal definitions of the supported analyzing wavelets and associated defaults.

Default: `'morl'`

`'padmode'`

Signal extension mode. See `dwtmode` for supported extension modes. By default, `cwtft` does not extend the signal prior to computing the CWT. In a Fourier-transform-based

CWT algorithm, extending a signal can mitigate wrap-around effects. The number of CWT coefficients in each row of the output matrix `cwtstruct.cfs` is truncated to match the length of the input signal.

Output Arguments

`cwtstruct`

A structure array with six fields. The fields of the structure array are:

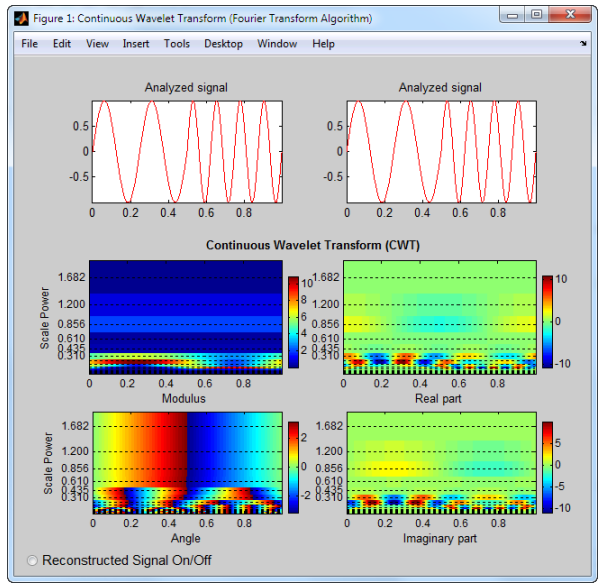
- `cfs` — The CWT coefficient matrix. `cwtstruct.cfs` is an `nb`-by-N matrix where `nb` is the number of scales and N is the length of the input signal.
- `scales` — Vector of scales at which the CWT is computed. The length of `cwtstruct.scales` is equal to the row dimension of `cwtstruct.cfs`.
- `frequencies` — Frequencies in cycles per unit time (or space) corresponding to the scales. If the sampling period units are seconds, the frequencies are in hertz. The elements of frequencies are in decreasing order to correspond to the elements in the scales vector. Use this field to examine the CWT in the time-frequency plane.
- `omega` — Vector of angular frequencies used in the Fourier transform of the wavelet. This field is used in `icwtft` and `icwtlin` for the inversion of the CWT for all wavelets except the bump wavelet.
- `meanSIG` — Mean of the analyzed signal
- `dt` — The sampling interval of the 1–D input signal
- `wav` — Analyzing wavelet

Examples

Compute and display the CWT of sine waves with disjoint support. The sampling interval is 1/1023.

```
N = 1024;
% Sampling interval is 1/1023
t = linspace(0,1,N);
y = sin(2*pi*4*t).*(t<=0.5)+sin(2*pi*8*t).*(t>0.5);
% Because the sampling interval differs from the default
% you must input it along with the signal
```

```
% Using cell array input
sig = {y,1/1023};
cwtS1 = cwtft(sig,'plot');
```

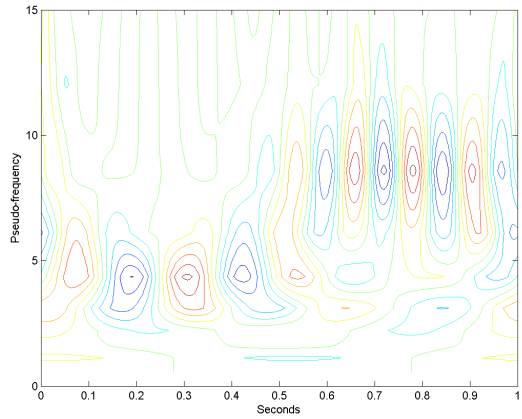


You can display or hide the reconstructed signal using the radio button at the bottom left of the figure. When you select the radio button, the maximum and quadratic relative errors are computed and displayed along with the reconstructed signal.

Reconstruct an approximation to a sum of disjoint sine waves in noise using `cwtft` to decompose the signal and `icwtft` to reconstruct the approximation. Use the CWT coefficients to identify the scales isolating the sinusoidal components. Reconstruct an approximation to the signal based on those scales using the inverse CWT. To ensure an accurate approximation to the based on select scales, use the default power of two spacing in the CWT.

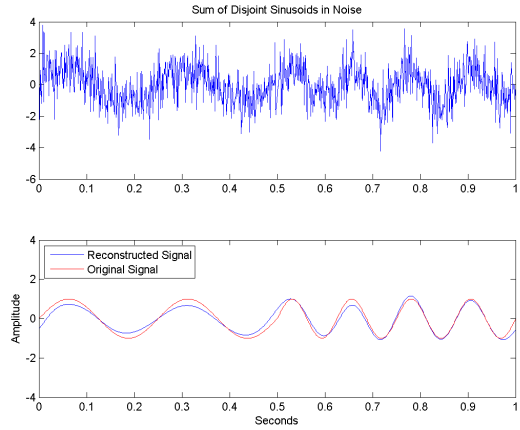
```
rng default % Reset random number generator for reproducible results
N = 1024;
% Sampling interval is 1/1023
t = linspace(0,1,N);
y = sin(2*pi*4*t).*(t<=0.5)+sin(2*pi*8*t).*(t>0.5);
ynoise = y+randn(size(t));
% Because the sampling interval differs from the default
```

```
% you must input it along with the signal
% Using structure array input
sig = struct('val',ynoise,'period',1/1023);
cwtS1 = cwtft(sig);
scales = cwtS1.scales;
MorletFourierFactor = 4*pi/(6+sqrt(2+6^2));
freq = 1./(scales.*MorletFourierFactor);
contour(t,freq,real(cwtS1.cfs));
xlabel('Seconds'); ylabel('Pseudo-frequency');
axis([0 t(end) 0 15]);
```



Extract the scales dominated by energy from the two sine waves and reconstruct a signal approximation using the inverse CWT.

```
cwtS2 = cwtS1;
cwtS2.cfs = zeros(size(cwtS1.cfs));
cwtS2.cfs(13:15,:) = cwtS1.cfs(13:15,:);
xrec = icwtft(cwtS2);
subplot(2,1,1);
plot(t,ynoise);
title('Sum of Disjoint Sinusoids in Noise');
subplot(2,1,2);
plot(t,xrec,'b'); hold on; axis([0 1 -4 4]);
plot(t,y,'r');
legend('Reconstructed Signal','Original Signal',...
    'Location','NorthWest');
xlabel('Seconds'); ylabel('Amplitude');
```



Alternatives

- `cwt` — Computes the CWT using convolutions. `cwt` supports a wider choice of analyzing wavelets than `cwtft`, but may be more computationally expensive. The output of `cwt` is not compatible with the inverse CWT implemented with `icwtft`. To use `icwtft`, obtain the CWT with `cwtft`.

More About

Morlet Wavelet

Both non-analytic and analytic Morlet wavelets are supported. The analytic Morlet wavelet, `'morl'`, is defined in the Fourier domain by:

$$\Psi(s\omega) = \pi^{-1/4} e^{-(s\omega-\omega_0)^2/2} U(s\omega)$$

where $U(\omega)$ is the Heaviside step function [5].

The non-analytic Morlet wavelet, `'morlex'`, is defined in the Fourier domain by:

$$\Psi(s\omega) = \pi^{-1/4} e^{-(s\omega-\omega_0)^2/2}$$

`'morl0'` defines a non-analytic Morlet wavelet in the Fourier domain with exact zero mean:

$$\Psi(s\omega) = \pi^{-1/4}\{e^{-(s\omega-\omega_0)^2/2} - e^{-\omega_0^2/2}\}$$

The default value of ω_0 is 6.

The default smallest scale for the Morlet wavelets is `s0 = 2*dt` where `dt` is the sampling period.

The default spacing between scales for the Morlet wavelets is `ds=0.4875`.

The default number of scales for the Morlet wavelets is `NbSc = fix(log2(length(sig)*dt/s0)/ds)`.

The default scales for the Morlet wavelet are `s0*2.^((0:NbSc-1)*ds)`.

m-th Order Derivative of Gaussian Wavelets

In the Fourier domain, the *m*-th order derivative of Gaussian wavelets, `'dog'`, are defined by:

$$\hat{\Psi}(s\omega) = -\frac{1}{\sqrt{\Gamma(m+1/2)}}(js\omega)^m e^{-(s\omega)^2/2}$$

where $\Gamma(\,)$ denotes the gamma function [5].

The derivative must be an even order. The default order of the derivative is 2, which is also known as the Mexican hat wavelet .

The default smallest scale for the DOG wavelet is `s0 = 2*dt` where `dt` is the sampling period.

The default spacing between scales for the DOG wavelet is `ds=0.4875`.

The default number of scales for the DOG wavelet is `NbSc = fix(log2(length(sig)*dt/s0)/ds)`.

The default scales for the DOG wavelet are `s0*2.^((0:NbSc-1)*ds)`.

Paul Wavelet

The Fourier transform of the analytic Paul wavelet, `'paul'`, of order *m* is:

$$\hat{\Psi}(s\omega) = 2^m \sqrt{m(2m-1)!}\,(s\omega)^m e^{-s\omega} U(sw)$$

where *U(ω)* is the Heaviside step function [5].

The default order of the Paul wavelet is 4.

The default smallest scale for the Paul wavelet is `s0 = 2*dt` where `dt` is the sampling period.

The default spacing between scales for the Paul wavelet is `ds=0.4875`.

The default number of scales for the Paul wavelet is `NbSc = fix(log2(length(sig)*dt/s0)/ds)`.

The default scales for the Paul wavelet are `s0*2.^((0:NbSc-1)*ds)`.

Bump wavelet

The Fourier transform of the analytic bump wavelet, `'bump'`, with parameters μ and σ is

$$\breve{\psi}(s\omega) = e^{\left(1-\frac{1}{1-(s\omega-\mu)^2/\sigma^2}\right)} \quad 1_{[(\mu-\sigma)/s,(\mu+\sigma)/s]}$$

where $1_{[(\mu-\sigma)/s,\,(\mu+\sigma)/s]}$ is the indicator function for the interval $(\mu-\sigma)/s \le \omega \le (\mu+\sigma)/s$.

Valid values for μ are [3,6]. Valid values for σ are [0.1, 1.2]. Smaller values of σ result in a wavelet with superior frequency localization but poorer time localization. Larger values of σ produce a wavelet with better time localization and poorer frequency localization.

The default values for μ and σ are 5 and 0.6 respectively.

The default smallest scale for the bump wavelet is `s0 = 2*dt` where `dt` is the sampling period.

The default spacing between scales for the bump wavelet is `ds=1/10`.

The default number of scales for the bump wavelet is `NbSc = fix(log2(length(sig)*dt/s0)/ds)`.

The default scales for the bump wavelet are `s0*2.^((0:NbSc-1)*ds)`.

Algorithms

`cwtft` implements the following algorithm:

- Obtain the discrete Fourier transform (DFT) of the signal using `fft`.
- Obtain the DFT of the analyzing wavelet at the appropriate angular frequencies. Scale the DFT of the analyzing wavelet at different scales to ensure different scales are directly comparable.
- Take the product of the signal DFT and the wavelet DFT over all the scales. Invert the DFT to obtain the CWT coefficients.

For a mathematical motivation for the FFT-based algorithm see "Continuous Wavelet Transform and Scale-Based Analysis".

- "Continuous and Discrete Wavelet Transforms"
- "Continuous Wavelet Transform and Scale-Based Analysis"
- "Inverse Continuous Wavelet Transform"
- "Time-Frequency Analysis with the Continuous Wavelet Transform"

References

[1] Daubechies, I. *Ten Lectures on Wavelets*, Philadelphia, PA: Society for Industrial and Applied Mathematics (SIAM), 1992.

[2] Farge, M. "Wavelet Transforms and Their Application to Turbulence", *Ann. Rev. Fluid. Mech.*, 1992, 24, 395–457.

[3] Mallat, S. *A Wavelet Tour of Signal Processing*, San Diego, CA: Academic Press, 1998.

[4] Sun,W. "Convergence of Morlet's Reconstruction Formula", *preprint*, 2010.

[5] Torrence, C. and G.P. Compo. "A Practical Guide to Wavelet Analysis", *Bull. Am. Meteorol. Soc.*, 79, 61–78, 1998.

See Also

`cwt` | `icwtft` | `icwtlin`

Introduced in R2011a

cwtftinfo

Valid analyzing wavelets for FFT-based CWT

Syntax

```
cwtftinfo
```

Description

`cwtftinfo` displays expressions for the Fourier transforms of valid analyzing wavelets for use with `cwtft`.

Examples

Display a list of Fourier transforms for all valid analyzing wavelets.

```
cwtftinfo
```

More About

Morlet Wavelet

Both non-analytic and analytic Morlet wavelets are supported. The analytic Morlet wavelet, `'morl'`, is defined in the Fourier domain by:

$$\Psi(s\omega) = \pi^{-1/4} e^{-(s\omega-\omega_0)^2/2} U(s\omega)$$

where $U(\omega)$ is the Heaviside step function.

The non-analytic Morlet wavelet, `'morlex'`, is defined in the Fourier domain by:

$$\Psi(s\omega) = \pi^{-1/4} e^{-(s\omega-\omega_0)^2/2}$$

`'morl0'` defines a non-analytic Morlet wavelet in the Fourier domain with exact zero mean:

$$\breve{\Psi}(s\omega) = \pi^{-1/4}\{e^{-(s\omega-\omega_0)^2/2} - e^{-\omega_0^2/2}\}$$

The default value of ω_0 is 6.

m-th Order Derivative of Gaussian Wavelets

In the Fourier domain, the *m*-th order derivative of Gaussian wavelets, `'dog'`, is defined by:

$$\hat{\Psi}(s\omega) = -\frac{1}{\sqrt{\Gamma(m+1/2)}}(js\omega)^m e^{-(s\omega)^2/2}$$

The derivative must be an even order. The default order of the derivative is 2, which is also known as the *Mexican hat* wavelet.

Because the unit imaginary, *j*, is always raised to an even power, the Fourier transform is real-valued.

Paul Wavelet

The Fourier transform of the Paul wavelet, `'paul'`, of order *m* is:

$$\hat{\Psi}(s\omega) = 2^m\sqrt{m(2m-1)!}(s\omega)^m e^{-s\omega}U(sw)$$

where $U(\omega)$ is the Heaviside step function. The Paul wavelet is analytic.

The default order of the Paul wavelet is 4.

Bump wavelet

The Fourier transform of the analytic bump wavelet, `'bump'`, with parameters μ and σ is

$$\breve{\psi}(s\omega) = e^{\left(1-\frac{1}{1-(s\omega-\mu)^2/\sigma^2}\right)} \quad 1_{[(\mu-\sigma)/s,(\mu+\sigma)/s]}$$

where $1_{[(\mu-\sigma)/s,\,(\mu+\sigma)/s]}$ is the indicator function for the interval $(\mu-\sigma)/s \le \omega \le (\mu+\sigma)/s$.

Valid values for μ are [3,6]. Valid values for σ are [0.1, 1.2]. Smaller values of σ result in a wavelet with superior frequency localization but poorer time localization. Larger values of σ produce a wavelet with better time localization and poorer frequency localization.

The default values for μ and σ are 5 and 0.6 respectively.

- "Continuous and Discrete Wavelet Transforms"
- "Continuous Wavelet Transform and Scale-Based Analysis"
- "Inverse Continuous Wavelet Transform"

References

[1] Daubechies, I. *Ten Lectures on Wavelets*, Philadelphia, PA: Society for Industrial and Applied Mathematics (SIAM), 1992.

[2] Farge, M. *Wavelet Transforms and Their Application to Turbulence*, Ann. Rev. Fluid. Mech., 1992, 24, 395–457.

[3] Mallat, S. *A Wavelet Tour of Signal Processing*, San Diego, CA: Academic Press, 1998.

[4] Torrence, C. and G.P. Compo *A Practical Guide to Wavelet Analysis*, Bull. Am. Meteorol. Soc., 79, 61–78, 1998.

See Also

`cwtft` | `icwtft` | `icwtlin`

Introduced in R2011a

cwtftinfo2

Supported 2-D CWT wavelets and Fourier transforms

Syntax

```
cwtftinfo2
cwtftinfo2(wname)
```

Description

`cwtftinfo2` lists the supported 2-D continuous wavelet transform (CWT) wavelets and corresponding parameters for use with `cwtft2`.

`cwtftinfo2(wname)` displays the equation for the 2-D Fourier transform of the wavelet, `wname`. The figure with the 2-D Fourier transform of the analyzing wavelet has a drop-down list from which you can select other wavelets.

Examples

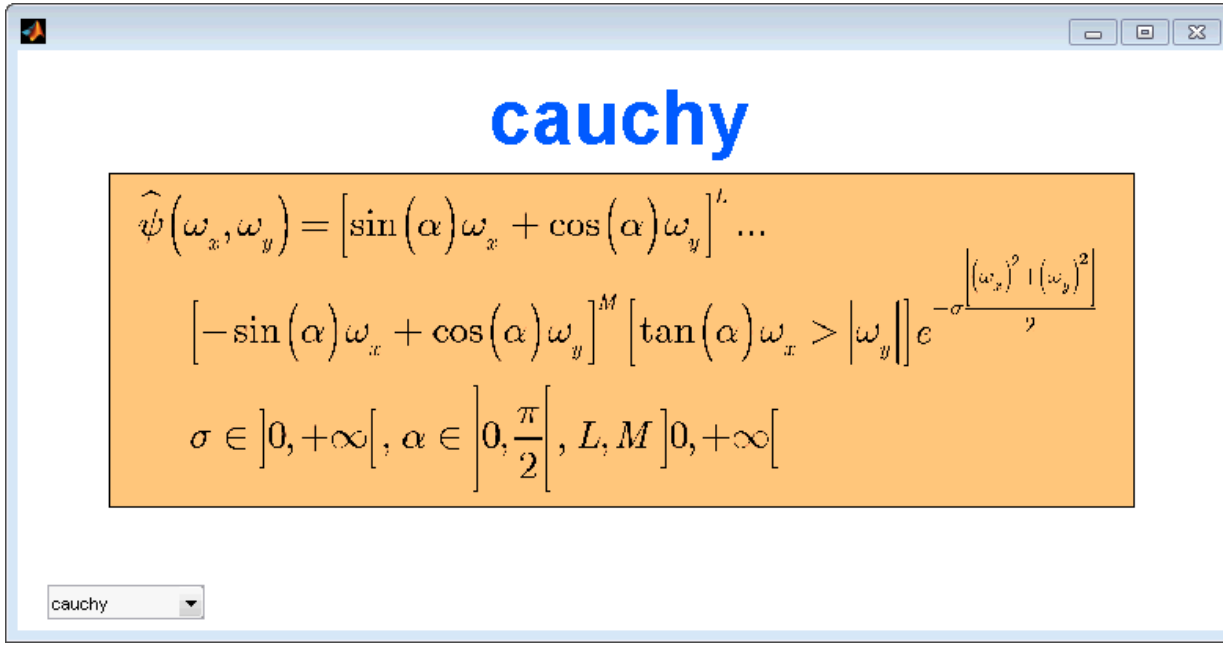
Available Wavelets with Parameters

```
cwtftinfo2
```

Display the Expression for the 2-D Fourier Transform

Display the expression for the 2-D Fourier transform of the Cauchy wavelet.

```
cwtftinfo2('cauchy')
```



After displaying the Fourier transform for any wavelet, you can use the drop-down list in the bottom left to view the Fourier transform for any supported wavelet.

Input Arguments

`wname` — Wavelet name

string

Wavelet name, specified as a string. The following table lists the supported wavelets for the 2-D CWT and associated parameters:

Wavelet name	Parameters
`'morl'`	`{'Omega0',6;'Sigma',1;'Epsilon',1}`
`'mexh'`	`{'p',2;'sigmax',1;'sigmay',1}`
`'paul'`	`{'p',4}`
`'dog'`	`{'alpha',1.25}`

Wavelet name	Parameters
'cauchy'	{'alpha','pi/6';'sigma',1;'L',4;'M',4}
'escauchy'	{'alpha','pi/6';'sigma',1;'L',4;'M',4}
'gaus'	{'p',1;'sigmax',1;'sigmay',1}
'wheel'	{'sigma',2}
'fan'	{'Omega0X',5.336;'Sigma',1;'Epsilon',1;'J',
'pethat'	None
'dogpow'	{'alpha',1.25;'p',2}
'esmorl'	{'Omega0',6;'Sigma',1;'Epsilon',1}
'esmexh'	{'Sigma',1;'Epsilon',0.5}
'gaus2'	{'p',1;'sigmax',1;'sigmay',1}
'gaus3'	{'A',1;'B',1;'p',1;'sigmax',1;'sigmay',1}
'isodog'	{'alpha',1.25,1.25}
'dog2'	{'alpha',1.25,1.25}
'isomorl'	{'Omega0',6;'Sigma',1}
'rmorl'	{'Omega0',6;'Sigma',1;'Epsilon',1}
'endstop1'	{'Omega0',6}
'endstop2'	{'Omega0',6;'Sigma',1}
'gabmexh'	{'Omega0',5.336;'Epsilon',1}
'sinc'	{'Ax',1;'Ay',1;'p',1;'Omega0X',0;'Omega0Y',

Example: `cwtftinfo2('paul')`

Data Types: `char`

See Also

`cwtft2`

Introduced in R2013b

cwtft2

2-D continuous wavelet transform

Syntax

```
cwtstruct = cwtft2(x)
cwtstruct = cwtft2(x,'plot')
cwtstruct = cwtft2(x,Name,Value)
```

Description

`cwtstruct = cwtft2(x)` returns the 2-D continuous wavelet transform (CWT) of the 2-D matrix, `x`. `cwtft2` uses a Fourier transform-based algorithm in which the 2-D Fourier transforms of the input data and analyzing wavelet are multiplied together and inverted.

`cwtstruct = cwtft2(x,'plot')` plots the data and the 2-D CWT.

`cwtstruct = cwtft2(x,Name,Value)` uses additional options specified by one or more Name,Value pair arguments.

Examples

Compare Isotropic and Anisotropic Wavelets

This example shows how an isotropic wavelet does not discern the orientation of features while an anisotropic wavelet does. The example uses the Mexican hat isotropic wavelet and the directional (anisotropic) Cauchy wavelet.

Load and view the hexagon image.

```
Im = imread('hexagon.jpg');
imagesc(Im); colormap(jet);
```

Obtain the scale-one 2-D CWT with both the Mexican hat and Cauchy wavelets. Specify a vector of angles going from 0 to 15π/8 in π/8 increments.

```
cwtcauchy = cwtft2(Im,'wavelet','cauchy','scales',1,...
```

```
    'angles',0:pi/8:2*pi-pi/8);

cwtmexh = cwtft2(Im,'wavelet','mexh','scales',1,...
    'angles',0:pi/8:2*pi-pi/8);
```

Visualize the scale-one 2-D CWT coefficient magnitudes at each angle.

```
angz = {'0', 'pi/8', 'pi/4', '3pi/8', 'pi/2', '5pi/8', '3pi/4', ...
    '7pi/8','pi', '9pi/8', '5pi/4', '11pi/8', '3pi/2', ...
    '13pi/8' '7pi/4', '15pi/8'};
for angn = 1:length(angz)
    subplot(211)
    imagesc(abs(cwtmexh.cfs(:,:,1,1,angn)));
    title(['Mexican hat at ' angz(angn) 'radians']);
    subplot(212)
    imagesc(abs(cwtcauchy.cfs(:,:,1,1,angn)));
    title(['Cauchy wavelet at ' angz(angn) 'radians']);
    pause(1);
end
```

Plot 2-D CWT

Load an image of a woman, obtain the 2-D CWT using the Morlet wavelet, and plot the CWT coefficients.

```
load woman;
cwtmorl = cwtft2(X,'scales',1:4,'angles',0:pi/2:3*pi/2,'plot');
```

2-D CWT with Morlet Wavelet

Obtain the 2-D CWT of the star image using the default Morlet wavelet, scales `2^(0:5)`, and an angle of 0.

```
Im = imread('star.jpg');
cwtout = cwtft2(Im);
```

- "Two-Dimensional CWT of Noisy Pattern"
- "2-D Continuous Wavelet Transform App"

Input Arguments

x — Input data

array

Input data, specified as a 2-D matrix or 3-D array. If the input data is a 3-D array, the input matrix is a truecolor image.

Example: `X = imread('stars.jpg');`

Data Types: `double` | `uint8`

Name-Value Pair Arguments

Specify optional comma-separated pairs of `Name,Value` arguments. `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside single quotes (`' '`). You can specify several name and value pair arguments in any order as `Name1,Value1,...,NameN,ValueN`.

Example: `'wavelet','paul','scales',2^(0:5)` specifies to use the Paul wavelet and a vector of scales.

`'angles'` — Angles

0 (default) | scalar | vector

Angles in radians, specified as a comma-separated pair consisting of `'angles'` and either a scalar or a vector.

Example: `'angles',[0 pi/2 pi]`

`'norm'` — Normalization

`'L2'` (default) | `'L1'` | `'L0'`

Normalization used in the 2-D CWT, specified as a comma-separated pair consisting of `'norm'` and one of these strings:

- `'L2'` — The Fourier transform of the analyzing wavelet at a given scale is multiplied by the corresponding scale. `'L2'` is the default normalization.
- `'L1'` — The Fourier transform of the analyzing wavelet is multiplied by 1 at all scales.
- `'L0'` — The Fourier transform of the analyzing wavelet at a given scale is multiplied by the square of the corresponding scale.

Example: `'norm','L1'`

`'scales'` — Scales

`2^(0:5)` (default) | scalar | vector

Scales, specified as a comma-separated pair consisting of `'scales'` and either a positive real-valued scalar or a vector of positive real numbers.

Example: `'scales',2^(1:6)`

`'wavelet'` — Analyzing wavelet

`'morl'` (default) | string | structure | cell array

Analyzing wavelet, specified as a comma-separated pair consisting of `'wavelet'` and a string, a structure, or a cell array. `cwtftinfo2` provides a comprehensive list of supported wavelets and associated parameters.

If you specify `'wavelet'` as a structure, the structure must contain two fields:

- `name` — the string corresponding to a supported wavelet.
- `param` — a cell array with the parameters of the wavelet.

If you specify `'wavelet'` as a cell array, `wav`, the cell array must contain two elements:

- `wav{1}` — the string corresponding to a supported wavelet.
- `wav{2}` — a cell array with the parameters of the wavelet.

Example: `'wavelet',{'morl',{6,1,1}}`

Example: `'wavelet',struct('name','paul','param',{'p',2})`

Output Arguments

`cwtstruct` — 2-D CWT

structure

The 2-D CWT, returned as a structure with the following fields:

`wav` — Analyzing wavelet and parameters

structure

Analyzing wavelet and parameters, returned as a structure with the following fields:

- `wname` — name
- `param` — parameters

`wav_norm` — Normalization constants
matrix

Normalization constants, returned as a *M*-by-*N* matrix where *M* is the number of scales and *N* is the number of angles.

`cfs` — CWT coefficients
array

CWT coefficients, returned as an N-D array. The row and column dimensions of the array equal the row and column dimensions of the input data. The third page of the array is equal to 1 or 3 depending on whether the input data is a grayscale or truecolor image. The fourth page of the array is equal to the number of scales and the fifth page of the array is equal to the number of angles.

`scales` — Scales
vector

Scales for the 2-D CWT, returned as a row vector.

`angles` — Angles
vector

Angles for the 2-D CWT, returned as a row vector.

`meanSIG` — Mean
scalar

Mean of the input data, returned as a scalar

See Also
`cwtftinfo2`

Introduced in R2013b

cwtext

Real or complex continuous 1-D wavelet coefficients using extension parameters

Syntax

```
COEFS = cwtext(S,SCALES,'wname')
COEFS = cwtext(S,SCALES,'wname',PropName1,ProVal1, ...)
EXTMODE = struct('Mode',ModeVAL,'Side',SideVAL,'Len',LenVAL);
EXTMODE = {ModeVAL,SideVAL,LenVAL};
COEFS = cwtext(...,'PlotMode',PLOTMODE)
```

Description

`COEFS = cwtext(S,SCALES,'wname')` computes the continuous wavelet coefficients of the vector S at real, positive SCALES, using a wavelet named `'wname'`. The signal S is real; the wavelet can be real or complex.

`COEFS = cwtext(S,SCALES,'wname',PropName1,ProVal1, ...)` computes and plots the continuous wavelet transform coefficients using extra parameters. Valid values for PropName are:

- `'ExtMode'`
- `'ExtSide'`
- `'ExtLen'`
- `'PlotMode'`
- `'xlim'`

The continuous wavelet transform coefficients are computed using the extension parameters:

- `'ExtMode'`
- `'ExtSide'`
- `'ExtLen'`

Valid values for *`ExtMode`* are:

- 'zpd' (zero padding)
- 'sp0' (smooth extension of order 0)
- 'sp1' (smooth extension of order 1)

etc.

Valid values for *ExtSide* are:

- ExtSide = 'l' (or 'u') for left (or up) extension
- ExtSide = 'r' (or 'd') for right (or down) extension
- ExtSide = 'b' for extension on both sides
- ExtSide = 'n' null extension

For the complete list of valid values for ExtMode and ExtSide, see wextend.

ExtLen is the length of extension.

Default values for extension parameters are 'zpd' and 'b'. ExtLen is computed using the maximum of SCALES.

Instead of three parameters, use the following syntaxes:

```
EXTMODE = struct('Mode',ModeVAL,'Side',SideVAL,'Len',LenVAL);
```

```
EXTMODE = {ModeVAL,SideVAL,LenVAL};
```

COEFS = cwtext(...,'PlotMode',PLOTMODE) computes and plots the continuous wavelet transform coefficients. Coefficients are colored using PLOTMODE:

- PLOTMODE = 'lvl' (By scale)
- PLOTMODE = 'glb' (All scales)
- PLOTMODE = 'abslvl' or 'lvlabs' (Absolute value and By scale)
- PLOTMODE = 'absglb' or 'glbabs' (Absolute value and All scales)

You get 3-D plots (surfaces) using the same keywords listed above for the PLOTMODE parameter, preceded by '3D', for example, PLOTMODE = '3Dlvl'.

When PLOTMODE = 'scal' or 'scalCNT' the continuous wavelet transform coefficients and the corresponding scalogram (percentage of energy for each coefficient) are computed.

When PLOTMODE is 'scal', a scaled image of scalogram is displayed. When PLOTMODE is 'scalCNT', a contour representation of scalogram is displayed.

If the XLIM parameter is given, the continuous wavelet transform coefficients are colored using PLOTMODE and XLIM.

XLIM = [x1 x2] with 1 <= x1 < x2 <= length(S).

For each given scale a within the vector SCALES, the wavelet coefficients C(a,b) are computed for b = 1 to ls = length(S), and are stored in COEFS(i,:) if a = SCALES(i).

Output argument COEFS is a la-by-ls matrix where la is the length of SCALES. COEFS is a real or complex matrix depending on the wavelet type.

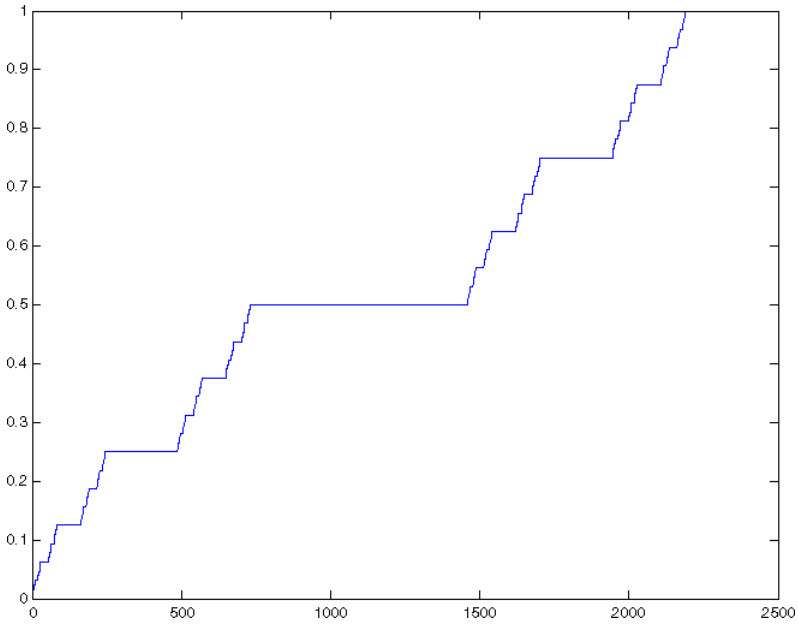
Examples of valid use are as follows:

```
t = linspace(-1,1,512);
s = 1-abs(t);
c = cwtext(s,1:32,'cgau4');
c = cwtext(s,[64 32 16:-2:2],'morl');
c = cwtext(s,[3 18 12.9 7 1.5],'db2');
c = cwtext(s,1:32,'sym2','plotMode','lvl');
c = cwtext(s,1:64,'sym4','plotMode','abslvl','xlim',[100 400]);

[c,Sc] = cwtext(s,1:64,'sym4','plotMode','scal');
[c,Sc] = cwtext(s,1:64,'sym4','plotMode','scalCNT');
[c,Sc] = cwtext(s,1:64,'sym4','plotMode','scalCNT', ...
                              'extMode','sp1');

c = cwtext(s,1:64,'sym4','plotMode','lvl','extMode','sp0');
c = cwtext(s,1:64,'sym4','plotMode','lvl','extMode','sp1');
c = cwtext(s,1:64,'sym4','plotMode','lvl', ...
                         'extMode',{'sp1','b',300});

ext = struct('Mode','sp1','Side','b','Len',300);
c = cwtext(s,1:64,'sym4','plotMode','lvl','extMode',ext);
```

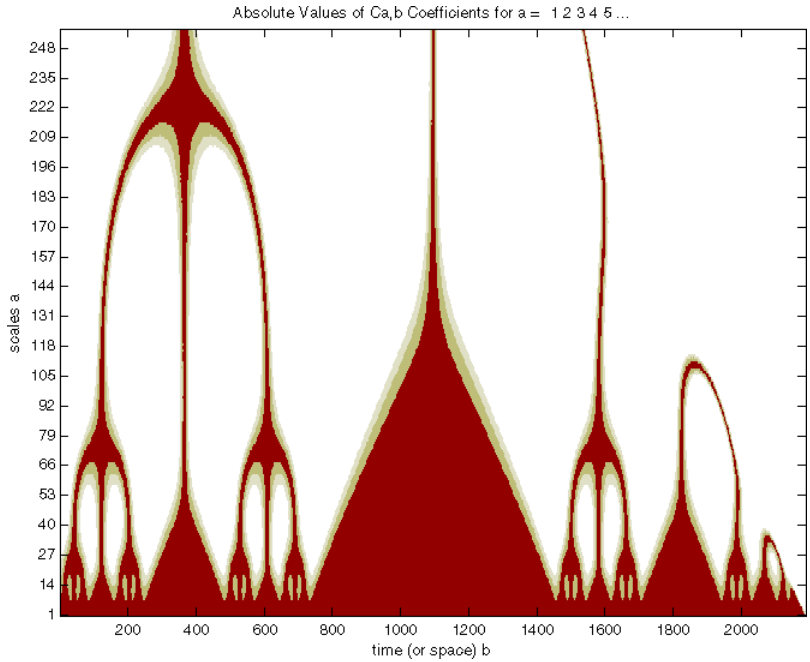
Examples

This example demonstrates the difference between a continuous wavelet transform which deals with signal extension and one which does not.

```
% Load and plot the signal
load wcantor
plot(wcantor)
```

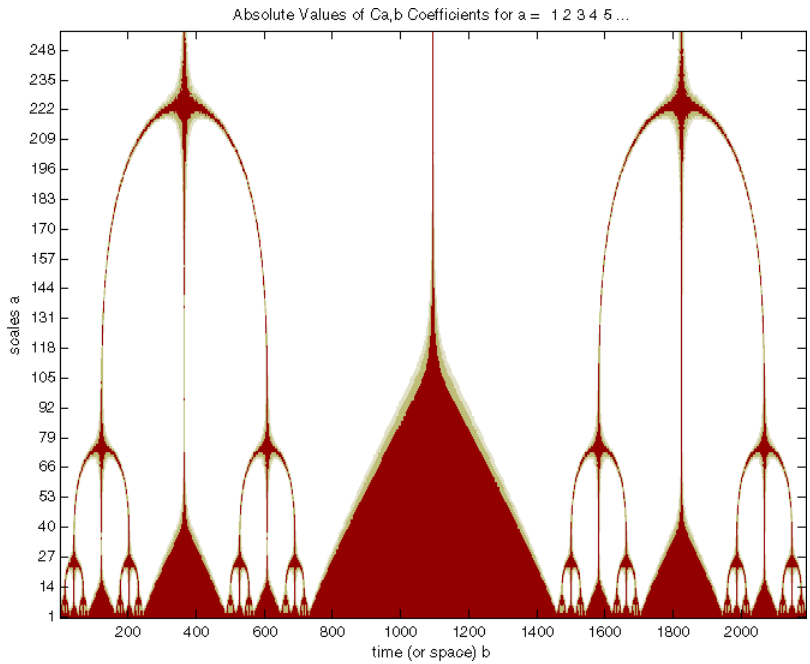



```
% Compute and plot the coefficients
cwt(wcantor,(1:256),'mexh','absglb');
colormap(pink(4))
```



In this figure above, which is produced by the `cwt` function, the values of coefficients are tremendously affected by the boundary effect due to the discontinuity of the signal on the right. The default (zero-padding) extension mode on the right explains this important discontinuity because the last value is 1. On the left there is no effect because the first value is 0.

```
% Compute and plot the coefficients with adapted extension mode
figure;
cwtext(wcantor,(1:256),'mexh','extmode','sp0','extLen',2000, ...
                                'plotMode','absglb');
colormap(pink(4))
```



In this figure, produced by the `cwtext` function, the suitable extension mode of the signal is very efficient, giving as it can be seen, a good result.

See Also

`cwt` | `wavedec` | `wavefun` | `waveinfo` | `wcodemat`

Introduced in R2008a

dbaux

Daubechies wavelet filter computation

Syntax

```
W = dbaux(N,SUMW)
W = dbaux(N)
W = dbaux(N,0)
```

Description

`W = dbaux(N,SUMW)` is the order *N* Daubechies scaling filter such that `sum(W) = SUMW`. Possible values for *N* are 1, 2, 3, ...

Note Instability may occur when *N* is too large.

`W = dbaux(N)` is equivalent to `W = dbaux(N,1)`

`W = dbaux(N,0)` is equivalent to `W = dbaux(N,1)`

Daubechies' Extremal Phase Scaling Filter with Specified Sum

This example shows to determine the Daubechies' extremal phase scaling filter with a specified sum. The two most common values for the sum are $\sqrt{2}$ and 1.

Construct two versions of the `'db4'` scaling filter. One scaling filter sums to $\sqrt{2}$ and the other version sums to 1.

```
NumVanishingMoments = 4;
h = dbaux(NumVanishingMoments,sqrt(2));
m0 = dbaux(NumVanishingMoments,1);
```

The filter with sum equal to $\sqrt{2}$ is the synthesis (reconstruction) filter returned by `wfilters` and used in the discrete wavelet transform.

```
[LoD,HiD,LoR,HiR] = wfilters('db4');
max(abs(LoR-h))
```

```
ans =

   4.2599e-13
```

For orthogonal wavelets, the analysis (decomposition) filter is the time-reverse of the synthesis filter.

```
max(abs(LoD-fliplr(h)))
```

```
ans =

   4.2599e-13
```

Limitations

The computation of the `dbN` Daubechies scaling filter requires the extraction of the roots of a polynomial of order `4N`. Instability may occur when `N` is too large.

More About

Algorithms

The algorithm used is based on a result obtained by Shensa (see "References"), showing a correspondence between the "Lagrange à trous" filters and the convolutional squares of the Daubechies wavelet filters.

The computation of the order *N* Daubechies scaling filter *w* proceeds in two steps: compute a "Lagrange à trous" filter *P*, and extract a square root. More precisely:

- P the associated "Lagrange à trous" filter is a symmetric filter of length 4N-1. P is defined by

P = [a(N) 0 a(N-1) 0 ... 0 a(1) 1 a(1) 0 a(2) 0 ... 0 a(N)]

- where

$$a(k) = \frac{\prod_{\substack{i=-N+1 \\ i \neq k}}^{N} \left(\frac{1}{2} - i\right)}{\prod_{\substack{i=-N+1 \\ i \neq k}}^{N} (k-i)} \quad \text{for } k = 1,\ldots,N$$

- Then, if w denotes db$N$ Daubechies scaling filter of sum $\sqrt{2}$, w is a square root of P:

  P = `conv(wrev(`w`),`w`)` where w is a filter of length 2N.

  The corresponding polynomial has N zeros located at −1 and N−1 zeros less than 1 in modulus.

Note that other methods can be used; see various solutions of the spectral factorization problem in Strang-Nguyen (p. 157).

References

Daubechies, I. (1992), *Ten lectures on wavelets*, CBMS-NSF conference series in applied mathematics, SIAM Ed.

Shensa, M.J. (1992), "The discrete wavelet transform: wedding the a trous and Mallat Algorithms," *IEEE Trans. on Signal Processing*, vol. 40, 10, pp. 2464-2482.

Strang, G.; T. Nguyen (1996), *Wavelets and Filter Banks*, Wellesley-Cambridge Press.

See Also

`dbwavf` | `wfilters`

Introduced before R2006a

dbwavf

Daubechies wavelet filter

Syntax

```
F = dbwavf(W)
```

Description

`F = dbwavf(W)` returns the scaling filter associated with Daubechies wavelet specified by the string *W* where `W = 'dbN'`. Possible values for `N` are 1, 2, 3, ..., 45.

Examples

```
% Set Daubechies wavelet name.
wname = 'db4';

% Compute the corresponding scaling filter.
f = dbwavf(wname)

f =
Columns 1 through 7
0.1629  0.5055  0.4461 -0.0198 -0.1323  0.0218  0.0233
Column 8
-0.0075
```

See Also

`dbaux` | `waveinfo` | `wfilters`

Introduced before R2006a

ddencmp

Default values for denoising or compression

Syntax

```
[THR,SORH,KEEPAPP,CRIT] = ddencmp(IN1,IN2,X)
[THR,SORH,KEEPAPP] = ddencmp(IN1,'wv',X)
[THR,SORH,KEEPAPP,CRIT] = ddencmp(IN1,'wp',X)
```

Description

`ddencmp` returns default values for denoising or compression for the critically-sampled discrete wavelet or wavelet packet transform.

You can use `ddencmp` for 1-D signals or 2-D images.

`[THR,SORH,KEEPAPP,CRIT] = ddencmp(IN1,IN2,X)` returns default values for denoising or compression, using wavelets or wavelet packets, of an input vector or matrix *X*, which can be a one- or two-dimensional signal. `THR` is the threshold, `SORH` is for soft or hard thresholding, `KEEPAPP` allows you to keep approximation coefficients, and `CRIT` (used only for wavelet packets) is the entropy name (see `wentropy` for more information).

`IN1` is `'den'` for denoising or `'cmp'` for compression.

`IN2` is `'wv'` for wavelet or `'wp'` for wavelet packet.

For wavelets (three output arguments):

`[THR,SORH,KEEPAPP] = ddencmp(IN1,'wv',X)` returns default values for denoising (if `IN1 = 'den'`) or compression (if `IN1 = 'cmp'`) of *X*. These values can be used for `wdencmp`.

For wavelet packets (four output arguments):

`[THR,SORH,KEEPAPP,CRIT] = ddencmp(IN1,'wp',X)` returns default values for denoising (if `IN1 = 'den'`) or compression (if `IN1 = 'cmp'`) of *X*. These values can be used for `wdencmp`.

Examples

Default Global Threshold for Wavelet Denoising

Determine the default global denoising threshold for an N(0,1) white noise input.

Create an N(0,1) white noise input. Set the random number generator to the default initial settings for reproducible results.

```
dwtmode('per');
rng default;
x = randn(512,1);
```

Use `ddencmp` to obtain the default global threshold for wavelet denoising. Demonstrate that the threshold is equal to the universal threshold of Donoho and Johnstone scaled by a robust estimate of the variance.

```
[thr,sorh,keepapp] = ddencmp('den','wv',x);
[A,D] = dwt(x,'db1');
noiselev = median(abs(D))/0.6745;
thresh = sqrt(2*log(length(x)))*noiselev;
```

Compare the value of the variable `thr` to the value of `thresh`.

Default Global Threshold for Wavelet Packet Compression

Determine the default global compression threshold for an N(0,1) white noise input.

Create an N(0,1) white noise input. Set the random number generator to the default initial settings for reproducible results.

```
dwtmode('per');
rng default;
x = randn(512,1);
```

Use `ddencmp` with the `'cmp'` and `'wp'` input arguments to return the default global compression threshold for a wavelet packet transform.

```
[thr,sorh,keepapp,crit] = ddencmp('den','wp',x);
```

References

Donoho, D.L. (1995), "De-noising by soft-thresholding," *IEEE, Trans. on Inf. Theory*, 41, 3, pp. 613–627.

Donoho, D.L.; I.M. Johnstone (1994), "Ideal spatial adaptation by wavelet shrinkage," *Biometrika*, vol 81, pp. 425–455.

Donoho, D.L.; I.M. Johnstone (1994), "Ideal de-noising in an orthonormal basis chosen from a library of bases," *C.R.A.S. Paris*, *Ser. I*, t. 319, pp. 1317–1322.

See Also

`wdencmp` | `wenergy` | `wpdencmp`

Introduced before R2006a

dddtree

Dual-tree and double-density 1-D wavelet transform

Syntax

```
wt = dddtree(typetree,x,level,fdf,df)
wt = dddtree(typetree,x,level,fname)
wt = dddtree(typetree,x,level,fname1,fname2)
```

Description

`wt = dddtree(typetree,x,level,fdf,df)` returns the `typetree` discrete wavelet transform (DWT) of the 1-D input signal, `x`, down to level, `level`. The wavelet transform uses the decomposition (analysis) filters, `fdf`, for the first level and the analysis filters, `df`, for subsequent levels. Supported wavelet transforms are the critically sampled DWT, double-density, dual-tree complex, and dual-tree double-density complex wavelet transform. The critically sampled DWT is a filter bank decomposition in an orthogonal or biorthogonal basis (nonredundant). The other wavelet transforms are oversampled filter banks.

`wt = dddtree(typetree,x,level,fname)` uses the filters specified by `fname` to obtain the wavelet transform. Valid filter specifications depend on the type of wavelet transform. See `dtfilters` for details.

`wt = dddtree(typetree,x,level,fname1,fname2)` uses the filters specified in `fname1` for the first stage of the dual-tree wavelet transform and the filters specified in `fname2` for subsequent stages of the dual-tree wavelet transform. Specifying different filters for stage 1 is valid and necessary only when `typetree` is `'cplxdt'` or `'cplxdddt'`.

Examples

Complex Dual-Tree Wavelet Transform

Obtain the complex dual-tree wavelet transform of the noisy Doppler signal. The FIR filters in the first and subsequent stages result in an approximately analytic wavelet as required.

Create the first-stage analysis filters for the two trees.

```
  Faf{1} = [0         0
   -0.0884   -0.0112
    0.0884    0.0112
    0.6959    0.0884
    0.6959    0.0884
    0.0884   -0.6959
   -0.0884    0.6959
    0.0112   -0.0884
    0.0112   -0.0884
         0         0];
Faf{2} = [ 0.0112  0
    0.0112         0
   -0.0884   -0.0884
    0.0884   -0.0884
    0.6959    0.6959
    0.6959   -0.6959
    0.0884    0.0884
   -0.0884    0.0884
         0    0.0112
         0   -0.0112];
```

Create the analysis filters for subsequent stages of the multiresolution analysis.

```
af{1} = [ 0.0352         0
         0         0
   -0.0883   -0.1143
    0.2339         0
    0.7603    0.5875
    0.5875   -0.7603
         0    0.2339
   -0.1143    0.0883
         0         0
         0   -0.0352];
```

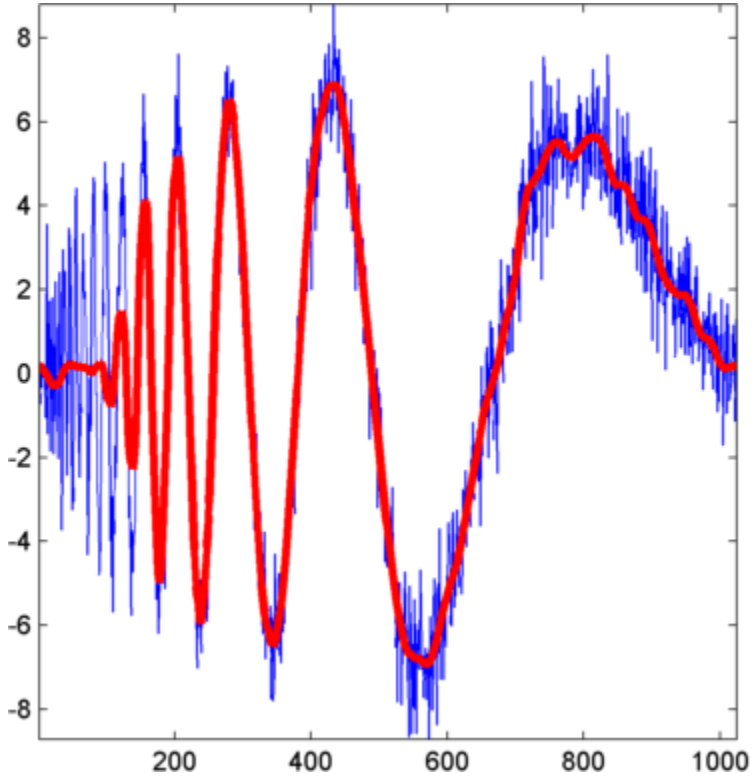
```
af{2} = [0   -0.0352
         0         0
   -0.1143    0.0883
         0    0.2339
    0.5875   -0.7603
    0.7603    0.5875
    0.2339         0
   -0.0883   -0.1143
         0         0
    0.0352         0];
```

Load the noisy Doppler signal and obtain the complex dual-tree wavelet transform down to level 4.

```
load noisdopp;
wt = dddtree('cplxdt',noisdopp,4,Faf,af);
```

Plot an approximation based on the level-four approximation coefficients.

```
xapp = dddtreecfs('r',wt,'scale',{5});
plot(noisdopp);  hold on;
plot(cell2mat(xapp),'r','linewidth',3);
axis tight;
```



Double-Density Wavelet Transform

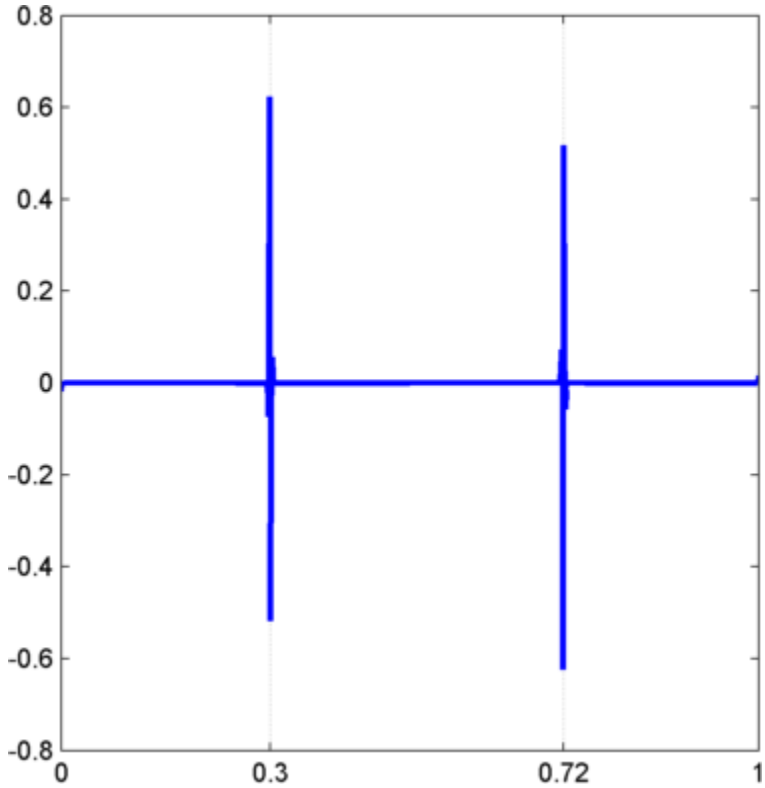
Obtain the double-density wavelet transform of a signal with two discontinuities. Use the level-one detail coefficients to localize the discontinuities.

Create a signal consisting of a 2-Hz sine wave with a duration of 1 second. The sine wave has discontinuities at 0.3 and 0.72 seconds.

```
N = 1024;
t = linspace(0,1,1024);
x = 4*sin(4*pi*t);
x = x - sign(t - .3) - sign(.72 - t);
plot(t,x); xlabel('t'); ylabel('x');
title('Original Signal');
```

Obtain the double-density wavelet transform of the signal, reconstruct an approximation based on the level-one detail coefficients, and plot the result.

```
wt = dddtree('ddt',x,1,'filters1');
wt.cfs{2} = zeros(1,512);
xrec = idddtree(wt);
plot(t,xrec,'linewidth',2)
set(gca,'xtick',[0 0.3 0.72 1]); set(gca,'xgrid','on');
```



First-Level Detail Coefficients Approximation — Complex Dual-Tree

Obtain the complex dual-tree wavelet transform of a signal with two discontinuities. Use the first-level detail coefficients to localize the discontinuities.

Create a signal consisting of a 2-Hz sine wave with a duration of 1 second. The sine wave has discontinuities at 0.3 and 0.72 seconds.

```
N = 1024;
```

```
t = linspace(0,1,1024);
x = 4*sin(4*pi*t);
x = x - sign(t - .3) - sign(.72 - t);
plot(t,x); xlabel('t'); ylabel('x');
title('Original Signal');
```

Obtain the dual-tree wavelet transform of the signal, reconstruct an approximation based on the level-one detail coefficients, and plot the result.

```
wt = dddtree('cplxdt',x,1,'FSfarras','qshift06');
wt.cfs{2} = zeros(1,512,2);
xrec = idddtree(wt);
plot(t,xrec,'linewidth',2)
set(gca,'xtick',[0 0.3 0.72 1]); set(gca,'xgrid','on');
```

- "Analytic Wavelets Using the Dual-Tree Wavelet Transform"

Input Arguments

`typetree` — Type of wavelet decomposition

`'dwt' | 'ddt' | 'cplxdt' | 'cplxdddt'`

Type of wavelet decomposition, specified as one of `'dwt'`, `'ddt'`, `'cplxdt'`, or `'cplxdddt'`. The type, `'dwt'`, gives a critically sampled (nonredundant) discrete wavelet transform. The other decomposition types produce oversampled wavelet transforms. `'ddt'` produces a double-density wavelet transform. `'cplxdt'` produces a dual-tree complex wavelet transform. `'cplxdddt'` produces a double-density dual-tree complex wavelet transform.

`x` — Input signal

vector

Input signal, specified as an even-length row or column vector. If L is the value of the `level` of the wavelet decomposition, 2^L must divide the length of `x`. Additionally, the length of the signal must be greater than or equal to the product of the maximum length of the decomposition (analysis) filters and $2^{(L-1)}$.

Data Types: `double`

`level` — Level of wavelet decomposition

positive integer

Level of the wavelet decomposition, specified as an integer. If L is the value of `level`, 2^L must divide the length of `x` . Additionally, the length of the signal must be greater than or equal to the product of the maximum length of the decomposition (analysis) filters and $2^{(L-1)}$.

Data Types: `double`

`fdf` — Level-one analysis filters

matrix | cell array

The level-one analysis filters, specified as a matrix or cell array of matrices. Specify `fdf` as a matrix when `typetree` is `'dwt'` or `'ddt'`. The size and structure of the matrix depend on the `typetree` input as follows:

- `'dwt'` — This is the critically sampled discrete wavelet transform. In this case, `fdf` is a two-column matrix with the lowpass (scaling) filter in the first column and the highpass (wavelet) filter in the second column.
- `'ddt'` — This is the double-density wavelet transform. The double-density DWT is a three-channel perfect reconstruction filter bank. `fdf` is a three-column matrix with the lowpass (scaling) filter in the first column and the two highpass (wavelet) filters in the second and third columns. In the double-density wavelet transform, the single lowpass and two highpass filters constitute a three-channel perfect reconstruction filter bank. This is equivalent to the three filters forming a tight frame. You cannot arbitrarily choose the two wavelet filters in the double-density DWT. The three filters together must form a tight frame.

Specify `fdf` as a 1-by-2 cell array of matrices when `typetree` is a dual-tree transform, `'cplxdt'` or `'cplxdddt'`. The size and structure of the matrix elements depend on the `typetree` input as follows:

- For the dual-tree complex wavelet transform, `'cplxdt'`, `fdf{1}` is a two-column matrix containing the lowpass (scaling) filter and highpass (wavelet) filters for the first tree. The scaling filter is the first column and the wavelet filter is the second column. `fdf{2}` is a two-column matrix containing the lowpass (scaling) and highpass (wavelet) filters for the second tree. The scaling filter is the first column and the wavelet filter is the second column.
- For the double-density dual-tree complex wavelet transform, `'cplxdddt'`, `fdf{1}` is a three-column matrix containing the lowpass (scaling) and two highpass (wavelet) filters for the first tree and `fdf{2}` is a three-column matrix containing the lowpass (scaling) and two highpass (wavelet) filters for the second tree.

Data Types: `double`

`df` — Analysis filters for levels > 1

matrix | cell array

Analysis filters for levels > 1, specified as a matrix or cell array of matrices. Specify `df` as a matrix when `typetree` is `'dwt'` or `'ddt'`. The size and structure of the matrix depend on the `typetree` input as follows:

- `'dwt'` — This is the critically sampled discrete wavelet transform. In this case, `df` is a two-column matrix with the lowpass (scaling) filter in the first column and the highpass (wavelet) filter in the second column. For the critically sampled orthogonal or biorthogonal DWT, the filters in `df` and `fdf` must be identical.
- `'ddt'` — This is the double-density wavelet transform. The double-density DWT is a three-channel perfect reconstruction filter bank. `df` is a three-column matrix with the lowpass (scaling) filter in the first column and the two highpass (wavelet) filters in the second and third columns. In the double-density wavelet transform, the single lowpass and two highpass filters must constitute a three-channel perfect reconstruction filter bank. This is equivalent to the three filters forming a tight frame. For the double-density DWT, the filters in `df` and `fdf` must be identical.

Specify `df` as a 1-by-2 cell array of matrices when `typetree` is a dual-tree transform, `'cplxdt'` or `'cplxdddt'`. For dual-tree transforms, the filters in `fdf` and `df` must be different. The size and structure of the matrix elements in the cell array depend on the `typetree` input as follows:

- For the dual-tree complex wavelet transform, `'cplxdt'`, `df{1}` is a two-column matrix containing the lowpass (scaling) and highpass (wavelet) filters for the first tree. The scaling filter is the first column and the wavelet filter is the second column. `df{2}` is a two-column matrix containing the lowpass (scaling) and highpass (wavelet) filters for the second tree. The scaling filter is the first column and the wavelet filter is the second column.
- For the double-density dual-tree complex wavelet transform, `'cplxdddt'`, `df{1}` is a three-column matrix containing the lowpass (scaling) and two highpass (wavelet) filters for the first tree and `df{2}` is a three-column matrix containing the lowpass (scaling) and two highpass (wavelet) filters for the second tree.

Data Types: `double`

`fname` — Filter name

string

Filter name, specified as a string. For the critically sampled DWT, specify any valid orthogonal or biorthogonal wavelet filter. See `wfilters` for details. For the double-density wavelet transform, `'ddt'`, valid choices are `'filters1'` and `'filters2'`. For the complex dual-tree wavelet transform, valid choices are `'dtfP'` with P = 1, 2, 3, 4. For the double-density dual-tree wavelet transform, the only valid choice is `'dddtf1'`. See `dtfilters` for more details on valid filter strings for the oversampled wavelet filter banks.

Data Types: `char`

`fname1` — First-stage filter name

string

First-stage filter name, specified as a string. Specifying a different filter for the first stage is valid and necessary only in the dual-tree transforms, `'cplxdt'` and `'cplxddt'`. In the complex dual-tree wavelet transform, you can use any valid wavelet filter for the first stage. In the double-density dual-tree wavelet transform, the first-stage filters must form a three-channel perfect reconstruction filter bank.

Data Types: `char`

`fname2` — Filter name for stages > 1

string

Filter name for stages > 1, specified as a string. You must specify a first-level filter that is different from the wavelet and scaling filters in subsequent levels when using the dual-tree wavelet transforms, `'cplxdt'` or `'cplxdddt'`. See `dtfilters` for valid choices.

Data Types: `char`

Output Arguments

`wt` — Wavelet transform

structure

Wavelet transform, returned as a structure with these fields:

`type` — Type of wavelet decomposition (filter bank)

`'dwt' | 'ddt' | 'cplxdt' | 'cplxdddt'`

Type of wavelet decomposition (filter bank) used in the analysis, returned as one of `'dwt'`, `'ddt'`, `'cplxdt'`, or `'cplxdddt'`. The type, `'dwt'`, gives a critically

sampled discrete wavelet transform. The other types correspond to oversampled wavelet transforms. `'ddt'` is a double-density wavelet transform, `'cplxdt'` is a dual-tree complex wavelet transform, and `'cplxdddt'` is a double-density dual-tree complex wavelet transform.

`level` — Level of the wavelet decomposition

positive integer

Level of wavelet decomposition, returned as a positive integer.

`filters` — Decomposition (analysis) and reconstruction (synthesis) filters

structure

Decomposition (analysis) and reconstruction (synthesis) filters, returned as a structure with these fields:

`Fdf` — First-stage analysis filters

matrix | cell array

First-stage analysis filters, returned as an *N*-by-2 or *N*-by-3 matrix for single-tree wavelet transforms, or a cell array of two *N*-by-2 or *N*-by-3 matrices for dual-tree wavelet transforms. The matrices are *N*-by-3 for the double-density wavelet transforms. For an *N*-by-2 matrix, the first column of the matrix is the scaling (lowpass) filter and the second column is the wavelet (highpass) filter. For an *N*-by-3 matrix, the first column of the matrix is the scaling (lowpass) filter and the second and third columns are the wavelet (highpass) filters. For the dual-tree transforms, each element of the cell array contains the first-stage analysis filters for the corresponding tree.

`Df` — Analysis filters for levels > 1

matrix | cell array

Analysis filters for levels > 1, returned as an *N*-by-2 or *N*-by-3 matrix for single-tree wavelet transforms, or a cell array of two *N*-by-2 or *N*-by-3 matrices for dual-tree wavelet transforms. The matrices are *N*-by-3 for the double-density wavelet transforms. For an *N*-by-2 matrix, the first column of the matrix is the scaling (lowpass) filter and the second column is the wavelet (highpass) filter. For an *N*-by-3 matrix, the first column of the matrix is the scaling (lowpass) filter and the second and third columns are the wavelet (highpass) filters. For the dual-tree transforms, each element of the cell array contains the analysis filters for the corresponding tree.

`Frf` — First-level reconstruction filters

matrix | cell array

First-level reconstruction filters, returned as an *N*-by-2 or *N*-by-3 matrix for single-tree wavelet transforms, or a cell array of two *N*-by-2 or *N*-by-3 matrices for dual-tree wavelet transforms. The matrices are *N*-by-3 for the double-density wavelet transforms. For an *N*-by-2 matrix, the first column of the matrix is the scaling (lowpass) filter and the second column is the wavelet (highpass) filter. For an *N*-by-3 matrix, the first column of the matrix is the scaling (lowpass) filter and the second and third columns are the wavelet (highpass) filters. For the dual-tree transforms, each element of the cell array contains the first-stage synthesis filters for the corresponding tree.

`Rf` — Reconstruction filters for levels > 1

matrix | cell array

Reconstruction filters for levels > 1, returned as an *N*-by-2 or *N*-by-3 matrix for single-tree wavelet transforms, or a cell array of two *N*-by-2 or *N*-by-3 matrices for dual-tree wavelet transforms. The matrices are *N*-by-3 for the double-density wavelet transforms. For an *N*-by-2 matrix, the first column of the matrix is the scaling (lowpass) filter and the second column is the wavelet (highpass) filter. For an *N*-by-3 matrix, the first column of the matrix is the scaling (lowpass) filter and the second and third columns are the wavelet (highpass) filters. For the dual-tree transforms, each element of the cell array contains the synthesis filters for the corresponding tree.

`cfs` — Wavelet transform coefficients

cell array of matrices

Wavelet transform coefficients, returned as a 1-by-(`level`+1) cell array of matrices. The size and structure of the matrix elements of the cell array depend on the type of wavelet transform, `typetree`, as follows:

- `'dwt'` — `cfs{j}`
  - j = 1,2,... `level` is the level.
  - `cfs{level+1}` are the lowpass, or scaling, coefficients.
- `'ddt'` — `cfs{j}(:,:,k)`
  - j = 1,2,... `level` is the level.
  - k = 1,2 is the wavelet filter.
  - `cfs{level+1}(:,:)` are the lowpass, or scaling, coefficients.
- `'cplxdt'` — `cfs{j}(:,:,m)`
  - j = 1,2,... `level` is the level.

- m = 1,2 are the real and imaginary parts.
- `cfs{level+1}(:,:)` are the lowpass, or scaling, coefficients.

- `'cplxdddt'` — `cfs{j}(:,:,k,m)`
  - j = 1,2,... `level` is the level.
  - k = 1,2 is the wavelet filter.
  - m = 1,2 are the real and imaginary parts.
  - `cfs{level+1}(:,:)` are the lowpass, or scaling, coefficients.

More About

- "Critically Sampled and Oversampled Wavelet Filter Banks"

See Also

`dddtree2` | `dddtreecfs` | `dtfilters` | `idddtree`

Introduced in R2013b

dddtreecfs

Extract dual-tree/double-density wavelet coefficients or projections

Syntax

```
out = dddtreecfs(outputtype,wt,outputspec,outputindices)
out = dddtreecfs(outputtype,wt,outputspec,outputindices,'plot')
```

Description

`out = dddtreecfs(outputtype,wt,outputspec,outputindices)` extracts the coefficients or subspace projections from the 1-D or 2-D wavelet decomposition, `wt`. If `outputtype` equals `'e'`, `out` contains wavelet or scaling coefficients. If `outputtype` equals `'r'`, `out` contains wavelet or scaling subspace projections (reconstructions).

`out = dddtreecfs(outputtype,wt,outputspec,outputindices,'plot')` plots the signal or image reconstruction or specified analysis coefficients. You can include the `'plot'` option anywhere after the `wt` input.

Examples

Reconstruction from 1-D Complex Dual-Tree Wavelet Transform

Obtain the complex dual-tree wavelet transform of the 1-D noisy Doppler signal. Reconstruct an approximation based on the level-three detail coefficients

Load the noisy Doppler signal. Obtain the complex dual-tree transform down to level 3.

```
load noisdopp;
wt = dddtree('cplxdt',noisdopp,3,'dtf1');
```

Plot a reconstruction of the original signal based on the level-three detail coefficients

```
xr = dddtreecfs('r',wt,'scale',{3},'plot');
```

Coefficients from 1-D Complex Dual-Tree Wavelet Transform

Load the noisy Doppler signal. Obtain the complex dual-tree transform down to level 3.

```
load noisdopp;
wt = dddtree('cplxdt',noisdopp,3,'dtf1');
```

Create a cell array of vectors to obtain the second- and third-level detail coefficients from each of the wavelet filter bank trees.

```
outputindices = {[2 1]; [2 2]; [3 1]; [3 2]};
```

The first element of each vector in the cell array denotes the level, or stage. The second element denotes the tree.

Extract the detail coefficients.

```
out = dddtreecfs('e',wt,'ind',outputindices);
```

`out` is a 1-by-4 cell array. The cell array elements contain the wavelet coefficients corresponding to the elements in `outputindices`. For example, `out{1}` contains the level-two detail coefficients from the first tree.

1-D Complex Dual-Tree Wavelet Transform Structure

Load the noisy Doppler signal. Obtain the complex dual-tree transform down to level 3.

```
load noisdopp;
wt = dddtree('cplxdt',noisdopp,3,'dtf1');
```

Create a cell array of vectors to obtain the second- and third-level detail coefficients from each of the wavelet filter bank trees.

```
outputindices = {[2 1]; [2 2]; [3 1];[3 2]};
```

The first element of each vector in the cell array denotes the level, or stage. The second element denotes the tree.

Create a structure array identical to the `wt` output of `dddtree` with all the coefficients equal to zero except the first- and second-level detail coefficients.

```
out = dddtreecfs('e',wt,'cumind',outputindices);
```

Extract Diagonal Features from Image

Use the complex dual-tree wavelet transform to isolate diagonal features in an image at +45 and –45 degrees.

Load and display the `xbox` image.

```
load xbox;
imagesc(xbox)
```

Obtain the complex dual-tree wavelet transform down to level 3.

```
fdf = dtfilters('FSfarras');
df = dtfilters('qshift1O');
wt = dddtree2('cplxdt',xbox,3,fdf,df);
```

Isolate the +45 and -45 diagonal image features in the level-one wavelet coefficients. Plot the result.

```
out = dddtreecfs('e',wt,'ind',{[1 3 1 2]; [1 3 2 2]},'plot');
```

Input Arguments

`outputtype` — Output type

`'e' | 'r'`

Output type, specified as `'e'` or `'r'`. Use `'e'` to obtain the scaling or wavelet coefficients. Use `'r'` to obtain a projection, or reconstruction, onto the appropriate scaling or wavelet subspace.

`wt` — Wavelet transform

structure

Wavelet transform, specified as a structure. The structure array is the output of `dddtree` or `dddtree2`.

`outputspec` — Output specification

`'lowpass' | 'scale' | 'ind' | 'cumind'`

Output specification, specified as one of `'lowpass'`, `'scale'`, `'ind'`, or `'cumind'`. The output specifications are defined as follows:

- `'lowpass'` — Outputs the lowpass, or scaling, coefficients or a signal/image approximation based on the scaling coefficients. If you set the output specification to `'lowpass'`, do not specify `outputindices`. If the `outputtype` is `'e'`, `out` is a structure array with fields identical to the input structure array `wt` except that all wavelet (detail) coefficients are equal to zero. If the `outputtype` is `'r'`, `out` is a signal or image approximation based on the scaling coefficients. The signal or image approximation is equal in size to the original input to `dddtree` or `dddtree2`.

- `'scale'` — Outputs the coefficients or a signal/image approximation based on the scales specified in `outputindices`. If the `outputtype` is `'e'`, `out` is a cell array of structure arrays. The fields of the structure arrays in `out` are identical to the fields of the input structure array `wt`. The coefficients in the `cfs` field are all equal to zero except the coefficients corresponding to the scales in `outputindices`. If the `outputtype` is `'r'`, `out` is a signal or image approximation based on the scales in `outputindices`. The signal or image approximation is equal in size to the original input to `dddtree` or `dddtree2`.
- `'ind'` — Outputs the coefficients or a signal/image approximation based on the tree-position indices specified in `outputindices`. If the `outputtype` is `'e'`, `out` is a cell array of vectors or matrices containing the coefficients specified by the tree-position indices in `outputindices`. If the `outputtype` is `'r'`, `out` is a cell array of vectors or matrices containing signal or image approximations based on the corresponding tree-position indices in `outputindices`.
- `'cumind'` — Outputs the coefficients or a signal/image approximation based on the tree-position indices specified in `outputindices`. If the `outputtype` is `'e'`, `out` is a structure array. The fields of the structure array are identical to the fields of the input structure array `wt`. The coefficients in the `cfs` field are all equal to zero except the coefficients corresponding to the tree positions in `outputindices`. If the `outputtype` is `'r'`, `out` is a signal or image approximation based on the coefficients corresponding to the tree-position indices in `outputindices`.

Example: `'ind',{[1 1]; [1 2]}`

`outputindices` — Output indices

cell array

Output indices, specified as a cell array with scalar or vector elements. If `outputspec` equals `'scale'`, a scalar element selects the corresponding element in the `cfs` field of `wt`. If `outputspec` equals `'ind'` or `'cumind'`, the elements of `outputspec` are row vectors. The first element of the row vector corresponds to the element in the `cfs` field of `wt`. Subsequent elements in the row vector correspond to the indices of the array contained in the cell array element.

Example: `'scale',{1;2;3}`

Output Arguments

`out` — Signal or image reconstruction or coefficients

cell array | structure | vector | matrix

Signal or image reconstruction or coefficients, returned as a vector, matrix, structure array, cell array of vectors or matrices, or cell array of structure arrays. The form of `out` depends on the value of `outputspec` and `outputindices`.

See Also

`dddtree` | `dddtree2` | `plotdt`

Introduced in R2013b

dddtree2

Dual-tree and double-density 2-D wavelet transform

Syntax

```
wt = dddtree2(typetree,x,level,fdf,df)
wt = dddtree2(typetree,x,level,fname)
wt = dddtree2(typetree,x,level,fname1,fname2)
```

Description

`wt = dddtree2(typetree,x,level,fdf,df)` returns the `typetree` discrete wavelet transform of the 2-D input image, `x`, down to level, `level`. The wavelet transform uses the decomposition (analysis) filters, `fdf`, for the first level and the analysis filters, `df`, for subsequent levels. Supported wavelet transforms are the critically sampled DWT, double-density, real oriented dual-tree, complex oriented dual-tree, real oriented dual-tree double-density, and complex oriented dual-tree double-density wavelet transform. The critically sampled DWT is a filter bank decomposition in an orthogonal or biorthogonal basis (nonredundant). The other wavelet transforms are oversampled filter banks with differing degrees of directional selectivity.

`wt = dddtree2(typetree,x,level,fname)` uses the filters specified by `fname` to obtain the wavelet transform. Valid filter specifications depend on the type of wavelet transform. See `dtfilters` for details.

`wt = dddtree2(typetree,x,level,fname1,fname2)` uses the filters specified in `fname1` for the first stage of the dual-tree wavelet transform and the filters specified in `fname2` for subsequent stages of the dual-tree wavelet transform. Specifying different filters for stage 1 is valid and necessary only when `typetree` is `'realdt'`, `'cplxdt'`, `'realdddt'`, or `'cplxdddt'`.

Examples

Real Oriented Dual-Tree Wavelets

Visualize the six directional wavelets of the real oriented dual-tree wavelet transform.

Create the first-stage analysis filters for the two trees.

```
  Faf{1} = [0         0
   -0.0884   -0.0112
    0.0884    0.0112
    0.6959    0.0884
    0.6959    0.0884
    0.0884   -0.6959
   -0.0884    0.6959
    0.0112   -0.0884
    0.0112   -0.0884
         0         0];
Faf{2} = [ 0.0112  0
    0.0112         0
   -0.0884   -0.0884
    0.0884   -0.0884
    0.6959    0.6959
    0.6959   -0.6959
    0.0884    0.0884
   -0.0884    0.0884
         0    0.0112
         0   -0.0112];
```

Create the analysis filters for subsequent stages of the multiresolution analysis.

```
af{1} = [ 0.0352         0
         0         0
   -0.0883   -0.1143
    0.2339         0
    0.7603    0.5875
    0.5875   -0.7603
         0    0.2339
   -0.1143    0.0883
         0         0
         0   -0.0352];

af{2} = [0   -0.0352
         0         0
   -0.1143    0.0883
         0    0.2339
    0.5875   -0.7603
    0.7603    0.5875
    0.2339         0
   -0.0883   -0.1143
         0         0
```

```
    0.0352         0];
```

Obtain the real dual-tree wavelet transform of an image of zeros down to level 4.

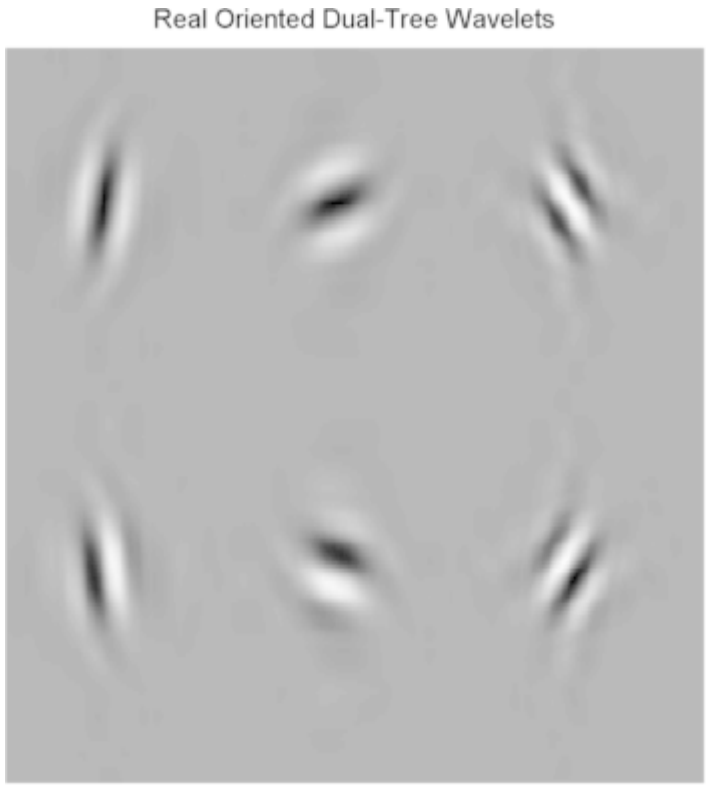
```
J = 4;
L = 3*2^(J+1);
N = L/2^J;
x = zeros(2*L,3*L);
wt = dddtree2('realdt',x,J,Faf,af);
```

Insert a 1 in one position of the six subbands and invert the wavelet transform.

```
wt.cfs{4}(N/2,N/2+0*N,1,1) = 1;
wt.cfs{4}(N/2,N/2+1*N,2,1) = 1;
wt.cfs{4}(N/2,N/2+2*N,3,1) = 1;
wt.cfs{4}(N/2+N,N/2+0*N,1,2) = 1;
wt.cfs{4}(N/2+N,N/2+1*N,2,2) = 1;
wt.cfs{4}(N/2+N,N/2+2*N,3,2) = 1;
xrec = idddtree2(wt);
```

Visualize the six directional wavelets.

```
imagesc(xrec);
colormap gray; axis off;
title('Real Oriented Dual-Tree Wavelets')
```

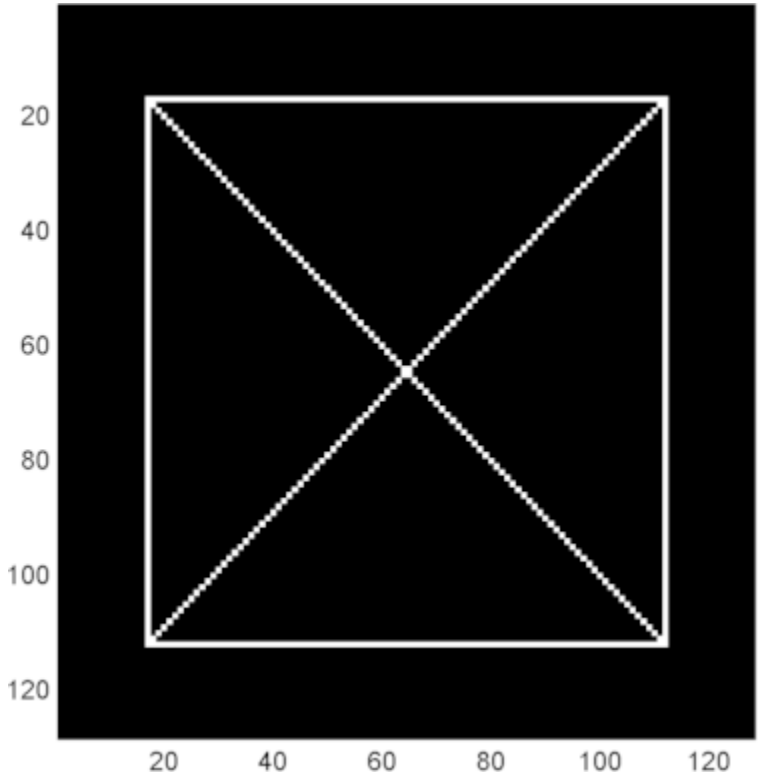


Double-Density Wavelet Transform

Obtain the double-density wavelet transform of an image.

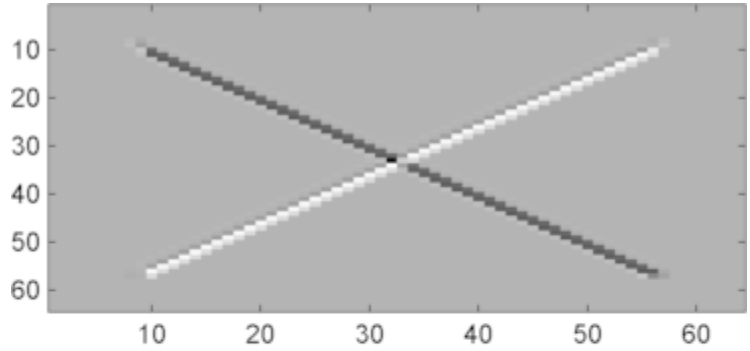
Load the image and obtain the double-density wavelet transform.

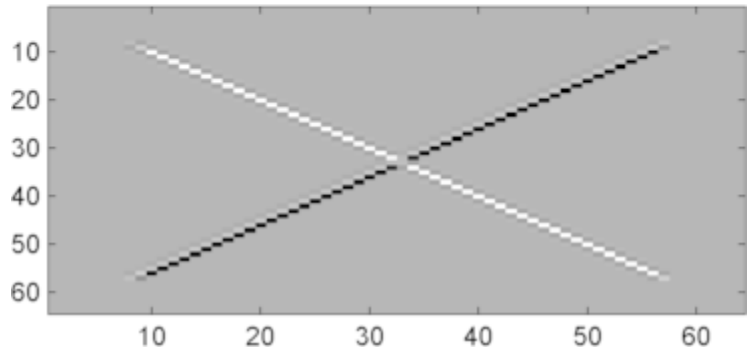
```
load xbox;
imagesc(xbox); colormap gray;
wt = dddtree2('ddt',xbox,1,'filters1');
```



Visualize the diagonal details in the two wavelet HH subbands.

```
HH1 = wt.cfs{1}(:,:,5);
HH2 = wt.cfs{1}(:,:,8);
subplot(211)
imagesc(HH1);
colormap gray;
subplot(212);
imagesc(HH2);
```



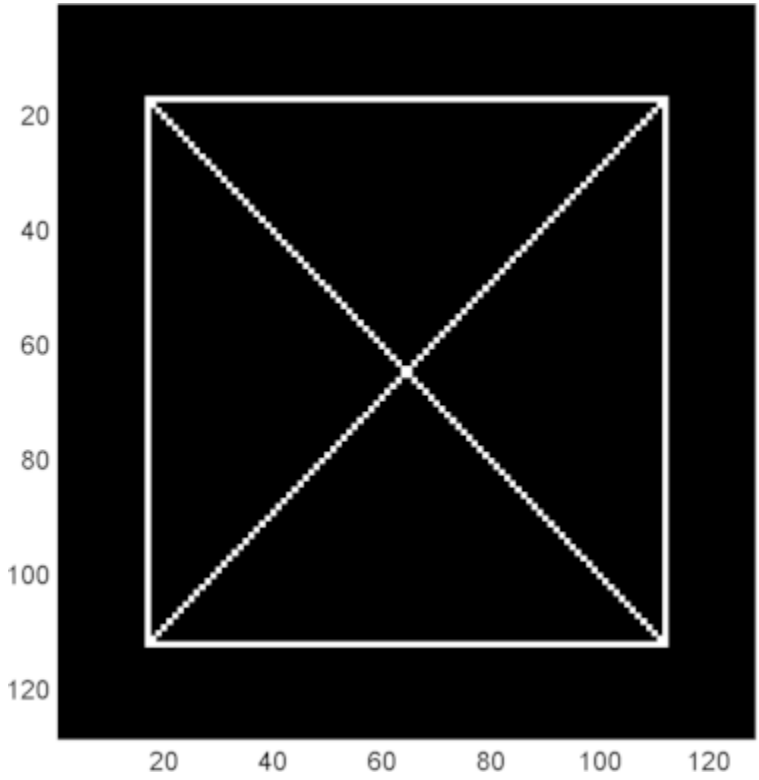


Complex Dual-Tree Wavelet Transform

Obtain the complex dual-tree wavelet transform of an image. Show that the complex dual-tree wavelet transform can detect the two different diagonal directions.

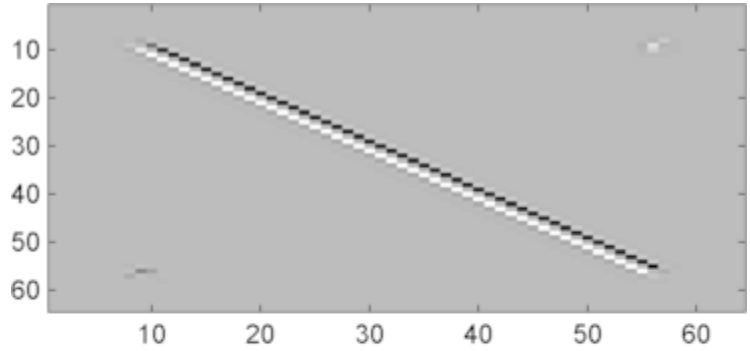
Load the image and obtain the complex dual-tree wavelet transform.

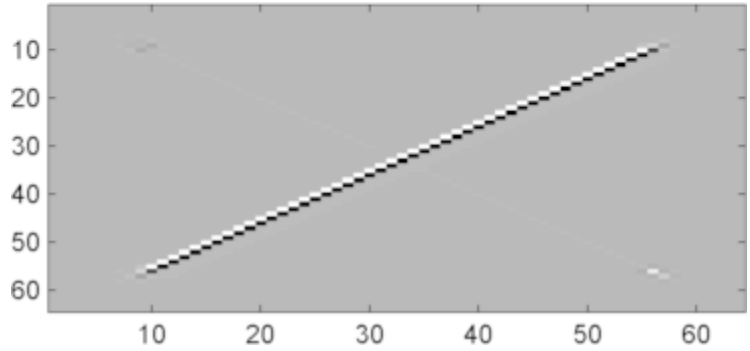
```
load xbox;
imagesc(xbox); colormap gray;
wt = dddtree2('cplxdt',xbox,1,'FSfarras','qshift10');
```



Obtain and display the imaginary parts of the 2 trees.

```
waveletcfs = wt.cfs{1};
subplot(211)
imagesc(waveletcfs(:,:,3,1,2));
colormap gray;
subplot(212)
imagesc(waveletcfs(:,:,3,2,2));
```





- “Analytic Wavelets Using the Dual-Tree Wavelet Transform”

Input Arguments

`typetree` — Type of wavelet decomposition

`'dwt' | 'ddt' | 'realdt' | 'cplxdt' | 'realdddt' | 'cplxdddt'`

Type of wavelet decomposition, specified as one of `'dwt'`, `'ddt'`, `'realdt'`, `'cplxdt'`, `'realdddt'`, or `'cplxdddt'`. The type, `'dwt'`, produces a critically sampled (nonredundant) discrete wavelet transform. The other decomposition types produce oversampled wavelet transforms. `'ddt'` produces a double-density wavelet transform with one scaling and two wavelet filters for both row and column filtering. The double-density wavelet transform uses the same filters at all stages. `'realdt'` and `'cplxdt'` produce oriented dual-tree wavelet transforms consisting of two and four separable

wavelet transforms. `'realdddt'` and `'cplxdddt'` produce double-density dual-tree wavelet transforms. The dual-tree wavelet transforms use different filters for the first stage (level).

`x` — Input image

matrix

Input image, specified as a matrix with even-length row and column dimensions. Both the row and column dimensions must be divisible by 2^L, where L is the level of the wavelet transform. Additionally, the minimum of the row and column dimensions of the image must be greater than or equal to the product of the maximum length of the decomposition (analysis) filters and $2^{(L-1).}$

Data Types: `double`

`level` — Level of wavelet decomposition

integer

Level of the wavelet decomposition, specified as a positive integer. If L is the value of `level`, 2^L must divide both the row and column dimensions of `x`. Additionally, the minimum of the row and column dimensions of the image must be greater than or equal to the product of the maximum length of the decomposition (analysis) filters and $2^{(L-1)}$.

`fdf` — Level-one analysis filters

matrix | cell array

The level-one analysis filters, specified as a matrix or cell array of matrices. Specify `fdf` as a matrix when `typetree` is `'dwt'` or `'ddt'`. The size and structure of the matrix depend on the `typetree` input as follows:

- `'dwt'` — This is the critically sampled discrete wavelet transform. In this case, `fdf` is a two-column matrix with the lowpass (scaling) filter in the first column and the highpass (wavelet) filter in the second column.
- `'ddt'` — This is the double-density wavelet transform. The double-density DWT is a three-channel perfect reconstruction filter bank. `fdf` is a three-column matrix with the lowpass (scaling) filter in the first column and the two highpass (wavelet) filters in the second and third columns. In the double-density wavelet transform, the single lowpass and two highpass filters constitute a three-channel perfect reconstruction filter bank. This is equivalent to the three filters forming a tight frame. You cannot arbitrarily choose the two wavelet filters in the double-density DWT. The three filters together must form a tight frame.

Specify `fdf` as a 1-by-2 cell array of matrices when `typetree` is a dual-tree transform, `'realdt'`, `'cplxdt'`, `'realdddt'`, or `'cplxdddt'`. The size and structure of the matrix elements in the cell array depend on the `typetree` input as follows:

- For the dual-tree complex wavelet transforms, `'realdt'` and `'cplxdt'`, `fdf{1}` is an *N*-by-2 matrix containing the lowpass (scaling) and highpass (wavelet) filters for the first tree and `fdf{2}` is an *N*-by-2 matrix containing the lowpass (scaling) and highpass (wavelet) filters for the second tree.
- For the double-density dual-tree complex wavelet transforms, `'realdddt'` and `'cplxdddt'`, `fdf{1}` is an *N*-by-3 matrix containing the lowpass (scaling) and two highpass (wavelet) filters for the first tree and `fdf{2}` is an *N*-by-3 matrix containing the lowpass (scaling) and two highpass (wavelet) filters for the second tree.

`df` — Analysis filters for levels > 1

matrix | cell array

Analysis filters for levels > 1, specified as a matrix or cell array of matrices. Specify `df` as a matrix when `typetree` is `'dwt'` or `'ddt'`. The size and structure of the matrix depend on the `typetree` input as follows:

- `'dwt'` — This is the critically sampled discrete wavelet transform. In this case, `df` is a two-column matrix with the lowpass (scaling) filter in the first column and the highpass (wavelet) filter in the second column. For the critically sampled orthogonal or biorthogonal DWT, the filters in `df` and `fdf` must be identical.
- `'ddt'` — This is the double-density wavelet transform. The double-density DWT is a three-channel perfect reconstruction filter bank. `df` is a three-column matrix with the lowpass (scaling) filter in the first column and the two highpass (wavelet) filters in the second and third columns. In the double-density wavelet transform, the single lowpass and two highpass filters constitute a three-channel perfect reconstruction filter bank. This is equivalent to the three filters forming a tight frame. For the double-density DWT, the filters in `df` and `fdf` must be identical.

Specify `df` as a 1-by-2 cell array of matrices when `typetree` is a dual-tree transform, `'realdt'`, `'cplxdt'`, `'realdddt'`, or `'cplxdddt'`. For dual-tree transforms, the filters in `fdf` and `df` must be different. The size and structure of the matrix elements in the cell array depend on the `typetree` input as follows:

- For the dual-tree wavelet transforms, `'realdt'` and `'cplxdt'`, `df{1}` is an *N*-by-2 matrix containing the lowpass (scaling) and highpass (wavelet) filters for the first tree and `df{2}` is an *N*-by-2 matrix containing the lowpass (scaling) and highpass (wavelet) filters for the second tree.

- For the double-density dual-tree complex wavelet transforms, `'realdddt'` and `'cplxdddt'`, `df{1}` is an *N*-by-3 matrix containing the lowpass (scaling) and two highpass (wavelet) filters for the first tree and `df{2}` is an *N*-by-3 matrix containing the lowpass (scaling) and two highpass (wavelet) filters for the second tree.

`fname` — Filter name

string

Filter name, specified as a string. For the critically sampled DWT, specify any valid orthogonal or biorthogonal wavelet filter. See `wfilters` for details. For the redundant wavelet transforms, see `dtfilters` for valid filter names.

`fname1` — First-stage filter name

string

First-stage filter name, specified as a string. Specifying a first-level filter that is different from the wavelet and scaling filters in subsequent levels is valid and necessary only with the dual-tree wavelet transforms, `'realdt'`, `'cplxdt'`, `'realdddt'`, and `'cplxdddt'`.

`fname2` — Filter name for stages > 1

string

Filter name for stages > 1, specified as a string. Specifying a different filter for stages > 1 is valid and necessary only with the dual-tree wavelet transforms, `'realdt'`, `'cplxdt'`, `'realdddt'`, and `'cplxdddt'`.

Output Arguments

`wt` — Wavelet transform

structure

Wavelet transform, returned as a structure with these fields:

`type` — Type of wavelet decomposition (filter bank)

`'dwt' | 'ddt' | 'realdt' | 'cplxdt' | 'realdddt' | 'cplxdddt'`

Type of wavelet decomposition used in the analysis returned as one of `'dwt'`, `'ddt'`, `'realdt'`, `'cplxdt'`, `'realdddt'`, or `'cplxdddt'`. `'dwt'` is the critically sampled DWT. `'ddt'` produces a double-density wavelet transform with one scaling and two wavelet filters for both row and column filtering. `'realdt'` and `'cplxdt'`

produce oriented dual-tree wavelet transforms consisting of 2 and 4 separable wavelet transforms. `'realdddt'` and `'cplxdddt'` produce double-density dual-tree wavelet transforms consisting of two and four separable wavelet transforms.

`level` — Level of wavelet decomposition

positive integer

Level of wavelet decomposition, returned as a positive integer.

`filters` — Decomposition (analysis) and reconstruction (synthesis) filters

structure

Decomposition (analysis) and reconstruction (synthesis) filters, returned as a structure with these fields:

`Fdf` — First-stage analysis filters

matrix | cell array

First-stage analysis filters, returned as an *N*-by-2 or *N*-by-3 matrix for single-tree wavelet transforms, or a 1-by-2 cell array of two *N*-by-2 or *N*-by-3 matrices for dual-tree wavelet transforms. The matrices are *N*-by-3 for the double-density wavelet transforms. For an *N*-by-2 matrix, the first column of the matrix is the scaling (lowpass) filter and the second column is the wavelet (highpass) filter. For an *N*-by-3 matrix, the first column of the matrix is the scaling (lowpass) filter and the second and third columns are the wavelet (highpass) filters. For the dual-tree transforms, each element of the cell array contains the first-stage analysis filters for the corresponding tree.

`Df` — Analysis filters for levels > 1

matrix | cell array

Analysis filters for levels > 1, returned as an *N*-by-2 or *N*-by-3 matrix for single-tree wavelet transforms, or a 1-by-2 cell array of two *N*-by-2 or *N*-by-3 matrices for dual-tree wavelet transforms. The matrices are *N*-by-3 for the double-density wavelet transforms. For an *N*-by-2 matrix, the first column of the matrix is the scaling (lowpass) filter and the second column is the wavelet (highpass) filter. For an *N*-by-3 matrix, the first column of the matrix is the scaling (lowpass) filter and the second and third columns are the wavelet (highpass) filters. For the dual-tree transforms, each element of the cell array contains the analysis filters for the corresponding tree.

`Frf` — First-level reconstruction filters

matrix | cell array

First-level reconstruction filters, returned as an *N*-by-2 or *N*-by-3 matrix for single-tree wavelet transforms, or a 1-by-2 cell array of two *N*-by-2 or *N*-by-3 matrices for dual-tree wavelet transforms. The matrices are *N*-by-3 for the double-density wavelet transforms. For an *N*-by-2 matrix, the first column of the matrix is the scaling (lowpass) filter and the second column is the wavelet (highpass) filter. For an *N*-by-3 matrix, the first column of the matrix is the scaling (lowpass) filter and the second and third columns are the wavelet (highpass) filters. For the dual-tree transforms, each element of the cell array contains the first-stage synthesis filters for the corresponding tree.

`Rf` — Reconstruction filters for levels > 1

matrix | cell array

Reconstruction filters for levels > 1, returned as an N-by-2 or N-by-3 matrix for single-tree wavelet transforms, or a 1-by-2 cell array of two N-by-2 or N-by-3 matrices for dual-tree wavelet transforms. The matrices are N-by-3 for the double-density wavelet transforms. For an N-by-2 matrix, the first column of the matrix is the scaling (lowpass) filter and the second column is the wavelet (highpass) filter. For an N-by-3 matrix, the first column of the matrix is the scaling (lowpass) filter and the second and third columns are the wavelet (highpass) filters. For the dual-tree transforms, each element of the cell array contains the first-stage analysis filters for the corresponding tree.

`cfs` — Wavelet transform coefficients

cell array of matrices

Wavelet transform coefficients, specified as a 1-by-(`level`+1) cell array of matrices. The size and structure of the matrix elements of the cell array depend on the type of wavelet transform, `typetree` as follows:

- `'dwt'` — `cfs{j}(:,:,d)`
  - j = 1,2,... `level` is the level.
  - d = 1,2,3 is the orientation.
  - `cfs{level+1}(:,:)` are the lowpass, or scaling, coefficients.
- `'ddt'` — `cfs{j}(:,:,d)`
  - j = 1,2,... `level` is the level.
  - d = 1,2,3,4,5,6,7,8 is the orientation.
  - `cfs{level+1}(:,:)` are the lowpass, or scaling, coefficients.
- `'realddt'` — `cfs{j}(:,:,d,k)`

- j = 1,2,... `level` is the level.
- d = 1,2,3 is the orientation.
- k = 1,2 is the wavelet transform tree.
- `cfs{level+1}(:,:)` are the lowpass, or scaling, coefficients.

- `'cplxdt'` — `cfs{j}(:,:,d,k,m)`
  - j = 1,2,... `level` is the level.
  - d = 1,2,3 is the orientation.
  - k = 1,2 is the wavelet transform tree.
  - m = 1,2 are the real and imaginary parts.
  - `cfs{level+1}(:,:)` are the lowpass, or scaling, coefficients.
- `'realdddt'` — `cfs{j}(:,:,d,k)`
  - j = 1,2,... `level` is the level.
  - d = 1,2,3 is the orientation.
  - k = 1,2 is the wavelet transform tree.
  - `cfs{level+1}(:,:)` are the lowpass, or scaling, coefficients.
- `'cplxdddt'` — `cfs{j}(:,:,d,k,m)`
  - j = 1,2,... `level` is the level.
  - d = 1,2,3 is the orientation.
  - k = 1,2 is the wavelet transform tree.
  - m = 1,2 are the real and imaginary parts.
  - `cfs{level+1}(:,:)` are the lowpass, or scaling, coefficients.

More About

- "Critically Sampled and Oversampled Wavelet Filter Banks"

See Also

`dddtree` | `dddtreecfs` | `dtfilters` | `idddtree2`

Introduced in R2013b

depo2ind

Node depth-position to node index

Syntax

Description

`depo2ind` is a tree-management utility.

For a tree of order `ORD`, `N = depo2ind(ORD,[D P])` computes the indices `N` of the nodes whose depths and positions are encoded within `[D,P]`.

The nodes are numbered from left to right and from top to bottom. The root index is 0.

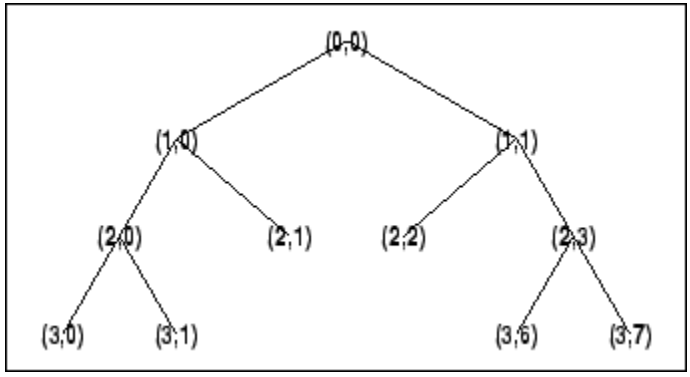
D and P are column vectors. The values of depths D and positions P must be such that $D \geq 0$ and $0 \leq P \leq ORD^{D-1}$.

Output indices N are such that $0 \leq N < (ORD^{max(D)}-1)/ORD-1$.

Note that for a column vector `X`, we have `depo2ind(0,X) = X`.

Examples

```
% Create initial tree.
ord = 2;
t = ntree(ord,3);     % binary tree of depth 3.
t = nodejoin(t,5);
t = nodejoin(t,4);
plot(t)
```




```
% List t nodes (Depth_Position).
aln_depo = allnodes(t,'deppos')
aln_depo =
     0     0
     1     0
     1     1
     2     0
     2     1
     2     2
     2     3
     3     0
     3     1
     3     6
     3     7

% Switch from Depth_Position to index.
aln_ind = depo2ind(ord,aln_depo)
aln_ind =
     0
     1
     2
     3
     4
     5
     6
     7
     8
    13
    14
```

See Also

`ind2depo`

Introduced before R2006a

detcoef

1-D detail coefficients

Syntax

```
D = detcoef(C,L,N)
D = detcoef(C,L)
```

Description

`detcoef` is a one-dimensional wavelet analysis function.

`D = detcoef(C,L,N)` extracts the detail coefficients at level *N* from the wavelet decomposition structure `[C,L]`. See `wavedec` for more information on *C* and *L*.

Level `N` must be an integer such that $1 \le N \le NMAX$ where `NMAX = length(L)-2`.

`D = detcoef(C,L)` extracts the detail coefficients at last level `NMAX`.

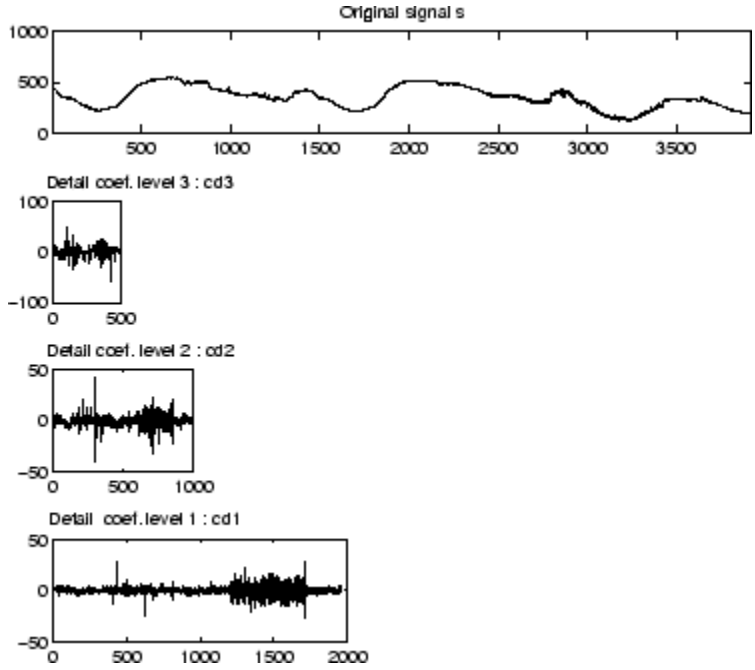
If *N* is a vector of integers such that $1 \le N(j) \le NMAX$:

- `DCELL = detcoef(C,L,N,'cells')` returns a cell array where `DCELL{j}` contains the coefficients of detail `N(j)`.
- If `length(N) > 1`, `DCELL = detcoef(C,L,N)` is equivalent to `DCELL = detcoef(C,L,N,'cells')`.
- `DCELL = detcoef(C,L,'cells')` is equivalent to `DCELL = detcoef(C,L,[1:NMAX])`.
- `[D1, ... ,Dp] = detcoef(C,L,[N(1), ... ,N(p)])` extracts the details coefficients at levels `[N(1), ... ,N(p)]`.

Examples

```
% The current extension mode is zero-padding (see dwtmode).

% Load original one-dimensional signal.
```

```
load leleccum;
s = leleccum(1:3920);

% Perform decomposition at level 3 of s using db1.
[c,l] = wavedec(s,3,'db1');

% Extract detail coefficients at levels
% 1, 2 and 3, from wavelet decomposition
% structure [c,l].
[cd1,cd2,cd3] = detcoef(c,l,[1 2 3]);

% Using some plotting commands,
% the following figure is generated.
```



See Also

appcoef | wavedec

Introduced before R2006a

detcoef2

2-D detail coefficients

Syntax

```
D = detcoef2(O,C,S,N)
```

Description

`detcoef2` is a two-dimensional wavelet analysis function.

`D = detcoef2(O,C,S,N)` extracts from the wavelet decomposition structure `[C,S]` the horizontal, vertical, or diagonal detail coefficients for `O = 'h'` (or `'v'` or `'d'`, respectively), at level *N*, where *N* must be an integer such that `1 ≤ N ≤ size(S,1)-2`. See `wavedec2` for more information on `C` and `S`.

`[H,V,D] = detcoef2('all',C,S,N)` returns the horizontal `H`, vertical `V`, and diagonal `D` detail coefficients at level `N`.

`D = detcoef2('compact',C,S,N)` returns the detail coefficients at level `N`, stored row-wise.

`detcoef2('a',C,S,N)` is equivalent to `detcoef2('all',C,S,N)`.

`detcoef2('c',C,S,N)` is equivalent to `detcoef2('compact',C,S,N)`.

Examples

```
% The current extension mode is zero-padding (see dwtmode).

% Load original image.
load woman;

% X contains the loaded image.
```

```
% Perform decomposition at level 2
% of X using db1.
[c,s] = wavedec2(X,2,'db1');
sizex = size(X)
sizex =
    256    256

sizec = size(c)
sizec =
    1    65536

val_s = s
val_s =
     64     64
     64     64
    128    128
    256    256

% Extract details coefficients at level 2
% in each orientation, from wavelet decomposition
% structure [c,s].
[chd2,cvd2,cdd2] = detcoef2('all',c,s,2);
sizecd2 = size(chd2)
sizecd2 =
     64     64

% Extract details coefficients at level 1
% in each orientation, from wavelet decomposition
% structure [c,s].
[chd1,cvd1,cdd1] = detcoef2('all',c,s,1);
sizecd1 = size(chd1)
sizecd1 =
    128    128
```

More About

Tips

If `C` and `S` are obtained from an indexed image analysis or a truecolor image analysis, `D` is an `m`-by-`n` matrix or an `m`-by-`n`-by-3 array, respectively.

For more information on image formats, see the `image` and `imfinfo` reference pages.

See Also

`appcoef2` | `wavedec2`

Introduced before R2006a

disp

WPTREE information

Syntax

```
disp(T)
```

Description

`disp(T)` displays the content of the WPTREE object *T*.

Examples

```
% Compute a wavelet packets tree
x = rand(1,1000);
t = wpdec(x,2,'db2');
disp(t)

 Wavelet Packet Object Structure
================================
 Size of initial data       : [1 1000]
 Order                      : 2
 Depth                      : 2
 Terminal nodes             : [3  4  5  6]
------------------------------------------------
 Wavelet Name               : db2
 Low Decomposition filter   : [-0.1294  0.2241  0.8365  0.483]
 High Decomposition filter  : [ -0.483  0.8365 -0.2241 -0.1294]
 Low Reconstruction filter  : [  0.483  0.8365  0.2241 -0.1294]
 High Reconstruction filter : [-0.1294 -0.2241  0.8365 -0.483]
------------------------------------------------
 Entropy Name               : shannon
 Entropy Parameter          : 0
------------------------------------------------
```

See Also

`get` | `read` | `set` | `write`

Introduced before R2006a

displs

Display lifting scheme

Syntax

```
S = displs(LS,FRM)
```

Description

`S = displs(LS,FRM)` returns a string describing the lifting scheme *`LS`*. The format string *`FRM`* (see `sprintf`) builds S.

`displs(LS)` is equivalent to DISPLS(LS,'%12.8f')

For more information about lifting schemes, see `lsinfo`.

Examples

```
% Start from the Haar wavelet and get the
% corresponding lifting scheme.
lshaar = liftwave('haar');

% Visualize the obtained lifting scheme.
displs(lshaar);

lshaar = {...
'd'             [ -1.00000000]  [0]
'p'             [  0.50000000]  [0]
[  1.41421356]  [  0.70710678]  []
};

% Add a primal ELS to the lifting scheme.
els = {'p',[-0.125 0.125],0};
lsnew = addlift(lshaar,els);
displs(lsnew);

lsnew = {...
```

```
'd'             [ -1.00000000]              [0]
'p'             [  0.50000000]              [0]
'p'             [ -0.12500000  0.12500000]  [0]
[  1.41421356]  [  0.70710678]              []
};
```

See Also

`lsinfo`

Introduced before R2006a

drawtree

Draw wavelet packet decomposition tree (GUI)

Syntax

```
drawtree(T)
F = drawtree(T)
drawtree(T,F)
```

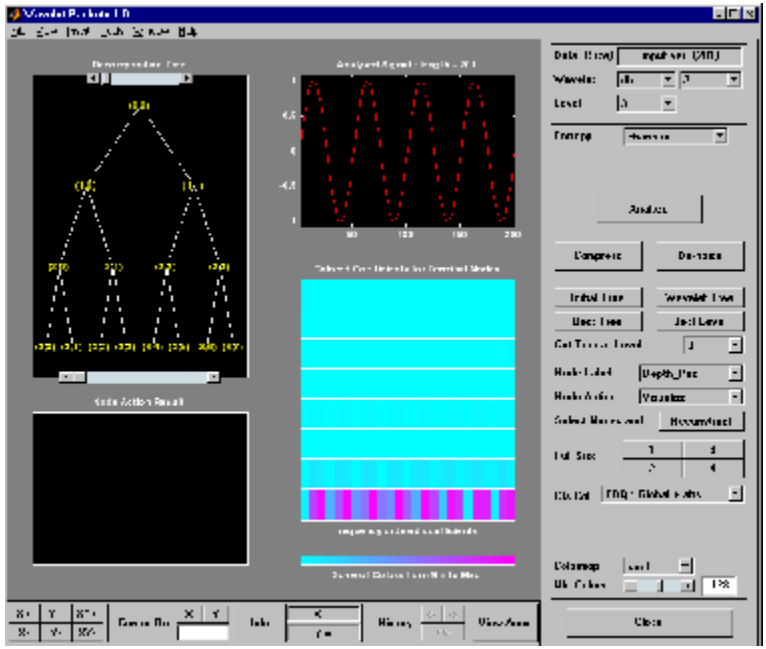
Description

`drawtree(T)` draws the wavelet packet tree *T*, and `F = drawtree(T)` also returns the figure's handle.

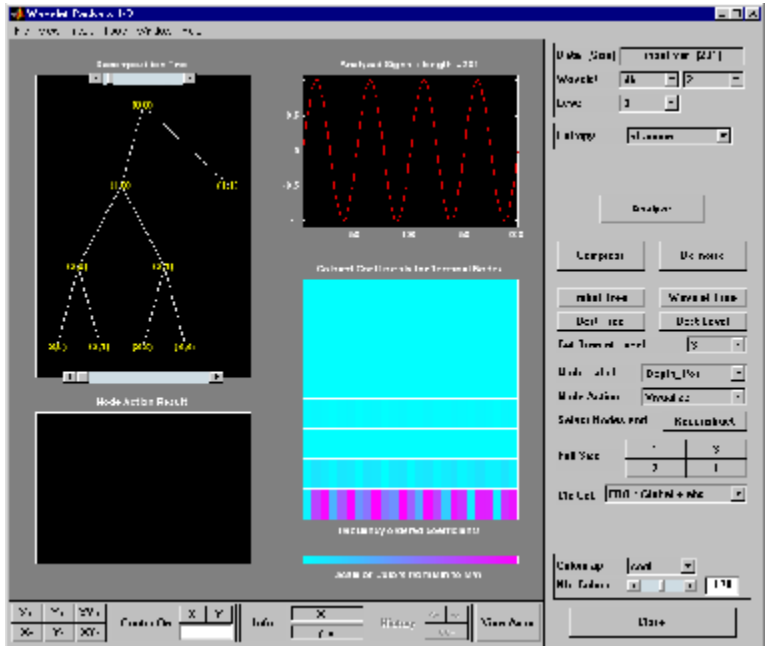
For an existing figure F produced by a previous call to the drawtree function, `drawtree(T,F)` draws the wavelet packet tree T in the figure whose handle is F.

Examples

```
x   = sin(8*pi*[0:0.005:1]);
t   = wpdec(x,3,'db2');
fig = drawtree(t);
```



```
%---------------------------------------
% Use command line function to modify t.
%---------------------------------------
t   = wpjoin(t,2);
drawtree(t,fig);
```



See Also

`readtree`

Introduced before R2006a

dtfilters

Analysis and synthesis filters for oversampled wavelet filter banks

Syntax

```
df = dtfilters(name)
[df,rf] = dtfilters(name)
```

Description

`df = dtfilters(name)` returns the decomposition (analysis) filters corresponding to the string, `name`.

`[df,rf] = dtfilters(name)` returns the reconstruction (synthesis) filters corresponding to the string, `name`.

Examples

Filters for Complex Dual-Tree Wavelet Transform

Obtain valid filters for the complex dual-tree wavelet transform. The transform uses Farras nearly symmetric filters for the first stage and Kingsbury Q-shift filters with 10 taps for subsequent stages.

Load the noisy Doppler signal. Obtain the filters for the first and subsequent stages of the complex dual-tree wavelet transform. Demonstrate perfect reconstruction using the complex dual-tree wavelet transform.

```
load noisdopp;
df = dtfilters('dtf2');
dt = dddtree('cplxdt',noisdopp,5,df{1},df{2});
xrec = idddtree(dt);
max(abs(noisdopp-xrec))
```

Filters for Double-Density Wavelet Transform

Obtain valid filters for the double-density wavelet transform.

Load the noisy Doppler signal. Obtain the filters for the double-density wavelet transform. The double-density wavelet transform uses the same filters at all stages. Demonstrate perfect reconstruction using the double-density wavelet transform.

```
df = dtfilters('filters1');
load noisdopp;
dt = dddtree('ddt',noisdopp,5,df,df);
xrec = idddtree(dt);
max(abs(noisdopp-xrec))
```

Input Arguments

`name` — Filter name

`'dtf1'` | `'dddtf1'` | `'self1'` | `'self2'` | ...

Filter name, specified as a string. Valid entries for `name` are:

- Any valid orthogonal or biorthogonal wavelet name. See `wfilters` for details. An orthogonal or biorthogonal wavelet is only valid when the filter bank type is `'dwt'`, or when you use the filter as the first stage in a complex dual-tree transform, `'realdt'` or `'cplxdt'`. An orthogonal or biorthogonal wavelet filter is not a valid filter if you have a double-density, `'ddt'` or dual-tree double-density, `'realdddt'` or `'cplxdddt'`, filter bank. An orthogonal or biorthogonal wavelet filter is not a valid filter for complex dual-tree filter banks for stages greater than 1.
- `'dtfP'` — With P equal to 1, 2, 3, or 4 returns the first-stage Farras filters (`'FSfarras'`) and Kingsbury Q-shift filters (`'qshiftN'` for subsequent stages. This input is only valid for a dual-tree transform, `'realdt'` or `'cplxdt'`. Setting P= 1, 2, 3, or 4 specifies the Kingsbury Q-shift filters with N = 6, 10, 14, or 18 taps respectively.
- `'dddtf1'` — Returns the filters for the first and subsequent stages of the double-density dual-tree transform. This input is only valid for the double-density dual-tree transforms, `'realdddt'` and `'cplxdddt'`.
- `'self1'` — Returns 10-tap filters for the double-density wavelet transform. This option is only valid for double-density wavelet transforms, `'ddt'`, `'realdddt'`, and `'cplxdddt'`.
- `'self2'` — Returns 16-tap filters for the double-density wavelet transform. This option is only valid for double-density wavelet transforms, `'ddt'`, `'realdddt'`, and `'cplxdddt'`.

- `'filters1'` — Returns 6-tap filters for the double-density wavelet transform, `'ddt'`.
- `'filters2'` — Returns 12-tap filters for the double-density wavelet transform, `'ddt'`.
- `'farras'` — Farras nearly symmetric filters for a two-channel perfect reconstruction filter bank. This option is only valid for an orthogonal critically sampled wavelet transform, `'dwt'`.
- he `'FSfarras'` — Farras nearly symmetric first-stage filters for a dual-tree wavelet transform.
- `'qshiftN'` — Kingsbury Q-shift N-tap filters with N = 6,10,14, or 18. The Kingsbury Q-shift filters are most commonly used in dual-tree wavelet transforms for stages greater than 1.
- `'doubledualfilt'` — Filters for one stage of the double-density dual-tree wavelet transforms, `'realdddt'` or `'cplxdddt'`.

Output Arguments

`df` — Decomposition (analysis) filters

matrix | cell array

Decomposition (analysis) filters, returned as a matrix or cell array of matrices.

`rf` — Reconstruction (synthesis) filters

matrix | cell array

Reconstruction (synthesis) filters, returned as a matrix or cell array of matrices.

See Also

`dddtree` | `dddtree2`

Introduced in R2013b

dtree

DTREE constructor

Syntax

```
T = dtree(ORD,D,X)
T = dtree(ORD,D,X,U)
[T,NB] = dtree(...)
[T,NB] = dtree('PropName1',PropValue1,'PropName2',PropValue2,...)
```

Description

`T = dtree(ORD,D,X)` returns a complete data tree (`DTREE`) object of order *`ORD`* and depth *`D`*. The data associated with the tree *`T`* is *`X`*.

With `T = dtree(ORD,D,X,U)` you can set a user data field.

`[T,NB] = dtree(...)` returns also the number of terminal nodes (leaves) of *`T`*.

`[T,NB] = dtree('PropName1',PropValue1,'PropName2',PropValue2,...)` is the most general syntax to construct a `DTREE` object.

The valid choices for `'PropName'` are

`'order'`	Order of the tree
`'depth'`	Depth of the tree
`'data'`	Data associated to the tree
`'spsch'`	Split scheme for nodes
`'ud'`	User data field

The split scheme field is an order `ORD` by 1 logical array. The root of the tree can be split and it has `ORD` children. If `spsch(j) = 1`, you can split the `j`-th child. Each node that you can split has the same property as the root node.

For more information on object fields, type `help dtree/get`.

Class DTREE (Parent class: NTREE)

Fields

dtree	Parent object
allNI	All nodes information
terNI	Terminal nodes information

Examples

```
% Create a data tree.
x = [1:10];
t = dtree(3,2,x);
t = nodejoin(t,2);
```

See Also

ntree | wtbo

Introduced before R2006a

dwt

Single-level discrete 1-D wavelet transform

Syntax

```
[cA,cD] = dwt(X,'wname')
[cA,cD] = dwt(X,Lo_D,Hi_D)
[cA,cD] = dwt(...,'mode',MODE)
```

Description

The `dwt` command performs a single-level one-dimensional wavelet decomposition. Compare this function to `wavedec`, which may be more useful for your application. The decomposition is done with respect to either a particular wavelet (`'wname'`, see `wfilters` for more information) or particular wavelet decomposition filters (`Lo_D` and `Hi_D`) that you specify.

`[cA,cD] = dwt(X,'wname')` computes the approximation coefficients vector `cA` and detail coefficients vector `cD`, obtained by a wavelet decomposition of the vector `X`. The string `'wname'` contains the wavelet name.

`[cA,cD] = dwt(X,Lo_D,Hi_D)` computes the wavelet decomposition as above, given these filters as input:

- `Lo_D` is the decomposition low-pass filter.
- `Hi_D` is the decomposition high-pass filter.

`Lo_D` and `Hi_D` must be the same length.

Let `lx` = the length of `X` and `lf` = the length of the filters `Lo_D` and `Hi_D`; then `length(cA) = length(cD) = la` where `la = ceil(lx/2)`, if the DWT extension mode is set to periodization. For the other extension modes, `la = floor(lx+lf-1)/2`.

For more information about the different Discrete Wavelet Transform extension modes, see `dwtmode`.

`[cA,cD] = dwt(...,'mode',MODE)` computes the wavelet decomposition with the extension mode `MODE` that you specify. `MODE` is a string containing the desired extension mode.

Example:

```
[cA,cD] = dwt(x,'db1','mode','sym');
```

Examples

DWT Using Wavelet Name

Obtain the level-1 DWT of the noisy Doppler signal using a wavelet name.

```
load noisdopp;
[A,D] = dwt(noisdopp,'sym4');
```

DWT Using Wavelet and Scaling Filters

Obtain the level-1 DWT of the noisy Doppler signal using wavelet and scaling filters.

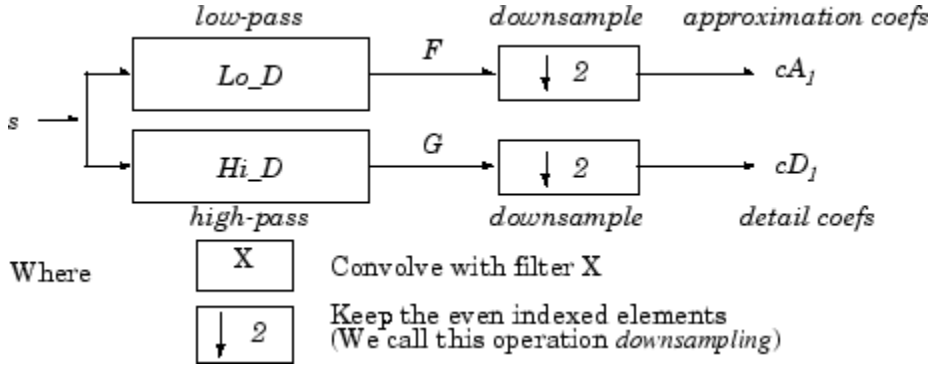
```
load noisdopp;
[Lo_D,Hi_D] = wfilters('bior3.5','d');
[A,D] = dwt(noisdopp,Lo_D,Hi_D);
```

More About

Algorithms

Starting from a signal s of length N, two sets of coefficients are computed: approximation coefficients CA_1, and detail coefficients CD_1. These vectors are obtained by convolving s with the low-pass filter Lo_D for approximation and with the high-pass filter Hi_D for detail, followed by dyadic decimation.

More precisely, the first step is



The length of each filter is equal to $2L$. For signal of length N, the signals F and G are of length $N + 2L - 1$, and then the coefficients CA_1 and CD_1 are of length

$$\left\lfloor \frac{N-1}{2} + L \right\rfloor.$$

To deal with signal-end effects involved by a convolution-based algorithm, a global variable managed by `dwtmode` is used. This variable defines the kind of signal extension mode used. The possible options include zero-padding (used in the previous example) and symmetric extension, which is the default mode.

Note For the same input, this `dwt` function and the DWT block in the Signal Processing Toolbox™ do not produce the same results. The blockset is designed for real-time implementation while Wavelet Toolbox™ software is designed for analysis, so they produce handle boundary conditions and filter states differently.

To make the `dwt` function output match the DWT block output, set the function boundary condition to zero-padding by typing `dwtmode('zpd')` at the MATLAB® command prompt. To match the latency of the DWT block, which is implemented using FIR filters, add zeros to the input of the `dwt` function. The number of zeros you add must be equal to half the filter length.

References

Daubechies, I. (1992), *Ten lectures on wavelets*, CBMS-NSF conference series in applied mathematics. SIAM Ed.

Mallat, S. (1989), "A theory for multiresolution signal decomposition: the wavelet representation," *IEEE Pattern Anal. and Machine Intell.*, vol. 11, no. 7, pp. 674–693.

Meyer, Y. (1990), *Ondelettes et opérateurs*, Tome 1, Hermann Ed. (English translation: *Wavelets and operators*, Cambridge Univ. Press. 1993.)

See Also

`dwtmode` | `idwt` | `wavedec` | `waveinfo`

Introduced before R2006a

dwt2

Single-level discrete 2-D wavelet transform

Syntax

```
[cA,cH,cV,cD] = dwt2(X,'wname')
[cA,cH,cV,cD] = dwt2(X,Lo_D,Hi_D)
[cA,cH,cV,cD] = dwt2(...,'mode',MODE)
```

Description

The `dwt2` command performs a single-level two-dimensional wavelet decomposition. Compare this function to `wavedec2`, which may be more useful for your application. The decomposition is done with respect to either a particular wavelet (`'wname'`, see `wfilters` for more information) or particular wavelet decomposition filters (`Lo_D` and `Hi_D`) you specify.

`[cA,cH,cV,cD] = dwt2(X,'wname')` computes the approximation coefficients matrix `cA` and details coefficients matrices `cH`, `cV`, and `cD` (horizontal, vertical, and diagonal, respectively), obtained by wavelet decomposition of the input matrix `X`. The `'wname'` string contains the wavelet name.

`[cA,cH,cV,cD] = dwt2(X,Lo_D,Hi_D)` computes the two-dimensional wavelet decomposition as above, based on wavelet decomposition filters that you specify.

- `Lo_D` is the decomposition low-pass filter.
- `Hi_D` is the decomposition high-pass filter.

`Lo_D` and `Hi_D` must be the same length.

Let `sx = size(X)` and `lf =` the length of filters; then `size(cA) = size(cH) = size(cV) = size(cD) = sa` where `sa = ceil(sx/2)`, if the DWT extension mode is set to periodization. For the other extension modes, `sa = floor((sx+lf-1)/2)`.

For information about the different Discrete Wavelet Transform extension modes, see `dwtmode`.

`[cA,cH,cV,cD] = dwt2(...,'mode',MODE)` computes the wavelet decomposition with the extension mode `MODE` that you specify.

`MODE` is a string containing the desired extension mode.

An example of valid use is

```
[cA,cH,cV,cD] = dwt2(x,'db1','mode','sym');
```

2-D DWT of Image

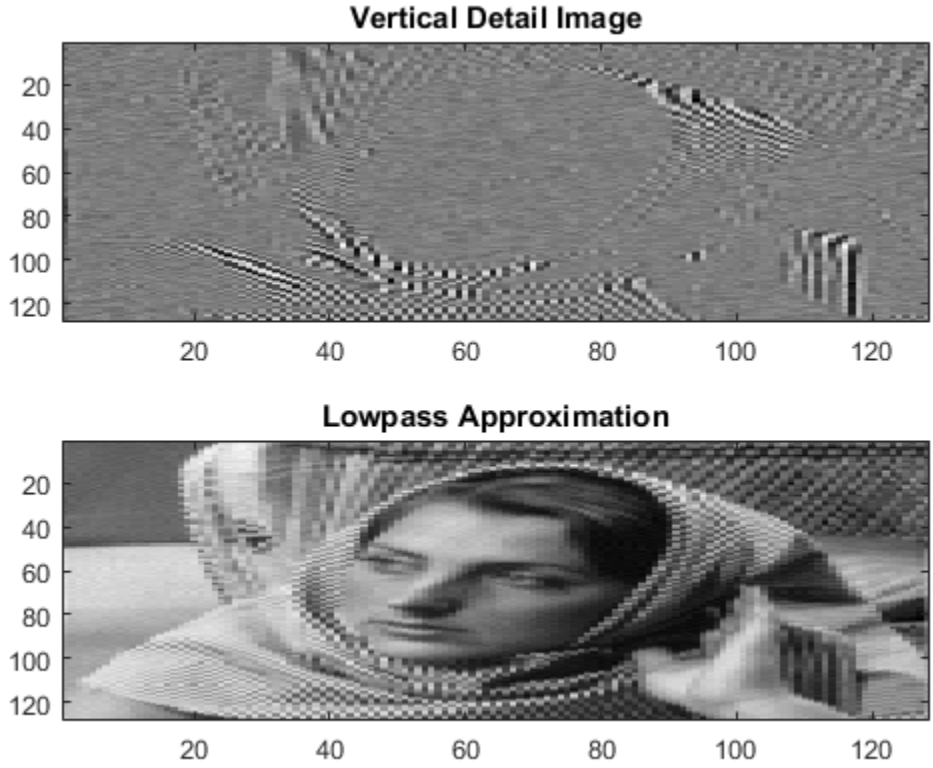
This example shows how to obtain the 2-D DWT of an image.

Load the "woman" image and obtain the 2-D DWT using the 'sym4' wavelet. Use the periodic extension mode.

```
load woman;
wname = 'sym4';
[CA,CH,CV,CD] = dwt2(X,wname,'mode','per');
```

Display the vertical detail image and the lowpass approximation.

```
subplot(211)
imagesc(CV); title('Vertical Detail Image');
colormap gray;
subplot(212)
imagesc(CA); title('Lowpass Approximation');
```



More About

Tips

When X represents an indexed image, then X, as well as the output arrays cA,cH,cV,cD are `m`-by-`n` matrices. When X represents a truecolor image, it is an `m`-by-`n`-by-3 array, where each `m`-by-`n` matrix represents a red, green, or blue color plane concatenated along the third dimension.

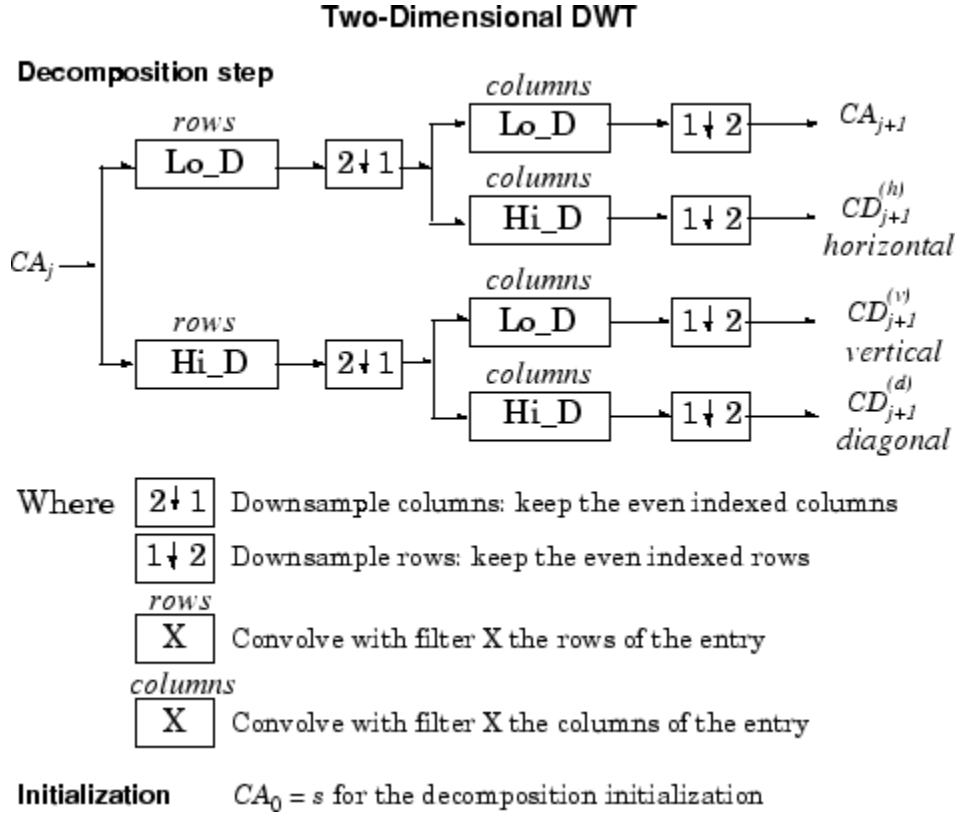
For more information on image formats, see the `image` and `imfinfo` reference pages.

Algorithms

For images, there exist an algorithm similar to the one-dimensional case for two-dimensional wavelets and scaling functions obtained from one- dimensional ones by tensorial product.

This kind of two-dimensional DWT leads to a decomposition of approximation coefficients at level j in four components: the approximation at level $j + 1$, and the details in three orientations (horizontal, vertical, and diagonal).

The following chart describes the basic decomposition steps for images:



Note To deal with signal-end effects involved by a convolution-based algorithm, a global variable managed by `dwtmode` is used. This variable defines the kind of signal extension

mode used. The possible options include zero-padding (used in the previous example) and symmetric extension, which is the default mode.

References

Daubechies, I. (1992), *Ten lectures on wavelets*, CBMS-NSF conference series in applied mathematics. SIAM Ed.

Mallat, S. (1989), "A theory for multiresolution signal decomposition: the wavelet representation," *IEEE Pattern Anal. and Machine Intell.*, vol. 11, no. 7, pp. 674–693.

Meyer, Y. (1990), *Ondelettes et opérateurs*, Tome 1, Hermann Ed. (English translation: *Wavelets and operators*, Cambridge Univ. Press. 1993.)

See Also

`dwtmode` | `idwt2` | `wavedec2` | `waveinfo`

Introduced before R2006a

dwt3

Single-level discrete 3-D wavelet transform

Syntax

```
WT = dwt3(X,'wname')
WT = dwt3(X,'wname','mode','ExtM')
WT = dwt3(X,W,...)
WT = dwt3(X,WF,...)
```

Description

`dwt3` performs a single-level three-dimensional wavelet decomposition using either a particular wavelet (`'wname'`) or the wavelet decomposition and reconstruction filters you specify. The decomposition also uses the specified DWT extension mode (see `dwtmode`).

`WT = dwt3(X,'wname')` returns the 3-D wavelet transform of the 3-D array `X` `'wname'` is a string containing the wavelet name. The default extension mode is `'sym'`. For more information on *`wname`*, see `wfilters`.

`WT = dwt3(X,'wname','mode','ExtM')` uses the extension mode `'ExtM'`.

`WT` is a structure with the following fields shown in the table.

`sizeINI`	Size of the three-dimensional array `X`.
`mode`	Name of the wavelet transform extension mode.
`filters`	Structure with four fields: `LoD`, `HiD`, `LoR`, `HiR`, which are the filters used for DWT.
`dec`	2 x 2 x 2 cell array containing the coefficients of the decomposition. `dec{i,j,k}`, `i`,`j`,`k = 1` or `2` contains the coefficients obtained by low-pass filtering (for `i` or `j` or `k = 1`) or high-pass filtering (for `i` or `j` or `k = 2`)

`WT = dwt3(X,W,...)` specify three wavelets, one for each direction. `W = {'wname1','wname2','wname3'}` or `W` is a structure with 3 fields `'w1'`, `'w2'`, `'w3'` containing strings that are the names of wavelets.

`WT = dwt3(X,WF,...)` specify four filters, two for decomposition, and two for reconstructionm or 3 x 4 filters (one quadruplet by direction). `WF` is either a cell array (1 x 4) or (3 x 4) : `{LoD,HiD,LoR,HiR}` or a structure with the four fields `'LoD','HiD','LoR','HiR'`.

Examples

```
% Define the original 3-D data.
X = reshape(1:64,4,4,4)

X(:,:,1) =

     1     5     9    13
     2     6    10    14
     3     7    11    15
     4     8    12    16

X(:,:,2) =

    17    21    25    29
    18    22    26    30
    19    23    27    31
    20    24    28    32

X(:,:,3) =

    33    37    41    45
    34    38    42    46
    35    39    43    47
    36    40    44    48

X(:,:,4) =

    49    53    57    61
    50    54    58    62
    51    55    59    63
    52    56    60    64
```

```
% Perform single level decomposition of X using db1.
wt = dwt3(X,'db1')

wt =

    sizeINI: [4 4 4]
    filters: [1x1 struct]
       mode: 'sym'
        dec: {2x2x2 cell}

% Decompose X using db2.
[LoD,HiD,LoR,HiR] = wfilters('db2');
wt = dwt3(X,{LoD,HiD,LoR,HiR})

wt =

    sizeINI: [4 4 4]
    filters: [1x1 struct]
       mode: 'sym'
        dec: {2x2x2 cell}


% Decompose X using different wavelets, one for
% each orientation (db1, db2 and again db1).
WS = struct('w1','db1','w2','db2','w3','db1');
wt = dwt3(X,WS,'mode','per')

wt =

    sizeINI: [4 4 4]
    filters: [1x1 struct]
       mode: 'per'
        dec: {2x2x2 cell}

WF = wt.filters;

% Decompose X using the filters given by WF and
% set the extension mode to symmetric.
wtBIS = dwt3(X,WF,'mode','sym')

wtBIS =

    sizeINI: [4 4 4]
    filters: [1x1 struct]
```

```
       mode: 'sym'
        dec: {2x2x2 cell}
```

See Also

dwtmode | idwt3 | wavedec3 | waverec3 | waveinfo | wfilters

Introduced in R2010a

dwtmode

Discrete wavelet transform extension mode

Syntax

```
ST = dwtmode
ST = dwtmode('status')
dwtmode('mode')
```

Description

`dwtmode` sets the signal or image extension mode for discrete wavelet and wavelet packet transforms. The extension modes represent different ways of handling the problem of border distortion in signal and image analysis. For more information, see “Border Effects” , in the User's Guide.

`dwtmode` or `dwtmode('status')` display the current mode.

`ST = dwtmode` or `ST = dwtmode('status')` display and returns in `ST` the current mode.

`ST = dwtmode('status','nodisp')` returns in `ST` the current mode and no text (status or warning) is displayed in the MATLAB Command Window.

`dwtmode('mode')` sets the DWT extension mode according to the value of `'mode'`:

`'mode'`	DWT Extension Mode
`'sym'` or `'symh'`	Symmetric-padding (half-point): boundary value symmetric replication — default mode
`'symw'`	Symmetric-padding (whole-point): boundary value symmetric replication
`'asym'` or `'asymh'`	Antisymmetric-padding (half-point): boundary value antisymmetric replication
`'asymw'`	Antisymmetric-padding (whole-point): boundary value antisymmetric replication

'*mode*'	DWT Extension Mode
'zpd'	Zero-padding
'spd' or 'sp1'	Smooth-padding of order 1 (first derivative interpolation at the edges)
'sp0'	Smooth-padding of order 0 (constant extension at the edges)
'ppd'	Periodic-padding (periodic extension at the edges)

For more information on symmetric extension modes see “References”.

The DWT associated with these five modes is slightly redundant. But, the IDWT ensures a perfect reconstruction for any of the five previous modes whatever is the extension mode used for DWT.

`dwtmode('per')` sets the DWT mode to periodization.

This mode produces the smallest length wavelet decomposition. But, the extension mode used for IDWT must be the same to ensure a perfect reconstruction.

Using this mode, `dwt` and `dwt2` produce the same results as the obsolete functions `dwtper` and `dwtper2`, respectively.

All functions and GUI tools involving the DWT (1-D & 2-D) or Wavelet Packet transform (1-D & 2-D) use the specified DWT extension mode.

`dwtmode` updates a global variable allowing the use of these six signal extensions. The extension mode should only be changed using this function. Avoid changing the global variable directly.

The default mode is loaded from the file `DWTMODE.DEF` (in the current path) if it exists. If not, the file `DWTMODE.CFG` (in the `toolbox/wavelet/wavelet` folder) is used.

`dwtmode('save',MODE)` saves `MODE` as the new default mode in the file `DWTMODE.DEF` (in the current folder). If a file with the same name already exists in the current folder, it is deleted before saving.

`dwtmode('save')` is equivalent to `dwtmode('save',CURRENTMODE)`.

In these last two cases, the new default mode saved in the file `DWTMODE.DEF` will be active as default mode in the next MATLAB session.

Examples

```
% If the DWT extension mode global variable does not
% exist, default is Symmetrization.
clear global
dwtmode

****************************************
**  DWT Extension Mode: Symmetrization  **
****************************************

% Display current DWT signal extension mode.
dwtmode

****************************************
**  DWT Extension Mode: Symmetrization  **
****************************************
% Change to Periodization extension mode.
dwtmode('per')

!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!
!  WARNING: Change DWT Extension Mode  !
!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!

****************************************
**  DWT Extension Mode: Periodization  **
****************************************

% Display current DWT signal extension mode.
dwtmode

****************************************
**  DWT Extension Mode: Periodization  **
****************************************
```

Note You should change the extension mode only by using `dwtmode`. Avoid changing the global variable directly.

References

Strang, G.; T. Nguyen (1996), *Wavelets and filter banks*, Wellesley- Cambridge Press.

See Also

idwt | idwt2 | dwt | dwt2 | wextend

Introduced before R2006a

dyaddown

Dyadic downsampling

Syntax

```
Y = dyaddown(X,EVENODD)
Y = dyaddown(X)
Y = dyaddown(X,EVENODD,'type')
Y = dyaddown(X,'type',EVENODD)
Y = dyaddown(X)
Y = dyaddown(X,'type')
Y = dyaddown(X,0,'type')
Y = dyaddown(X,EVENODD)
Y = dyaddown(X,EVENODD,'c')
```

Description

`Y = dyaddown(X,EVENODD)` where *X* is a *vector*, returns a version of *X* that has been downsampled by 2. Whether `Y` contains the even- or odd-indexed samples of *X* depends on the value of positive integer `EVENODD`:

- If `EVENODD` is even, then `Y(k) = X(2k)`.
- If `EVENODD` is odd, then `Y(k) = X(2k+1)`.

`Y = dyaddown(X)` is equivalent to `Y = dyaddown(X,0)` (even-indexed samples).

`Y = dyaddown(X,EVENODD,'type')` or `Y = dyaddown(X,'type',EVENODD)`, where *X* is a *matrix*, returns a version of *X* obtained by suppressing one out of two:

Columns of *X*	If `'type'= 'c'`
Rows of *X*	If `'type'= 'r'`
Rows and columns of *X*	If `'type'= 'm'`

according to the parameter *EVENODD*, which is as above.

If you omit the *EVENODD* or '*type*' arguments, dyaddown defaults to EVENODD = 0 (even-indexed samples) and '*type*'= 'c' (columns).

Y = dyaddown(X) is equivalent to Y = dyaddown(X,0,'c').
Y = dyaddown(X,'*type*') is equivalent to Y = dyaddown(X,0,'*type*').
Y = dyaddown(X,EVENODD) is equivalent to Y = dyaddown(X,EVENODD,'c').

Examples

```
% For a vector.
s = 1:10
s =
   1   2   3   4   5   6   7   8   9  10

dse = dyaddown(s)   % Downsample elements with even indices.
dse =
   2   4   6   8  10
% or equivalently
dse = dyaddown(s,0)
dse =
   2   4   6   8  10

dso = dyaddown(s,1) % Downsample elements with odd indices.
dso =
   1   3   5   7   9

% For a matrix.
s = (1:3)'*(1:4)
s =
   1   2   3   4
   2   4   6   8
   3   6   9  12

dec = dyaddown(s,0,'c') % Downsample columns with even indices.
dec =
   2   4
   4   8
   6  12

der = dyaddown(s,1,'r') % Downsample rows with odd indices.
der =
 1   2   3   4
```

```
 3   6   9  12

dem = dyaddown(s,1,'m') % Downsample rows and columns
                        % with odd indices.
dem =
     1     3
     3     9
```

References

Strang, G.; T. Nguyen (1996), *Wavelets and Filter Banks*, Wellesley-Cambridge Press.

See Also

dyadup

Introduced before R2006a

dyadup

Dyadic upsampling

Syntax

```
Y = dyadup(X,EVENODD)
Y = dyadup(X)
Y = dyadup(X,EVENODD,'type')
Y = dyadup(X,'type',EVENODD)
Y = dyadup(X)
Y = dyaddown(X,1,'c')
Y = dyadup(X,'type')
Y = dyadup(X,1,'type')
Y = dyadup(X,EVENODD)
Y = dyadup(X,EVENODD,'c')
```

Description

`dyadup` implements a simple zero-padding scheme very useful in the wavelet reconstruction algorithm.

`Y = dyadup(X,EVENODD)`, where *`X`* is a *vector*, returns an extended copy of vector *`X`* obtained by inserting zeros. Whether the zeros are inserted as even- or odd-indexed elements of *`Y`* depends on the value of positive integer `EVENODD`:

- If *`EVENODD`* is even, then `Y(2k–1) = X(k), Y(2k) = 0`.
- If *`EVENODD`* is odd, then `Y(2k–1) = 0, Y(2k) = X(k)`.

`Y = dyadup(X)` is equivalent to `Y = dyadup(X,1)` (odd-indexed samples).

`Y = dyadup(X,EVENODD,'type')` or `Y = dyadup(X,'type',EVENODD)`, where *`X`* is a *matrix*, returns extended copies of *`X`* obtained by inserting

Columns in *`X`*	If *`'type'`*`= 'c'`
Rows in *`X`*	If *`'type'`*`= 'r'`

Rows and columns in *X*	If '*type*'= 'm'

according to the parameter *EVENODD*, which is as above.

If you omit the *EVENODD* or '*type*' arguments, dyadup defaults to EVENODD = 1 (zeros in odd-indexed positions) and '*type*'= 'c' (insert columns).

Y = dyadup(*X*) is equivalent to Y = dyaddown(X,1,'c').

Y = dyadup(*X*,'*type*') is equivalent to Y = dyadup(*X*,1,'*type*').
Y = dyadup(*X*,*EVENODD*) is equivalent to Y = dyadup(*X*,*EVENODD*,'c').

Examples

```
% For a vector.
s = 1:5
s =
    1 2 3 4 5

dse = dyadup(s) % Upsample elements at odd indices.
dse =
    0 1 0 2 0 3 0 4 0 5 0

% or equivalently
dse = dyadup(s,1)
dse =
    0 1 0 2 0 3 0 4 0 5 0

dso = dyadup(s,0) % Upsample elements at even indices.
dso =
    1 0 2 0 3 0 4 0 5

% For a matrix.
s = (1:2)'*(1:3)
s =
    1 2 3
    2 4 6

der = dyadup(s,1,'r') % Upsample rows at even indices.
der =
    0 0 0
    1 2 3
```

```
    0 0 0
    2 4 6
    0 0 0

doc = dyadup(s,0,'c') % Upsample columns at odd indices.
doc =
    1 0 2 0 3
    2 0 4 0 6
dem = dyadup(s,1,'m') % Upsample rows and columns
                      % at even indices.
dem =
     0     0     0     0     0     0     0
     0     1     0     2     0     3     0
     0     0     0     0     0     0     0
     0     2     0     4     0     6     0
     0     0     0     0     0     0     0

% Using default values for dyadup and dyaddown, we have:
% dyaddown(dyadup(s)) = s.
s = 1:5
s =
    1 2 3 4 5

uds = dyaddown(dyadup(s))
uds =
    1 2 3 4 5

% In general reversed identity is false.
```

References

Strang, G.; T. Nguyen (1996), *Wavelets and Filter Banks*, Wellesley-Cambridge Press.

See Also

dyaddown

Introduced before R2006a

entrupd

Entropy update (wavelet packet)

Syntax

```
T = entrupd(T,ENT)
T = entrupd(T,ENT,PAR)
```

Description

`entrupd` is a one- or two-dimensional wavelet packet utility.

`T = entrupd(T,ENT)` or `T = entrupd(T,ENT,PAR)` returns for a given wavelet packet tree *T*, the updated tree using the entropy function *ENT* with the optional parameter *PAR* (see `wenergy` for more information).

Examples

```
% The current extension mode is zero-padding (see dwtmode).

% Load signal.
load noisdopp; x = noisdopp;

% Decompose x at depth 2 with db1 wavelet packets
% using shannon entropy.
t = wpdec(x,2,'db1','shannon');

% Read entropy of all the nodes.
nodes = allnodes(t);
ent = read(t,'ent',nodes);
ent'
ent =
    1.0e+04 *
    -5.8615 -6.8204 -0.0350 -7.7901 -0.0497 -0.0205 -0.0138

% Update nodes entropy.
t = entrupd(t,'threshold',0.5);
```

```
nent = read(t,'ent');
nent'
nent =
    937 488 320 241 175 170 163
```

See Also

wenergy | wpdec | wpdec2

Introduced before R2006a

fbspwavf

Complex frequency B-spline wavelet

Syntax

```
[PSI,X] = fbspwavf(LB,UB,N,M,FB,FC)
```

Description

`[PSI,X] = fbspwavf(LB,UB,N,M,FB,FC)` returns values of the complex frequency B-Spline wavelet defined by the order parameter *M* (*M* is an integer such that $1 \leq M$), a bandwidth parameter `FB`, and a wavelet center frequency *FC*.

The function `PSI` is computed using the explicit expression

```
PSI(X) = (FB^0.5)*((sinc(FB*X/M).^M).*exp(2*i*pi*FC*X))
```

on an *N* point regular grid in the interval `[LB,UB]`.

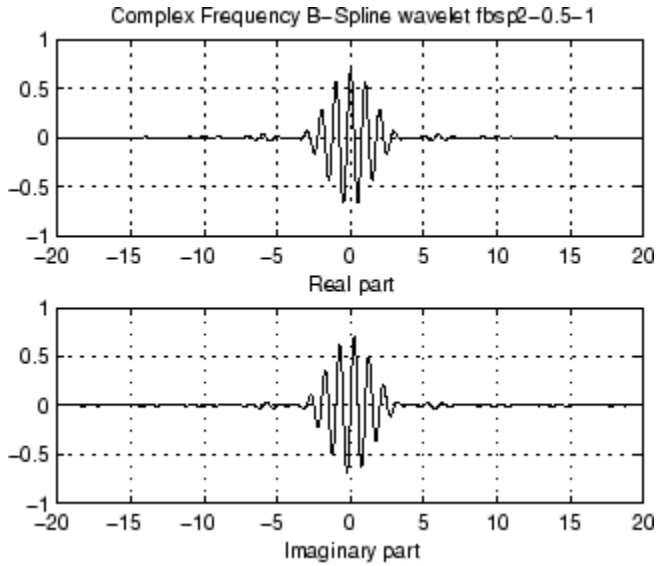
FB and *FC* must be such that `FC > 0` and > `FB > 0`.

Output arguments are the wavelet function `PSI` computed on the grid *X*.

Examples

```
% Set order, bandwidth and center frequency parameters.
m = 2; fb = 0.5; fc = 1;

% Set effective support and grid parameters.
lb = -20; ub = 20; n = 1000;

% Compute complex Frequency B-Spline wavelet fbsp2-0.5-1.
[psi,x] = fbspwavf(lb,ub,n,m,fb,fc);

% Plot complex Frequency B-Spline wavelet.
subplot(211)
plot(x,real(psi))
```

```
title('Complex Frequency B-Spline wavelet fbsp2-0.5-1')
xlabel('Real part'), grid
subplot(212)
plot(x,imag(psi))
xlabel('Imaginary part'), grid
```



References

Teolis, A. (1998), *Computational signal processing with wavelets*, Birkhauser, p. 63.

See Also

`waveinfo`

Introduced before R2006a

filt2ls

Transform quadruplet of filters to lifting scheme

Syntax

```
LS = filt2ls(LoD,HiD,LoR,HiR)
```

Description

`LS = filt2ls(LoD,HiD,LoR,HiR)` returns the lifting scheme `LS` associated with the four input filters `LoD`, `HiD`, `LoR`, and `HiR` that verify the perfect reconstruction condition.

Examples

```
[LoD,HiD,LoR,HiR] = wfilters('db2')

LoD =

   -0.1294    0.2241    0.8365    0.4830

HiD =

   -0.4830    0.8365   -0.2241   -0.1294

LoR =

    0.4830    0.8365    0.2241   -0.1294

HiR =

   -0.1294   -0.2241    0.8365   -0.4830

LS = filt2ls(LoD,HiD,LoR,HiR);
displs(LS);

LS = {...
'd'             [ -1.73205081]              [0]
```

```
'p'             [ -0.06698730  0.43301270]  [1]
'd'             [  1.00000000]              [-1]
[  1.93185165]  [  0.51763809]              []
};

LSref = liftwave('db2');
displs(LSref);

LSref = {...
'd'             [ -1.73205081]              [0]
'p'             [ -0.06698730  0.43301270]  [1]
'd'             [  1.00000000]              [-1]
[  1.93185165]  [  0.51763809]              []
};
```

See Also

`ls2filt` | `lsinfo`

Introduced before R2006a

fejerkorovkin

Fejer-Korovkin wavelet filters

Syntax

```
Lo = fejerkorovkin(wname)
```

Description

`Lo = fejerkorovkin(wname)` returns the Fejer-Korovkin scaling filter specified by `wname`. Valid entries for `wname` are `'fk4'`, `'fk6'`, `'fk8'`, `'fk14'`, `'fk18'`, and `'fk22'`. For information on the Fejer–Korovkin filters, see Nielson[1].

Examples

Fejer-Korovkin Filters

Construct and plot the Fejer-Korovkin (14) scaling function and wavelet.

Obtain the Fejer-Korovkin scaling filter and display its 14 coefficients.

```
Lo = fejerkorovkin('fk14')
```

```
Lo =

  Columns 1 through 7

    0.2604    0.6869    0.6116    0.0514   -0.2456   -0.0486    0.1243

  Columns 8 through 14

    0.0222   -0.0640   -0.0051    0.0298   -0.0033   -0.0093    0.0035
```

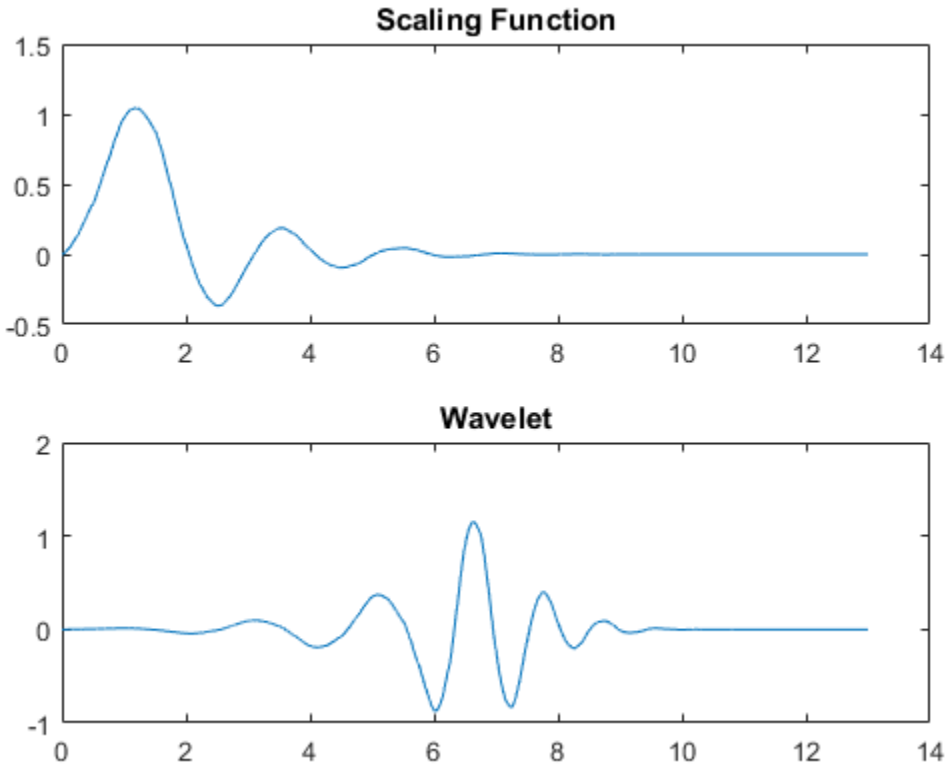
Use the scaling filter to obtain the wavelet filter and display its wavelet filter coefficients.

```
Hi = qmf(Lo)
```

```
Hi =

  Columns 1 through 7

    0.0035    0.0093   -0.0033   -0.0298   -0.0051    0.0640    0.0222

  Columns 8 through 14

   -0.1243   -0.0486    0.2456    0.0514   -0.6116    0.6869   -0.2604
```

`wavefun` provides an efficient way to construct and plot the scaling function and wavelet.

```
[phi,psi,xval] = wavefun('fk14');
subplot(2,1,1)
plot(xval,phi)
title('Scaling Function')
subplot(2,1,2)
plot(xval,psi)
title('Wavelet')
```



Input Arguments

wname — Filter name

`'fk4' | 'fk6' | 'fk8' | 'fk14' | 'fk18' | 'fk22'`

Filter name, specified as a string. The numeric value in each string is the number of Fejer-Korovkin filter coefficients.

Output Arguments

`Lo` — Scaling filter

vector

Scaling filter, returned as a vector.

References

[1] Nielsen, M. "On the construction and frequency localization of finite orthogonal quadrature filters." *Journal of Approximation Theory*. Vol. 108, pp. 36–52.

See Also

`coifwavf` | `dbwavf` | `symwavf`

Introduced in R2015b

gauswavf

Gaussian wavelet

Syntax

```
[PSI,X] = gauswavf(LB,UB,N)
[PSI,X] = gauswavf(LB,UB,N,P)
[PSI,X] = gauswavf(LB,UB,N,WAVNAME)
```

Description

`[PSI,X] = gauswavf(LB,UB,N)` returns the 1^{st} order derivative of the Gaussian wavelet, `PSI`, on an `N`-point regular grid, `X`, for the interval `[LB,UB]`. The effective support of the Gaussian wavelets is `[-5 5]`.

`[PSI,X] = gauswavf(LB,UB,N,P)` returns the P^{th} derivative. Valid values of `P` are integers from 1 to 8.

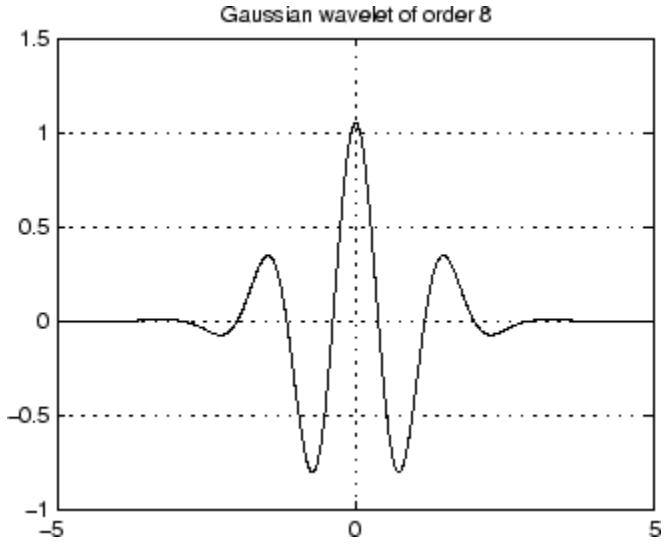
The Gaussian function is defined as $C_p e^{-x^2}$. C_p is such that the 2-norm of the P^{th} derivative of `PSI` is equal to 1.

`[PSI,X] = gauswavf(LB,UB,N,WAVNAME)` uses the valid wavelet family short name `WAVNAME` plus the order of the derivative in a string, such as `'gaus4'`. To see valid strings for Gaussian wavelets, use `waveinfo('gaus')` or use `wavemngr('read',1)` and refer to the Gaussian section.

Note: For visualizing the second or third order derivative of Gaussian wavelets, the convention is to use the negative of the normalized derivative. In the case of the second derivative, scaling by -1 produces a wavelet with its main lobe in the positive y direction. This scaling also makes the Gaussian wavelet resemble the Mexican hat wavelet. The validity of the wavelet is not affected by the -1 scaling factor.

Examples

```
% Set effective support and grid parameters.
lb = -5; ub = 5; n = 1000;

% Compute Gaussian wavelet of order 8.
[psi,x] = gauswavf(lb,ub,n,8);

% Plot Gaussian wavelet of order 8.
plot(x,psi),
title('Gaussian wavelet of order 8'), grid
```



See Also

waveinfo

Introduced before R2006a

get

WPTREE contents

Syntax

```
[FieldValue1,FieldValue2, ...] =
get(T,'FieldName1','FieldName2', ...)
[FieldValue1,FieldValue2, ...] = get(T)
```

Description

`[FieldValue1,FieldValue2, ...] = get(T,'FieldName1','FieldName2', ...)` returns the content of the specified fields for the WPTREE object T.

For the fields that are objects or structures, you can get the subfield contents, giving the name of these subfields as `'FieldName'` values. (See “Examples” below.)

`[FieldValue1,FieldValue2, ...] = get(T)` returns all the field contents of the tree *T*.

The valid choices for `'FieldName'` are

`'dtree'`	DTREE parent object
`'wavInfo'`	Structure (wavelet information)

The fields of the wavelet information structure, `'wavInfo'`, are also valid for `'FieldName'`:

`'wavName'`	Wavelet name
`'Lo_D'`	Low Decomposition filter
`'Hi_D'`	High Decomposition filter
`'Lo_R'`	Low Reconstruction filter
`'Hi_R'`	High Reconstruction filter

`'entInfo'`	Structure (entropy information)

The fields of the entropy information structure, `'entInfo'`, are also valid for `'FieldName'`:

`'entName'`	Entropy name
`'entPar'`	Entropy parameter

Or fields of DTREE parent object:

`'ntree'`	NTREE parent object
`'allNI'`	All nodes information
`'terNI'`	Terminal nodes information

Or fields of NTREE parent object:

`'wtbo'`	WTBO parent object
`'order'`	Order of the tree
`'depth'`	Depth of the tree
`'spsch'`	Split scheme for nodes
`'tn'`	Array of terminal nodes of the tree

Or fields of WTBO parent object:

`'wtboInfo'`	Object information
`'ud'`	Userdata field

Examples

```
% Compute a wavelet packets tree
x = rand(1,1000);
t = wpdec(x,2,'db2');
o = get(t,'order');
[o,tn] = get(t,'order','tn');
[o,allNI,tn] = get(t,'order','allNI','tn');
[o,wavInfo,allNI,tn] = get(t,'order','wavInfo','allNI','tn');
```

```
[o,tn,Lo_D,EntName] = get(t,'order','tn','Lo_D','EntName');
[wo,nt,dt] = get(t,'wtbo','ntree','dtree');
```

See Also

`disp` | `read` | `set` | `write`

Introduced before R2006a

icwtft

Inverse CWT

Syntax

```
xrec = icwtft(cwtstruct)
xrec = icwtft(cwtstruct,'plot')
xrec = icwtft(cwtstruct,'signal',SIG,'plot')
```

Description

`xrec = icwtft(cwtstruct)` returns the inverse continuous wavelet transform of the CWT coefficients contained in the `cfs` field of the structure array `cwtstruct`. Obtain the structure array `cwtstruct` as the output of `cwtft`.

`xrec = icwtft(cwtstruct,'plot')` plots the reconstructed signal.

`xrec = icwtft(cwtstruct,'signal',SIG,'plot')` places a radio button in the bottom left corner of the plot. Enabling the radio button superimposes the plot of the input signal `SIG` on the plot of the reconstructed signal. By default the radio button is not enabled and only the reconstructed signal is plotted.

Input Arguments

`cwtstruct`

Structure array containing six fields.

- `cfs` — CWT coefficient matrix
- `scales` — Vector of scales
- `frequencies` — frequencies in cycles per unit time (or space) corresponding to the scales. If the sampling period units are seconds, the frequencies are in hertz. The elements of frequencies are in decreasing order to correspond to the elements in the scales vector.

- `omega` — Angular frequencies used in the Fourier transform
- `meanSig` — Mean of the analyzed signal
- `dt` — The sampling period
- `wav` — Analyzing wavelet used in the CWT with parameters specified

`cwtstruct` is the output of `cwtft`.

Output Arguments

`xrec`

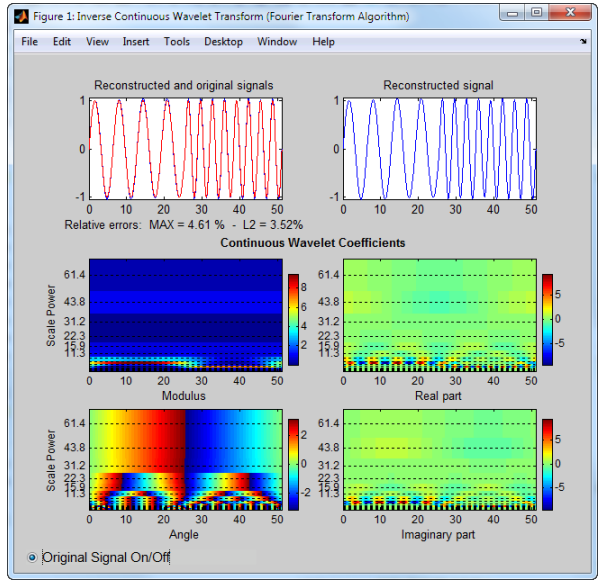
Reconstructed signal

Examples

Compute the CWT and inverse CWT of two sinusoids with disjoint support.

```
N = 1024;
t = linspace(0,1,N);
y = sin(2*pi*8*t).*(t<=0.5)+sin(2*pi*16*t).*(t>0.5);
dt = 0.05;
s0 = 2*dt;
ds = 0.4875;
NbSc = 20;
wname = 'morl';
sig = {y,dt};
sca = {s0,ds,NbSc};
wave = {wname,[]};
cwtsig = cwtft(sig,'scales',sca,'wavelet',wave);

% Compute inverse CWT and plot reconstructed signal with original
sigrec = icwtft(cwtsig,'signal',sig,'plot');
```

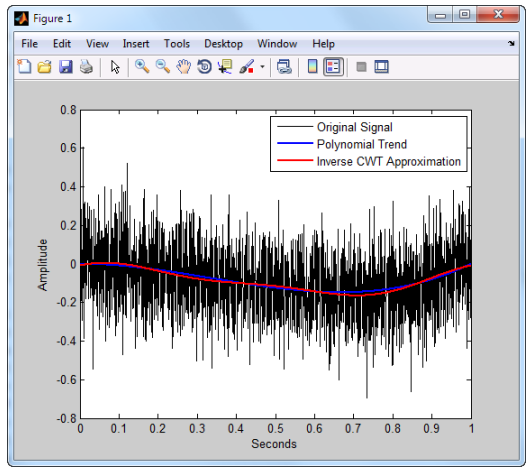
Select the radio button in the bottom left corner of the plot.

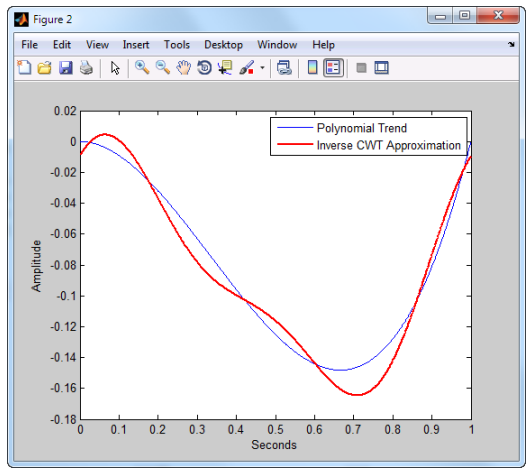


Use the inverse CWT to approximate a trend in a time series. Construct a time series consisting of a polynomial trend, a sinewave (oscillatory component), and additive white Gaussian noise. Obtain the CWT of the input signal and use the inverse CWT based on only the coarsest scales to reconstruct an approximation to the trend. To obtain an accurate approximation based on select scales use the default power of two spacing for the scales in the continuous wavelet transform. See `cwtft` for details.

```
t = linspace(0,1,1e3);
% Polynomial trend
x = t.^3-t.^2;
% Periodic term
x1 = 0.25*cos(2*pi*250*t);
% Reset random number generator for reproducible results
rng default
y = x+x1+0.1*randn(size(t));
% Obtain CWT of input time series
cwty = cwtft({y,0.001},'wavelet','morl');
% Zero out all but the coarsest scale CWT coefficients
cwty.cfs(1:16,:) = 0;
% Reconstruct a signal approximation based on the coarsest scales
xrec = icwtft(cwty);
plot(t,y,'k'); hold on;
xlabel('Seconds'); ylabel('Amplitude');
```

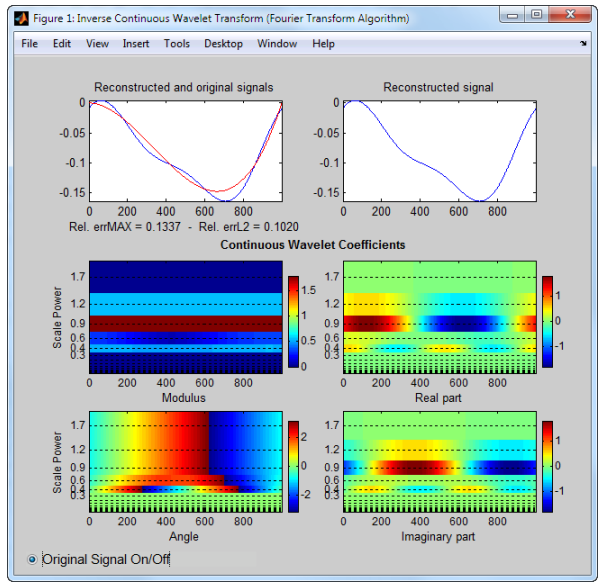
```
plot(t,x,'b','linewidth',2);
plot(t,xrec,'r','linewidth',2);
legend('Original Signal','Polynomial Trend',...
    'Inverse CWT Approximation');
figure
plot(t,x,'b'); hold on;
xlabel('Seconds'); ylabel('Amplitude');
plot(t,xrec,'r','linewidth',2);
legend('Polynomial Trend','Inverse CWT Approximation');
```





You can also use the following syntax to plot the approximation. Select the radio button to view the original polynomial trend superimposed on the wavelet approximation.

```
% Input the polynomial trend as the value of 'signal'
xrec = icwtft(cwty,'signal',x,'plot');
```



More About

Inverse CWT

`icwtft` computes the inverse CWT based on a discretized version of the single integral formula due to Morlet. The Wavelet Toolbox Getting Started Guide contains a brief description of the theoretical foundation for the single integral formula in “Inverse Continuous Wavelet Transform”. The discretized version of this integral is presented in [5]

- “Continuous and Discrete Wavelet Transforms”
- “Continuous Wavelet Transform and Scale-Based Analysis”
- “Inverse Continuous Wavelet Transform”

References

[1] Daubechies, I. *Ten Lectures on Wavelets*, Philadelphia, PA: Society for Industrial and Applied Mathematics (SIAM), 1992.

[2] Farge, M. “Wavelet Transforms and Their Application to Turbulence”, *Ann. Rev. Fluid. Mech*., 1992, 24, 395–457.

[3] Mallat, S. *A Wavelet Tour of Signal Processing*, San Diego, CA: Academic Press, 1998.

[4] Sun,W. “Convergence of Morlet's Reconstruction Formula”, *preprint*, 2010.

[5] Torrence, C. and G.P. Compo “A Practical Guide to Wavelet Analysis”, *Bull. Am. Meteorol. Soc.*, 79, 61–78, 1998.

See Also

`cwt` | `cwtft` | `icwtlin`

Introduced in R2011a

icwtlin

Inverse continuous wavelet transform (CWT) for linearly spaced scales

Syntax

```
xrec = icwtlin(cwtstruct)
xrec = icwtlin(wav,meanSIG,cfs,scales,dt)
xrec = icwtin(...,'plot')
xrec = icwtlin (...,'signal',SIG,'plot')
xrec = icwtlin(...,Name,Value)
```

Description

`xrec = icwtlin(cwtstruct)` returns the inverse continuous wavelet transform (CWT) of the CWT coefficients obtained at linearly spaced scales.

Note: To use `icwtlin` you must:

- Use linearly-spaced scales in the CWT. `icwtlin` does not verify that the scales are linearly-spaced.
- Use one of the supported wavelets. See "Input Arguments" on page 1-208 for a list of supported wavelets.

`xrec = icwtlin(wav,meanSIG,cfs,scales,dt)` returns the inverse CWT of the coefficients in `cfs`. The inverse CWT is obtained using the wavelet `wav`, the linearly spaced scales `scales`, the sampling period `dt`, and the mean signal value `meanSig`.

`xrec = icwtin(...,'plot')` plots the reconstructed signal `xrec` along with the CWT coefficients and CWT moduli. If the analyzing wavelet is complex-valued, the plot includes the real and imaginary parts of the CWT coefficients.

`xrec = icwtlin (...,'signal',SIG,'plot')` places a radio button in the bottom-left corner of the plot. Enabling the radio button superimposes the plot of the input signal `SIG` on the plot of the reconstructed signal. `SIG` can be a structure array, a cell array, or

a vector. If `SIG` is a structure array, there must be two fields: `val` and `period`. The `val` field contains the signal and the `period` field contains the sampling period. If `SIG` is a cell array, `SIG{1}` contains the signal and `SIG{2}` is the sampling period.

`xrec = icwtlin(...,Name,Value)` returns the inverse CWT transform with additional options specified by one or more `Name,Value` pair arguments.

Input Arguments

`cwtstruct`

A structure array that is the output of `cwtft` or constructed from the output of `cwt`. If you obtain `cwtstruct` from `cwtft`, the structure array contains seven fields:

- `cfs` — CWT coefficient matrix
- `scales` — Vector of linearly spaced scales. The scale vector must be linearly-spaced to ensure accurate reconstruction. `icwtlin` does not check that the spacing of your scale vector is linear.
- `frequencies` — frequencies in cycles per unit time (or space) corresponding to the scales. If the sampling period units are seconds, the frequencies are in hertz. The elements of frequencies are in decreasing order to correspond to the elements in the scales vector.
- `omega` — Angular frequencies used in the Fourier transform in radians/sample
- `MeanSIG` — Signal mean
- `dt` — Sampling period in seconds
- `wav` — Analyzing wavelet. `icwtlin` uses this wavelet as the reconstruction wavelet. The supported wavelets are:
  - `'dog'` — An m-th order derivative of Gaussian wavelet where m is a positive even integer
  - `'morl'` — Analytic Morlet wavelet
  - `'morlex'` — Nonanalytic Morlet wavelet
  - `'morl0'` — Nonanalytic Morlet wavelet with exact zero mean
  - `'mexh'` — Mexican-hat wavelet. This argument represents a special case of the derivative of Gaussian wavelet with m=2.
  - `'paul'` — Paul wavelet

- `'bump'` — Bump wavelet

If you create `cwtstruct` from the output of `cwt`, `cwtstruct` contains all of the preceding fields except `omega`.

Using `cwt` to obtain the CWT coefficients, the valid analyzing wavelets are:

- Coiflets — `'coif1'`,`'coif2'` ,`'coif3'` ,`'coif4'`, `'coif5'`
- Biorthogonal wavelets — `'bior2.2'`, `'bior2.4'`, `'bior2.6'`, `'bior2.8'`, `'bior4.4'`, `'bior5.5'`, `bior6.8`
- Reverse biorthogonal wavelets — `'rbio2.2'`, `'rbio2.4'`, `'rbio2.6'`, `'rbio2.8'`, `'rbio4.4'`, `'rbio5.5'`, `'rbio6.8'`
- Complex Gaussian wavelets — `'cgau2'`, `'cgau4'`, `'cgau6'`, `'cgau8'`

Name-Value Pair Arguments

`'IdxSc'`

Vector of scales to use in the signal reconstruction. Specifying a subset of scales results in a scale-localized approximation of the analyzed signal.

Output Arguments

`xrec`

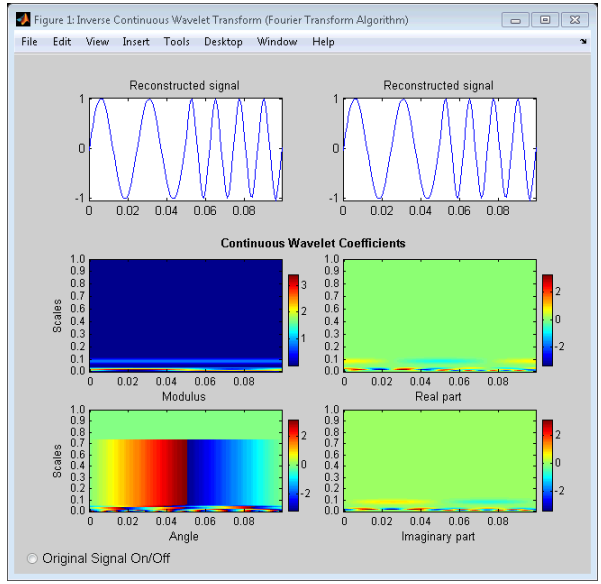
Reconstructed signal. Signal approximation based on the input CWT coefficient matrix, analyzing wavelet, selected scales, and sampling period.

The purpose of the CWT inversion algorithm is not to produce a perfect reconstruction of the input signal. The inversion preserves time and scale-localized features in the reconstructed signal. The amplitude scaling in the reconstructed signal, however, can be significantly different. This difference in scaling can occur whether or not you use all the CWT coefficients in the inversion.

Examples

Compute the inverse CWT of a sum of sine waves with disjoint support.

```
% Define the signal
N = 100;
t = linspace(0,1,N);
Y = sin(8*pi*t).*(t<=0.5) + sin(16*pi*t).*(t>0.5) ;

% Define parameters before analysis
dt = 0.001;
maxsca = 1; s0 = 2*dt; ds = 2*dt;
scales = s0:ds:maxsca;
wname = 'morl';
SIG = {Y,dt};
WAV = {wname,[]};

% Compute the CWT using cwtft with linear scales
cwtS = cwtft(SIG,'scales',scales,'wavelet',WAV);
% Compute inverse CWT using linear scales
Yrec = icwtlin(cwtS,'Signal',Y,'plot');
```

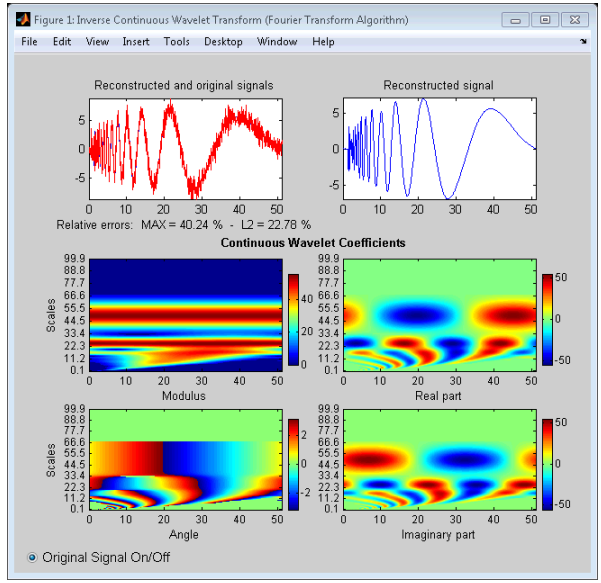


Reconstruct an approximation to a noisy Doppler signal based on thresholded coefficients. Use the universal threshold. Assume the sampling period is 0.05 seconds.

```
load noisdopp;
Y = noisdopp;
```

```
N = length(Y);

% Define parameters before analysis
% Assume sampling period is 0.05
dt = 0.05;
maxsca = 100; s0 = 2*dt; ds = 4*dt;
scales = s0:ds:maxsca;
wname = 'morl';
SIG = {Y,dt};
WAV = {wname,[]};

% Compute CWT
cwtS = cwtft(SIG,'scales',scales,'wavelet',WAV,'plot');

% Select subset of coefficients
cwtS1 = cwtS;
Hfreq = cwtS.cfs(1:10,:);
% Set threshold
thr = sqrt(2*log(N))*median(abs(Hfreq(:)))/0.6745;
newCFS = cwtS.cfs;
% Set coefficients smaller than threshold in absolute value to 0
newCFS(abs(newCFS)<thr) = 0;
cwtS1.cfs = newCFS;


% Reconstruction from the modified structure
YRDen = icwtlin(cwtS1,'signal',Y,'plot');
```

Enable the **Reconstructed Signal On/Off** radio button in the bottom-left corner.



Alternatives

- `icwtft` — Computes the inverse for the CWT obtained using `cwtft` with logarithmically spaced scales. If you use linearly spaced scales in `cwtft`, or you obtain the CWT with `cwt`, use `icwtlin` to compute the inverse.

More About

Algorithms

See [4] for a description of the inverse CWT algorithm for linearly spaced scales. The `icwtlin` function uses heuristic scaling factors for the analyzing wavelets. These scaling factors can result in significant differences in the amplitude scaling of the reconstructed signal.

- "Continuous and Discrete Wavelet Transforms"
- "Continuous Wavelet Transform and Scale-Based Analysis"
- "Inverse Continuous Wavelet Transform"

References

[1] Daubechies, I. *Ten Lectures on Wavelets*, Philadelphia, PA: Society for Industrial and Applied Mathematics (SIAM), 1992.

[2] Farge, M. “Wavelet Transforms and Their Application to Turbulence”, *Ann. Rev. Fluid. Mech*., 1992, 24, 395–457.

[3] Mallat, S. *A Wavelet Tour of Signal Processing*, San Diego, CA: Academic Press, 1998.

[4] Sun,W. “Convergence of Morlet's Reconstruction Formula”, *preprint*, 2010.

[5] Torrence, C. and G.P. Compo. “A Practical Guide to Wavelet Analysis”, *Bull. Am. Meteorol. Soc*., 79, 61–78, 1998.

See Also

`icwtft` | `cwtft` | `cwt`

Introduced in R2011b

idddtree

Inverse dual-tree and double-density 1-D wavelet transform

Syntax

```
xrec = idddtree(wt)
```

Description

`xrec = idddtree(wt)` returns the inverse wavelet transform of the wavelet decomposition (analysis filter bank), `wt`. `wt` is the output of `dddtree`.

Examples

Perfect Reconstruction Using the Dual-Tree Double-Density Wavelet Filter Bank

Demonstrate perfect reconstruction of a signal using a dual-tree double-density wavelet transform.

Load the noisy Doppler signal. Obtain the dual-tree double-density wavelet transform down to level 5. Invert the transform and demonstrate perfect reconstruction.

```
load noisdopp;
wt = dddtree('cplxdddt',noisdopp,5,'FSdoubledualfilt',...
     'doubledualfilt');
xrec = idddtree(wt);
max(abs(noisdopp-xrec))
```

- “Analytic Wavelets Using the Dual-Tree Wavelet Transform”

Input Arguments

`wt` — Wavelet transform

structure

Wavelet transform, returned as a structure from `dddtree` with these fields:

`type` — Type of wavelet decomposition (filter bank)

`'dwt' | 'ddt' | 'cplxdt' | 'cplxdddt'`

Type of wavelet decomposition (filter bank), specified as one of `'dwt'`, `'ddt'`, `'cplxdt'`, or `'cplxdddt'`. The type, `'dwt'`, gives a critically sampled discrete wavelet transform. The other types are oversampled wavelet transforms. `'ddt'` is a double-density wavelet transform, `'cplxdt'` is a dual-tree complex wavelet transform, and `'cplxdddt'` is a double-density dual-tree complex wavelet transform.

`level` — Level of wavelet decomposition

positive integer

Level of wavelet decomposition, specified as a positive integer.

`filters` — Decomposition (analysis) and reconstruction (synthesis) filters

structure

Decomposition (analysis) and reconstruction (synthesis) filters, specified as a structure with these fields:

`Fdf` — First-stage analysis filters

matrix | cell array

First-stage analysis filters, specified as an *N*-by-2 or *N*-by-3 matrix for single-tree wavelet transforms, or a cell array of two *N*-by-2 or *N*-by-3 matrices for dual-tree wavelet transforms. The matrices are *N*-by-3 for the double-density wavelet transforms. For an *N*-by-2 matrix, the first column of the matrix is the scaling (lowpass) filter and the second column is the wavelet (highpass) filter. For an *N*-by-3 matrix, the first column of the matrix is the scaling (lowpass) filter and the second and third columns are the wavelet (highpass) filters. For the dual-tree transforms, each element of the cell array contains the first-stage analysis filters for the corresponding tree.

`Df` — Analysis filters for levels > 1

matrix | cell array

Analysis filters for levels > 1, specified as an *N*-by-2 or *N*-by-3 matrix for single-tree wavelet transforms, or a cell array of two *N*-by-2 or *N*-by-3 matrices for dual-tree wavelet transforms. The matrices are *N*-by-3 for the double-density wavelet transforms. For an *N*-by-2 matrix, the first column of the matrix is the scaling (lowpass) filter and the second

column is the wavelet (highpass) filter. For an *N*-by-3 matrix, the first column of the matrix is the scaling (lowpass) filter and the second and third columns are the wavelet (highpass) filters. For the dual-tree transforms, each element of the cell array contains the analysis filters for the corresponding tree.

`Frf` — First-level reconstruction filters

matrix | cell array

First-level reconstruction filters, specified as an *N*-by-2 or *N*-by-3 matrix for single-tree wavelet transforms, or a cell array of two *N*-by-2 or *N*-by-3 matrices for dual-tree wavelet transforms. The matrices are *N*-by-3 for the double-density wavelet transforms. For an *N*-by-2 matrix, the first column of the matrix is the scaling (lowpass) filter and the second column is the wavelet (highpass) filter. For an *N*-by-3 matrix, the first column of the matrix is the scaling (lowpass) filter and the second and third columns are the wavelet (highpass) filters. For the dual-tree transforms, each element of the cell array contains the first-stage synthesis filters for the corresponding tree.

`Rf` — Reconstruction filters for levels > 1

matrix | cell array

Reconstruction filters for levels > 1, specified as an *N*-by-2 or *N*-by-3 matrix for single-tree wavelet transforms, or a cell array of two *N*-by-2 or *N*-by-3 matrices for dual-tree wavelet transforms. The matrices are N-by-3 for the double-density wavelet transforms. For an *N*-by-2 matrix, the first column of the matrix is the scaling (lowpass) filter and the second column is the wavelet (highpass) filter. For an *N*-by-3 matrix, the first column of the matrix is the scaling (lowpass) filter and the second and third columns are the wavelet (highpass) filters. For the dual-tree transforms, each element of the cell array contains the synthesis filters for the corresponding tree.

`cfs` — Wavelet transform coefficients

cell array of matrices

Wavelet transform coefficients, specified as a 1-by-(`level`+1) cell array of matrices. The size and structure of the matrix elements of the cell array depend on the type of wavelet transform as follows:

- `'dwt'` — `cfs{j}`
  - j = 1,2,... `level` is the level.
  - `cfs{level+1}` are the lowpass, or scaling, coefficients.
- `'ddt'` — `cfs{j}(:,:,k)`

  - j = 1,2,... `level` is the level.
  - k = 1,2 is the wavelet filter.
  - `cfs{level+1}(:,:)` are the lowpass, or scaling, coefficients.
- `'cplxdt'` — `cfs{j}(:,:,m)`
  - j = 1,2,... `level` is the level.
  - m = 1,2 are the real and imaginary parts.
  - `cfs{level+1}(:,:)` are the lowpass, or scaling, coefficients.
- `'cplxdddt'` — `cfs{j}(:,:,k,m)`
  - j = 1,2 `level` is the level.
  - k = 1,2 is the wavelet filter.
  - m = 1,2 are the real and imaginary parts.
  - `cfs{level+1}(:,:)` are the lowpass, or scaling, coefficients.

Output Arguments

`xrec` — Synthesized 1-D signal

vector

Synthesized 1-D signal, returned as a vector. The row or column orientation of `xrec` depends on the row or column orientation of the 1-D signal input to `dddtree`.

Data Types: `double`

More About

- "Critically Sampled and Oversampled Wavelet Filter Banks"

See Also

`dddtree` | `dddtreecfs` | `plotdt`

Introduced in R2013b

idddtree2

Inverse dual-tree and double-density 2-D wavelet transform

Syntax

```
xrec = idddtree2(wt)
```

Description

`xrec = idddtree2(wt)` returns the inverse wavelet transform of the 2-D decomposition (analysis filter bank), `wt`. `wt` is the output of `dddtree2`.

Examples

Perfect Reconstruction Using the Complex Oriented Dual-Tree Wavelet Filter Bank

Demonstrate perfect reconstruction of an image using a complex oriented dual-tree wavelet transform.

Load the image and obtain the complex oriented dual-tree wavelet transform down to level 5 using `dddtree2`. Reconstruct the image using `idddtree2` and demonstrate perfect reconstruction.

```
load woman;
wt = dddtree2('cplxdt',X,5,'dtf2');
xrec = idddtree2(wt);
max(max(abs(X-xrec)))
```

- "Analytic Wavelets Using the Dual-Tree Wavelet Transform"

Input Arguments

`wt` — Wavelet transform

structure

Wavelet transform, returned as a structure from `dddtree2` with these fields:

`type` — Type of wavelet decomposition (filter bank)

`'dwt' | 'ddt' | 'realdt' | 'cplxdt' | 'realdddt' | 'cplxdddt'`

Type of wavelet decomposition (filter bank), specified as one of `'dwt'`, `'ddt'`, `'realdt'`, `'cplxdt'`, `'realdddt'`, or `'cplxdddt'`. `'dwt'` is the critically sampled DWT. `'ddt'` produces a double-density wavelet transform with one scaling and two wavelet filters for both row and column filtering. `'realdt'` and `'cplxdt'` produce oriented dual-tree wavelet transforms consisting of two and four separable wavelet transforms. `'realdddt'` and `'cplxdddt'` produce double-density dual-tree wavelet transforms consisting of two and four separable wavelet transforms.

`level` — Level of the wavelet decomposition

positive integer

Level of the wavelet decomposition, specified as a positive integer.

`filters` — Decomposition (analysis) and reconstruction (synthesis) filters

structure

Decomposition (analysis) and reconstruction (synthesis) filters, specified as a structure with these fields:

`Fdf` — First-stage analysis filters

matrix | cell array

First-stage analysis filters, specified as an *N*-by-2 or *N*-by-3 matrix for single-tree wavelet transforms, or a 1-by-2 cell array of two *N*-by-2 or *N*-by-3 matrices for dual-tree wavelet transforms. The matrices are *N*-by-3 for the double-density wavelet transforms. For an *N*-by-2 matrix, the first column of the matrix is the scaling (lowpass) filter and the second column is the wavelet (highpass) filter. For an *N*-by-3 matrix, the first column of the matrix is the scaling (lowpass) filter and the second and third columns are the wavelet (highpass) filters. For the dual-tree transforms, each element of the cell array contains the first-stage analysis filters for the corresponding tree.

`Df` — Analysis filters for levels > 1

matrix | cell array

Analysis filters for levels > 1, specified as an *N*-by-2 or *N*-by-3 matrix for single-tree wavelet transforms, or a 1-by-2 cell array of two *N*-by-2 or *N*-by-3 matrices for dual-tree wavelet transforms. The matrices are *N*-by-3 for the double-density wavelet transforms. For an *N*-by-2 matrix, the first column of the matrix is the scaling (lowpass) filter and the second column is the wavelet (highpass) filter. For an *N*-by-3 matrix, the first column of the matrix is the scaling (lowpass) filter and the second and third columns are the

wavelet (highpass) filters. For the dual-tree transforms, each element of the cell array contains the analysis filters for the corresponding tree.

`Frf` — First-level reconstruction filters

matrix | cell array

First-level reconstruction filters, specified as an *N*-by-2 or *N*-by-3 matrix for single-tree wavelet transforms, or a 1-by-2 cell array of two *N*-by-2 or *N*-by-3 matrices for dual-tree wavelet transforms. The matrices are *N*-by-3 for the double-density wavelet transforms. For an *N*-by-2 matrix, the first column of the matrix is the scaling (lowpass) filter and the second column is the wavelet (highpass) filter. For an *N*-by-3 matrix, the first column of the matrix is the scaling (lowpass) filter and the second and third columns are the wavelet (highpass) filters. For the dual-tree transforms, each element of the cell array contains the first-stage synthesis filters for the corresponding tree.

`Rf` — Reconstruction filters for levels > 1

matrix | cell array

Reconstruction filters for levels > 1, specified as an *N*-by-2 or *N*-by-3 matrix for single-tree wavelet transforms, or a 1-by-2 cell array of two *N*-by-2 or *N*-by-3 matrices for dual-tree wavelet transforms. The matrices are *N*-by-3 for the double-density wavelet transforms. For an *N*-by-2 matrix, the first column of the matrix is the scaling (lowpass) filter and the second column is the wavelet (highpass) filter. For an *N*-by-3 matrix, the first column of the matrix is the scaling (lowpass) filter and the second and third columns are the wavelet (highpass) filters. For the dual-tree transforms, each element of the cell array contains the first-stage analysis filters for the corresponding tree.

`cfs` — Wavelet transform coefficients

cell array of matrices

Wavelet transform coefficients, specified as a 1-by-(`level`+1) cell array of matrices. The size and structure of the matrix elements of the cell array depend on the type of wavelet transform as follows:

- `'dwt'` — `cfs{j}(:,:,d)`
  - j = 1,2,... `level` is the level.
  - d = 1,2,3 is the orientation.
  - `cfs{level+1}(:,:)` are the lowpass, or scaling, coefficients.
- `'ddt'` — `cfs{j}(:,:,d)`
  - j = 1,2,... `level` is the level.

- d = 1,2,3,4,5,6,7,8 is the orientation.
- `cfs{level+1}(:,:)` are the lowpass, or scaling, coefficients.

- `'realddt'` — `cfs{j}(:,:,d,k)`
  - j = 1,2,... `level` is the level.
  - d = 1,2,3 is the orientation.
  - k = 1,2 is the wavelet transform tree.
  - `cfs{level+1}(:,:)` are the lowpass, or scaling, coefficients.
- `'cplxdt'` — `cfs{j}(:,:,d,k,m)`
  - j = 1,2,... `level` is the level.
  - d = 1,2,3 is the orientation.
  - k = 1,2 is the wavelet transform tree.
  - m = 1,2 are the real and imaginary parts.
  - `cfs{level+1}(:,:)` are the lowpass, or scaling, coefficients..
- `'realdddt'` — `cfs{j}(:,:,d,k)`
  - j = 1,2,... `level` is the level.
  - d = 1,2,3 is the orientation.
  - k = 1,2 is the wavelet transform tree.
  - `cfs{level+1}(:,:)` are the lowpass, or scaling, coefficients.
- `'cplxdddt'` — `cfs{j}(:,:,d,k,m)`
  - j = 1,2,... `level` is the level.
  - d = 1,2,3 is the orientation.
  - k = 1,2 is the wavelet transform tree.
  - m = 1,2 are the real and imaginary parts.
  - `cfs{level+1}(:,:)` are the lowpass, or scaling, coefficients.

Output Arguments

`xrec` — Synthesized 2-D image

matrix

Synthesized image, returned as a matrix.

Data Types: `double`

More About

- "Critically Sampled and Oversampled Wavelet Filter Banks"

See Also

`dddtree2` | `dddtreecfs`

Introduced in R2013b

idwt

Single-level inverse discrete 1-D wavelet transform

Syntax

```
X = idwt(cA,cD,'wname')
X = idwt(cA,cD,Lo_R,Hi_R)
X = idwt(cA,cD,'wname',L)
X = idwt(cA,cD,Lo_R,Hi_R,L)
idwt(cA,cD,'wname')
X = idwt(...,'mode',MODE)
X = idwt(cA,[],...)
X = idwt([],cD,...)
```

Description

The `idwt` command performs a single-level one-dimensional wavelet reconstruction with respect to either a particular wavelet (`'wname'`, see `wfilters` for more information) or particular wavelet reconstruction filters (`Lo_R` and `Hi_R`) that you specify.

`X = idwt(cA,cD,'wname')` returns the single-level reconstructed approximation coefficients vector `X` based on approximation and detail coefficients vectors `cA` and `cD`, and using the wavelet `'wname'`.

`X = idwt(cA,cD,Lo_R,Hi_R)` reconstructs as above using filters that you specify.

- `Lo_R` is the reconstruction low-pass filter.
- `Hi_R` is the reconstruction high-pass filter.

`Lo_R` and `Hi_R` must be the same length.

Let `la` be the length of `cA` (which also equals the length of `cD`) and `lf` the length of the filters `Lo_R` and `Hi_R`; then `length(X) = LX` where `LX = 2*la` if the DWT extension mode is set to periodization. For the other extension modes `LX = 2*la-lf+2`.

For more information about the different Discrete Wavelet Transform extension modes, see `dwtmode`.

`X = idwt(cA,cD,'wname',L)` or `X = idwt(cA,cD,Lo_R,Hi_R,L)` returns the length-`L` central portion of the result obtained using `idwt(cA,cD,'wname')`. `L` must be less than `LX`.

`X = idwt(...,'mode',MODE)` computes the wavelet reconstruction using the specified extension mode `MODE`.

`X = idwt(cA,[],...)` returns the single-level reconstructed approximation coefficients vector `X` based on approximation coefficients vector `cA`.

`X = idwt([],cD,...)` returns the single-level reconstructed detail coefficients vector `X` based on detail coefficients vector `cD`.

Examples

Inverse DWT Using Orthogonal Wavelet

Demonstrate perfect reconstruction using `dwt` and `idwt` with an orthonormal wavelet.

```
load noisdopp;
[A,D] = dwt(noisdopp,'sym4');
x = idwt(A,D,'sym4');
max(abs(noisdopp-x))
```

Inverse DWT Using Biorthgonal Wavelet

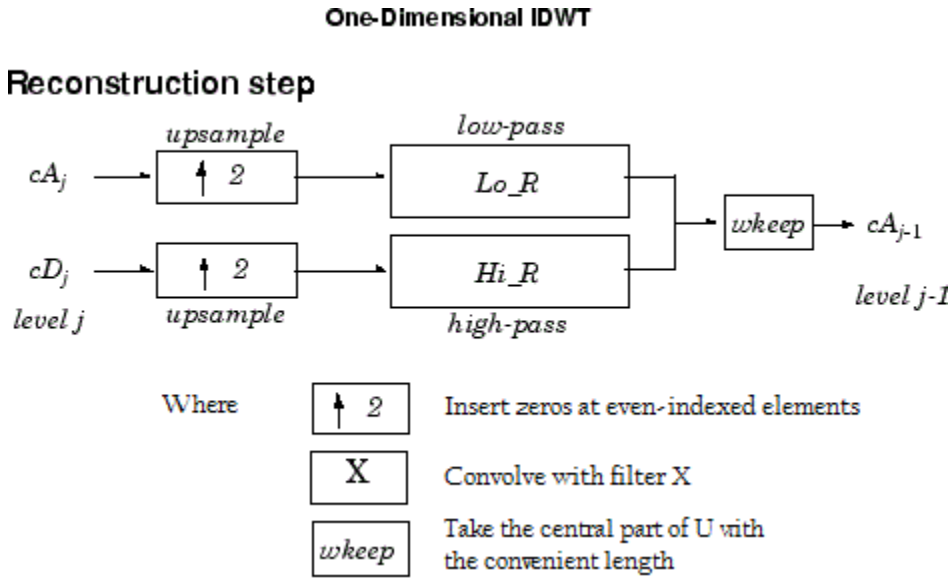
Demonstrate perfect reconstruction using `dwt` and `idwt` with a biorthogonal wavelet.

```
load noisdopp;
[Lo_D,Hi_D,Lo_R,Hi_R] = wfilters('bior3.5');
[A,D] = dwt(noisdopp,Lo_D,Hi_D);
x = idwt(A,D,Lo_R,Hi_R);
max(abs(noisdopp-x))
```

More About

Algorithms

Starting from the approximation and detail coefficients at level j, cAj and cD_j, the inverse discrete wavelet transform reconstructs cA_{j-1}, inverting the decomposition step by inserting zeros and convolving the results with the reconstruction filters.



References

Daubechies, I. (1992), *Ten lectures on wavelets*, CBMS-NSF conference series in applied mathematics. SIAM Ed.

Mallat, S. (1989), "A theory for multiresolution signal decomposition: the wavelet representation," *IEEE Pattern Anal. and Machine Intell.*, vol. 11, no. 7, pp. 674–693.

Meyer, Y. (1990), *Ondelettes et opérateurs*, Tome 1, Hermann Ed. (English translation: *Wavelets and operators*, Cambridge Univ. Press. 1993.)

See Also

`dwt` | `dwtmode` | `upwlev`

Introduced before R2006a

idwt2

Single-level inverse discrete 2-D wavelet transform

Syntax

```
X = idwt2(cA,cH,cV,cD,'wname')
X = idwt2(cA,cH,cV,cD,Lo_R,Hi_R)
X = idwt2(cA,cH,cV,cD,'wname',S)
X = idwt2(cA,cH,cV,cD,Lo_R,Hi_R,S)
idwt2(cA,cH,cV,cD,'wname')
X = idwt2(...,'mode',MODE)
X = idwt2(cA,[],[],[],...)
X = idwt2([],cH,[],[],...)
```

Description

The `idwt2` command performs a single-level two-dimensional wavelet reconstruction with respect to either a particular wavelet (`'wname'`, see `wfilters` for more information) or particular wavelet reconstruction filters (`Lo_R` and `Hi_R`) that you specify.

`X = idwt2(cA,cH,cV,cD,'wname')` uses the wavelet `'wname'` to compute the single-level reconstructed approximation coefficients matrix *X*, based on approximation matrix `cA` and details matrices `cH`,`cV`, and `cD` (horizontal, vertical, and diagonal, respectively).

`X = idwt2(cA,cH,cV,cD,Lo_R,Hi_R)` reconstructs as above, using filters that you specify.

- `Lo_R` is the reconstruction low-pass filter.
- `Hi_R` is the reconstruction high-pass filter.

`Lo_R` and `Hi_R` must be the same length.

Let `sa = size(cA) = size(cH) = size(cV) = size(cD)` and `lf` the length of the filters; then `size(X) = SX`, where `SX = 2* SA`, if the DWT extension mode is set to periodization. For the other extension modes, `SX = 2*size(cA)-lf+2`.

For more information about the different Discrete Wavelet Transform extension modes, see `dwtmode`.

`X = idwt2(cA,cH,cV,cD,'wname',S)` and `X = idwt2(cA,cH,cV,cD,Lo_R,Hi_R,S)` return the size-`S` central portion of the result obtained using the syntax `idwt2(cA,cH,cV,cD,'wname')`. `S` must be less than `SX`.

`X = idwt2(...,'mode',MODE)` computes the wavelet reconstruction using the extension mode `MODE` that you specify.

`X = idwt2(cA,[],[],[],...)` returns the single-level reconstructed approximation coefficients matrix `X` based on approximation coefficients matrix `cA`.

`X = idwt2([],cH,[],[],...)` returns the single-level reconstructed detail coefficients matrix `X` based on horizontal detail coefficients matrix `cH`.

The same result holds for `X = idwt2([],[],cV,[],...)` and `X = idwt2([],[],[],cD,...)`, based on vertical and diagonal details.

More generally, `X = idwt2(AA,HH,VV,DD,...)` returns the single-level reconstructed matrix `X`, where `AA` can be `cA` or `[]`, and so on.

`idwt2` is the inverse function of `dwt2` in the sense that the abstract statement `idwt2(dwt2(X,'wname'),'wname')` would give back `X`.

Examples

```
% The current extension mode is zero-padding (see dwtmode).

% Load original image.
load woman;

% X contains the loaded image.
sX = size(X);

% Perform single-level decomposition
% of X using db4.
[cA1,cH1,cV1,cD1] = dwt2(X,'db4');

% Invert directly decomposition of X
% using coefficients at level 1.
A0 = idwt2(cA1,cH1,cV1,cD1,'db4',sX);
```

```
% Check for perfect reconstruction.
max(max(abs(X-A0)))
ans =
    3.4176e-10
```

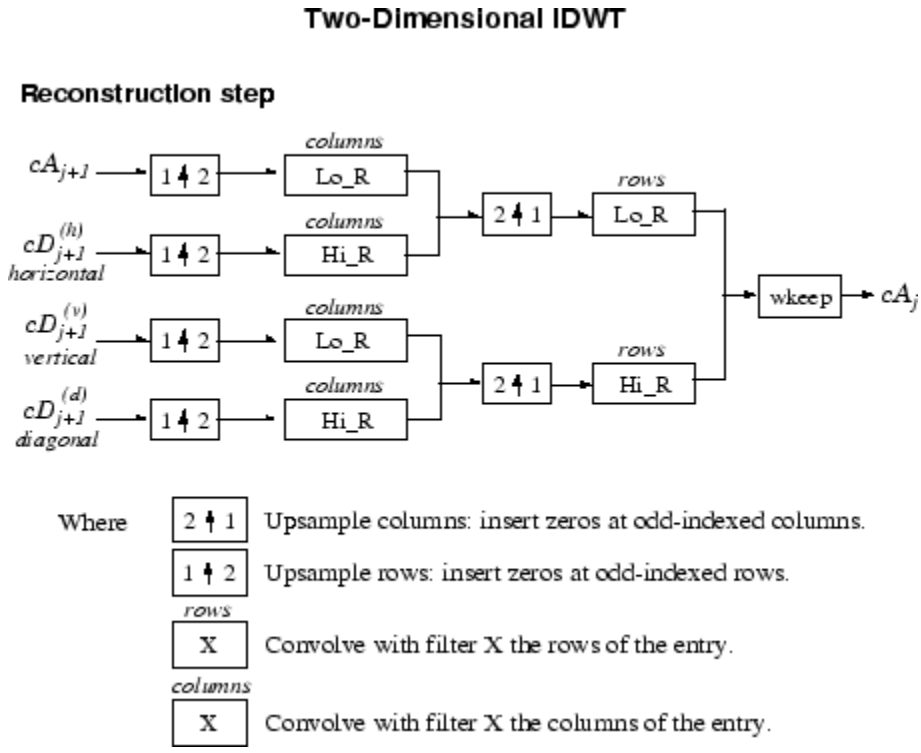
More About

Tips

If cA,cH,cV,cD are obtained from an indexed image analysis or a truecolor image analysis, they are `m`-by-`n` matrices or `m`-by-`n`-by-3 arrays, respectively.

For more information on image formats, see the `image` and `imfinfo` reference pages.

Algorithms



See Also

dwt2 | dwtmode | upwlev2

Introduced before R2006a

idwt3

Single-level inverse discrete 3-D wavelet transform

Syntax

```
X = idwt3(WT)
C = idwt3(WT,TYPE)
```

Description

The `idwt3` command performs a single-level three-dimensional wavelet reconstruction starting from a single-level three-dimensional wavelet decomposition.

`X = idwt3(WT)` computes the single-level reconstructed 3-D array X, based on the three-dimensional wavelet decomposition stored in the WT structure. This structure contains the following fields.

`sizeINI`	Size of the three-dimensional array `X`.
`mode`	Name of the wavelet transform extension mode.
`filters`	Structure with 4 fields, `LoD`, `HiD`, `LoR`, `HiR`, which contain the filters used for DWT.
`dec`	2 x 2 x 2 cell array containing the coefficients of the decomposition. `dec{i,j,k}`, `i`,`j`,`k = 1` or `2` contains the coefficients obtained by low-pass filtering (for `i` or `j` or `k = 1`) or high-pass filtering (for `i` or `j` or `k = 2`).

`C = idwt3(WT,TYPE)` computes the single-level reconstructed component based on the three-dimensional wavelet decomposition. Valid values for `TYPE` are:

- A group of three characters `'xyz'`, one per direction, with `'x'`,`'y'` and `'z'` selected in the set {`'a'`,`'d'`,`'l'`,`'h'`} or in the corresponding uppercase set {`'A'`,`'D'`,`'L'`,`'H'`}), where `'A'` (or `'L'`) specifies low-pass filter and `'D'` (or `'H'`) specifies high-pass filter.

- The char `'d'` (or `'h'` or `'D'` or `'H'`) which specifies the sum of all the components different from the low-pass component.

Examples

```
% Define original 3D data.
X  = reshape(1:64,4,4,4);

% Decompose X using db1.
wt = dwt3(X,'db1');

% Reconstruct X from coefficients.
XR = idwt3(wt);

% Compute reconstructed approximation, i.e. the
% low-pass component.
A  = idwt3(wt,'aaa');

% Compute the sum of all the components different
% from the low-pass component.
D  = idwt3(wt,'d');

% Reconstruct the component associated with low-pass in the
% X and Z directions and high-pass in the Y direction.
ADA  = idwt3(wt,'ada');
```

See Also

`dwt3` | `wavedec3` | `waverec3`

Introduced in R2010a

ilwt

Inverse 1-D lifting wavelet transform

Syntax

```
X = ilwt(AD_In_Place,W)
X = ilwt(CA,CD,W)
X = ilwt(AD_In_Place,W,LEVEL)
X = ILWT(CA,CD,W,LEVEL)
X = ilwt(AD_In_Place,W,LEVEL,'typeDEC',typeDEC)
X = ilwt(CA,CD,W,LEVEL,'typeDEC',typeDEC)
```

Description

`ilwt` performs a 1-D lifting wavelet reconstruction with respect to a particular lifted wavelet that you specify.

`X = ilwt(AD_In_Place,W)` computes the reconstructed vector `X` using the approximation and detail coefficients vector `AD_In_Place` obtained by a lifting wavelet reconstruction. `W` is a lifted wavelet name (see `liftwave`).

`X = ilwt(CA,CD,W)` computes the reconstructed vector `X` using the approximation coefficients vector `CA` and detail coefficients vector `CD` obtained by a lifting wavelet reconstruction.

`X = ilwt(AD_In_Place,W,LEVEL)` or `X = ILWT(CA,CD,W,LEVEL)` computes the lifting wavelet reconstruction, at level `LEVEL`.

`X = ilwt(AD_In_Place,W,LEVEL,'typeDEC',typeDEC)` or `X = ilwt(CA,CD,W,LEVEL,'typeDEC',typeDEC)` with `typeDEC = 'w'` or `'wp'` computes the wavelet or the wavelet packet decomposition using lifting, at level `LEVEL`.

Instead of a lifted wavelet name, you may use the associated lifting scheme `LS`: `X = ilwt(...,LS,...)` instead of `X = ILWT(...,W,...)`.

For more information about lifting schemes, see `lsinfo`.

Examples

```
% Start from the Haar wavelet and get the
% corresponding lifting scheme.
lshaar = liftwave('haar');

% Add a primal ELS to the lifting scheme.
els = {'p',[-0.125 0.125],0};
lsnew = addlift(lshaar,els);

% Perform LWT at level 1 of a simple signal.
x = 1:8;
[cA,cD] = lwt(x,lsnew);

% Perform integer LWT of the same signal.
lshaarInt = liftwave('haar','int2int');
lsnewInt = addlift(lshaarInt,els);
[cAint,cDint] = lwt(x,lsnewInt);

% Invert the two transforms.
xRec = ilwt(cA,cD,lsnew);
err = max(max(abs(x-xRec)))


err =

  4.4409e-016

xRecInt = ilwt(cAint,cDint,lsnewInt);
errInt = max(max(abs(x-xRecInt)))

errInt =

     0
```

See Also

lwt

Introduced before R2006a

ilwt2

Inverse 2-D lifting wavelet transform

Syntax

```
X = ilwt2(AD_In_Place,W)
X = ilwt2(CA,CH,CV,CD,W)
X = ilwt2(AD_In_Place,W,LEVEL)
X = ILWT2(CA,CH,CV,CD,W,LEVEL)
X = ilwt2(AD_In_Place,W,LEVEL,'typeDEC',typeDEC)
X = ilwt2(CA,CH,CV,CD,W,LEVEL,'typeDEC',typeDEC)
```

Description

`ilwt2` performs a 2-D lifting wavelet reconstruction with respect to a particular lifted wavelet that you specify.

`X = ilwt2(AD_In_Place,W)` computes the reconstructed matrix `X` using the approximation and detail coefficients matrix `AD_In_Place`, obtained by a lifting wavelet decomposition. `W` is a lifted wavelet name (see `liftwave`).

`X = ilwt2(CA,CH,CV,CD,W)` computes the reconstructed matrix `X` using the approximation coefficients vector `CA` and detail coefficients vectors `CH`, `CV`, and `CD` obtained by a lifting wavelet decomposition.

`X = ilwt2(AD_In_Place,W,LEVEL)` or `X = ILWT2(CA,CH,CV,CD,W,LEVEL)` computes the lifting wavelet reconstruction, at level `LEVEL`.

`X = ilwt2(AD_In_Place,W,LEVEL,'typeDEC',typeDEC)` or `X = ilwt2(CA,CH,CV,CD,W,LEVEL,'typeDEC',typeDEC)` with `typeDEC = 'w'` or `'wp'` computes the wavelet or the wavelet packet decomposition using lifting, at level `LEVEL`.

Instead of a lifted wavelet name, you may use the associated lifting scheme `LS`: `X = ilwt2(...,LS,...)` instead of `X = ilwt2(...,W,...)`.

For more information about lifting schemes, see `lsinfo`.

Examples

```
% Start from the Haar wavelet and get the
% corresponding lifting scheme.
lshaar = liftwave('haar');

% Add a primal ELS to the lifting scheme.
els = {'p',[-0.125 0.125],0};
lsnew = addlift(lshaar,els);

% Perform LWT at level 1 of a simple image.
x = reshape(1:16,4,4);
[cA,cH,cV,cD] = lwt2(x,lsnew);

% Perform integer LWT of the same image.
lshaarInt = liftwave('haar','int2int');
lsnewInt = addlift(lshaarInt,els);
[cAint,cHint,cVint,cDint] = lwt2(x,lsnewInt);

% Invert the two transforms.
xRec = ilwt2(cA,cH,cV,cD,lsnew);
err = max(max(abs(x-xRec)))

err =

     0

xRecInt = ilwt2(cAint,cHint,cVint,cDint,lsnewInt);
errInt = max(max(abs(x-xRecInt)))

errInt =

     0
```

More About

Tips

If `AD_In_Place` or cA,cH,cV,cD are obtained from an indexed image analysis or a truecolor image analysis, they are `m`-by-`n` matrices or `m`-by-`n`-by-3 arrays, respectively.

For more information on image formats, see the `image` and `imfinfo` reference pages.

See Also

`lwt2`

Introduced before R2006a

imodwpt

Inverse maximal overlap discrete wavelet packet transform

Syntax

```
xrec = imodwpt(coefs)
xrec = imodwpt(coefs,wname)
xrec = imodwpt(coefs,lo,hi)
```

Description

`xrec = imodwpt(coefs)` returns the inverse maximal overlap discrete wavelet packet transform (inverse MODWPT), in `xrec`. The inverse transform is for the terminal node coefficient matrix (`coefs`) obtained using `modwpt` with the default length 18 Fejer-Korovkin (`'fk18'`) wavelet.

`xrec = imodwpt(coefs,wname)` returns the inverse MODWPT using the orthogonal filter specified the string `wname`. This filter must be the same filter used in `modwpt`.

`xrec = imodwpt(coefs,lo,hi)` returns the inverse MODWPT using the orthogonal scaling filter, `lo`, and wavelet filter, `hi`.

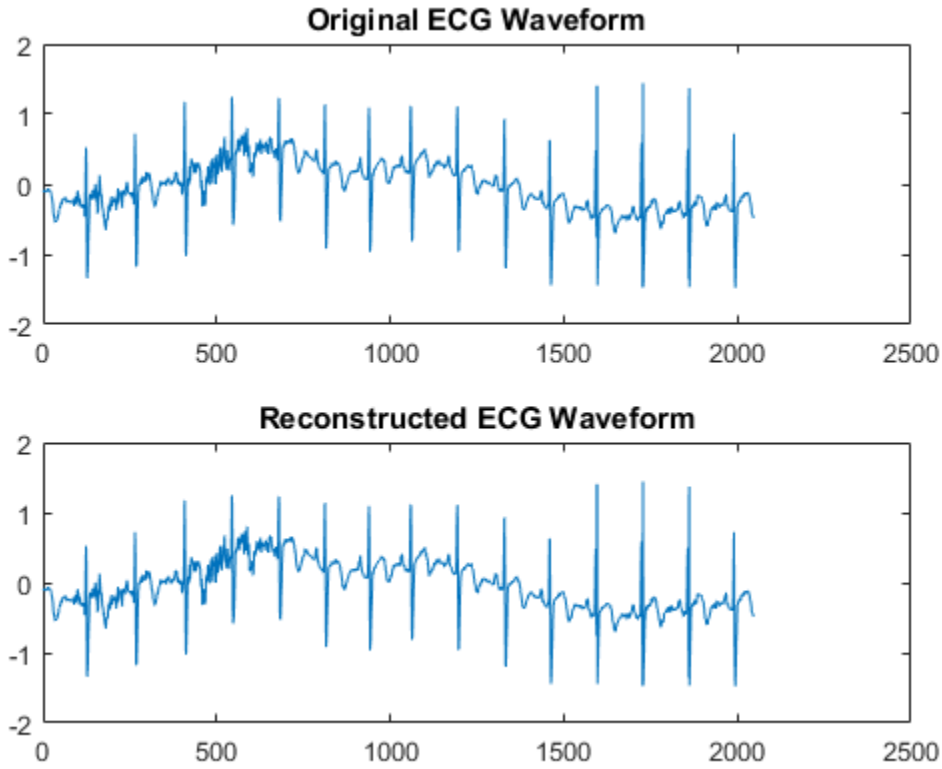
Examples

Perfect Reconstruction with the Inverse MODWPT

Obtain the MODWPT of an ECG waveform and demonstrate perfect reconstruction using the inverse MODWPT.

```
load wecg;
wpt = modwpt(wecg);
xrec = imodwpt(wpt);
subplot(2,1,1)
plot(wecg);
```

```
title('Original ECG Waveform');
subplot(2,1,2)
plot(xrec);
title('Reconstructed ECG Waveform');
```



Find the largest absolute difference between the original signal and the reconstruction. The difference is on the order of 10^{-11}, which demonstrates perfect reconstruction.

```
max(abs(wecg-xrec'))
```

```
ans =

   1.7903e-11
```

Inverse MODWPT Using Daubechies Extremal Phase Wavelet

Obtain the MODWPT of Southern Oscillation Index data using the Daubechies extremal phase wavelet with two vanishing moments (`'db2'`). Reconstruct the signal using the inverse MODWPT.

```
load soi;
wsoi = modwpt(soi,'db2');
xrec = imodwpt(wsoi,'db2');
```

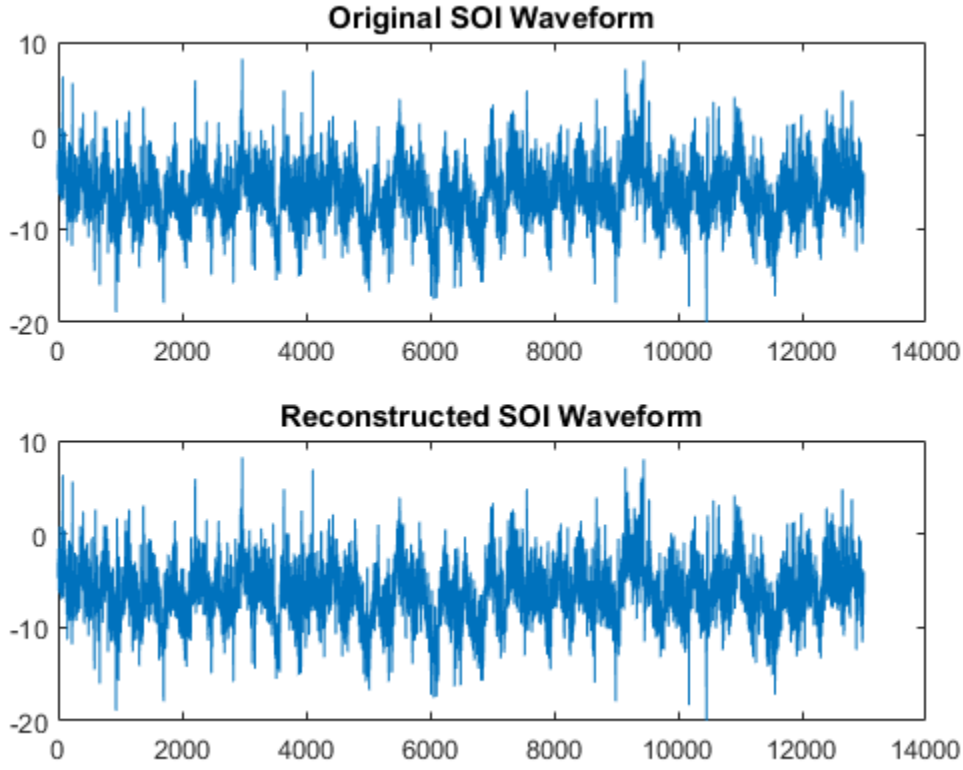
Inverse MODWPT Using Scaling and Wavelet Filters

Obtain the MODWPT of Southern Oscillation Index data using specified scaling and wavelets filters with the Daubechies extremal phase wavelet with two vanishing moments (`'db2'`).

```
load soi;
[lo,hi] = wfilters('db2');
wpt = modwpt(soi,lo,hi);
xrec = imodwpt(wpt,lo,hi);
```

Plot the original SOI waveform and the reconstructed waveform.

```
subplot(2,1,1)
plot(soi)
title('Original SOI Waveform');
subplot(2,1,2)
plot(xrec)
title('Reconstructed SOI Waveform')
```



Input Arguments

`coefs` — Terminal node coefficients

matrix

Terminal node coefficients of a wavelet packet tree, specified as a matrix. You must obtain the coefficient matrix from `modwpt` using the `'FullTree',false` option. `'FullTree',false` is the default value of `modwpt`.

Data Types: `double`

`wname` — Synthesizing wavelet filter

`fk18` (default) | string

Synthesizing wavelet filter used to invert the MODWPT, specified as a string. The specified wavelet must be the same wavelet as used in the analysis with `modwpt`.

`lo` — Scaling filter

even-length real-valued vector

Scaling filter, specified as an even-length real-valued vector. `lo` must be the same scaling filter as used in the analysis with `modwpt`. You cannot specify both a scaling-wavelet filter pair and a `wname` filter.

`hi` — Wavelet filter

even-length real-valued vector

Wavelet filter, specified as an even-length real-valued vector. `hi` must be the same wavelet filter used in the analysis with `modwpt`. You cannot specify both a scaling-wavelet filter pair and a `wname` filter.

Output Arguments

`xrec` — Inverse maximal overlap discrete wavelet packet transform

row vector

Inverse maximal overlap discrete wavelet packet transform, returned as a row vector. The inverse transform is the reconstructed version of the original signal based on the MODWPT terminal node coefficients. `xrec` has the same number of columns as the input `coefs` matrix.

References

[1] Percival, D. B., and A. T. Walden. *Wavelet Methods for Time Series Analysis*. Cambridge, UK: Cambridge University Press, 2000.

[2] Walden, A. T., and A. C. Cristan. "The Phase-Corrected Undecimated Discrete Wavelet Packet Transform and the Recurrence of High Latitude Interplanetary Shock Waves." *Statistics Section Technical Report TR-97-03*. London, UK: Dept. of Mathematics, Imperial College of Science, Technology & Medicine, 1997.

See Also

`modwpt` | `modwptdetails`

Introduced in R2016a

imodwt

Inverse maximal overlap discrete wavelet transform

Syntax

```
xrec = imodwt(w)
xrec = imodwt(w,wname)
xrec = imodwt(w,Lo,Hi)
xrec = imodwt(___,lev)
xrec = imodwt(___,'reflection')
```

Description

`xrec = imodwt(w)` returns in `xrec` a reconstructed version of the signal. The reconstructed signal is based on `w`, the maximal overlap discrete wavelet transform (MODWT) coefficients and on the level of reconstruction, which defaults to zero.

`xrec = imodwt(w,wname)` reconstructs the signal using `wname`, the orthogonal wavelet. `wname` must be the same wavelet used to analyze the signal input to `modwt`. The reconstruction is up to level 0, which is a perfect reconstruction of the original signal.

`xrec = imodwt(w,Lo,Hi)` reconstructs the signal using the orthogonal scaling filter `Lo` and the wavelet filter `Hi`. The `Lo` and `Hi` filters must be the same filters used to analyze the signal input to `modwt`. The reconstruction is up to level 0, which is a perfect reconstruction of the original signal.

`xrec = imodwt(___,lev)` reconstructs the signal up to level `lev`. `xrec` is a projection onto the scaling space at level `lev`.

`xrec = imodwt(___,'reflection')` uses the reflection boundary condition in the reconstruction. If you specify `'reflection'`, `imodwt` assumes that the length of the original signal length is one half the number of columns in the input coefficient matrix. By default, `imodwt` assumes periodic signal extension at the boundary.

Examples

Perfect Reconstruction with the Inverse MODWT

Obtain the MODWT of an ECG signal and demonstrate perfect reconstruction.

Load the ECG signal data and obtain the MODWT.

```
load wecg;
```

Obtain the MODWT and the Inverse MODWT.

```
w = modwt(wecg);
xrec = imodwt(w);
```

Use the L-infinity norm to show that the difference between the original signal and the reconstruction is extremely small. The largest absolute difference between the original signal and the reconstruction is on the order of 10^{-12}, which demonstrates perfect reconstruction.

```
norm(abs(xrec'-wecg),Inf)
```

```
ans =

   2.3253e-12
```

Inverse MODWT with Specified Wavelet

Obtain the MODWT of Deutsche Mark-U.S. Dollar exchange rate data and demonstrate perfect reconstruction.

Load the Deutsche Mark-U.S. Dollar exchange rate data.

```
load DM_USD;
```

Obtain the MODWT and the Inverse MODWT using the `'db2'` wavelet.

```
wdm = modwt(DM_USD,'db2');
xrec = imodwt(wdm,'db2');
```

Use the L-infinity norm to show that the difference between the original signal and the reconstruction is extremely small. The largest absolute difference between the original

signal and the reconstruction is on the order of 10^{-13}, which demonstrates perfect reconstruction.

```
norm(abs(xrec'-DM_USD),Inf)
```

```
ans =

   1.6362e-13
```

Inverse MODWT with Specified Filters

Obtain the MODWT of an ECG signal using the Fejer-Korovkin filters.

Load the ECG data.

```
load wecg;
```

Create the 8-coefficient Fejer-Korovkin filters.

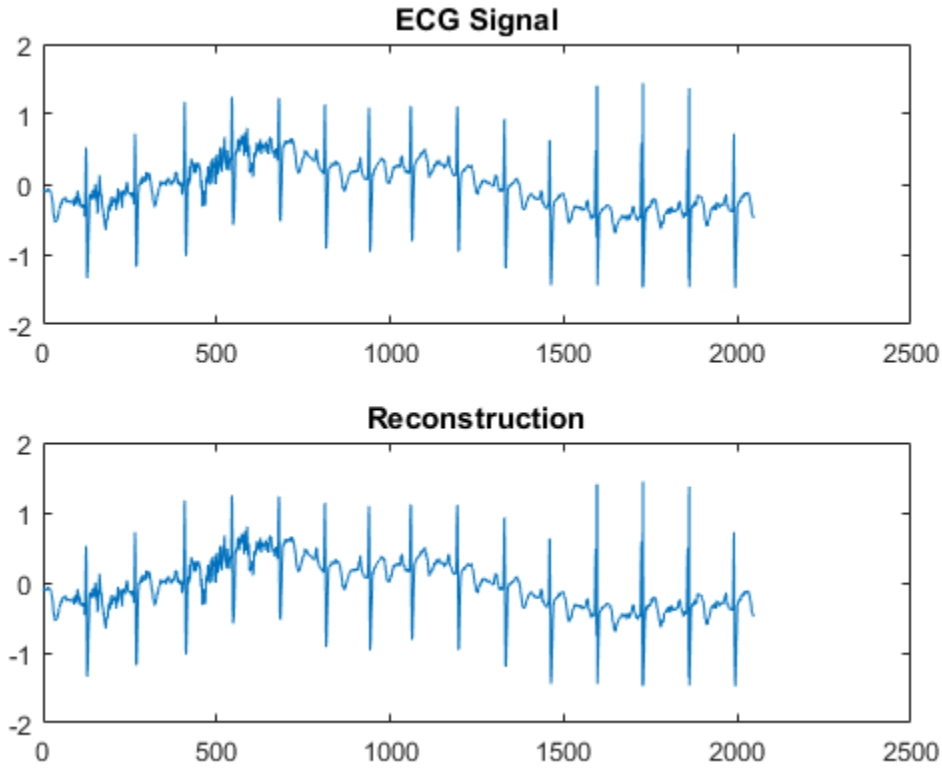
```
[Lo,Hi] = wfilters('fk8');
```

Obtain the MODWT and Inverse MODWT.

```
wtecg = modwt(wecg,Lo,Hi);
xrec = imodwt(wtecg,Lo,Hi);
```

Plot the original data and the reconstruction.

```
subplot(2,1,1)
plot(wecg)
title('ECG Signal');
subplot(2,1,2)
plot(xrec)
title('Reconstruction')
```



Obtain Projection onto Scaling Space

Obtain the MODWT of an ECG signal down to the maximum level and obtain the projection of the ECG signal onto the scaling space at level 3.

Load the ECG data.

```
load wecg;
```

Obtain the MODWT.

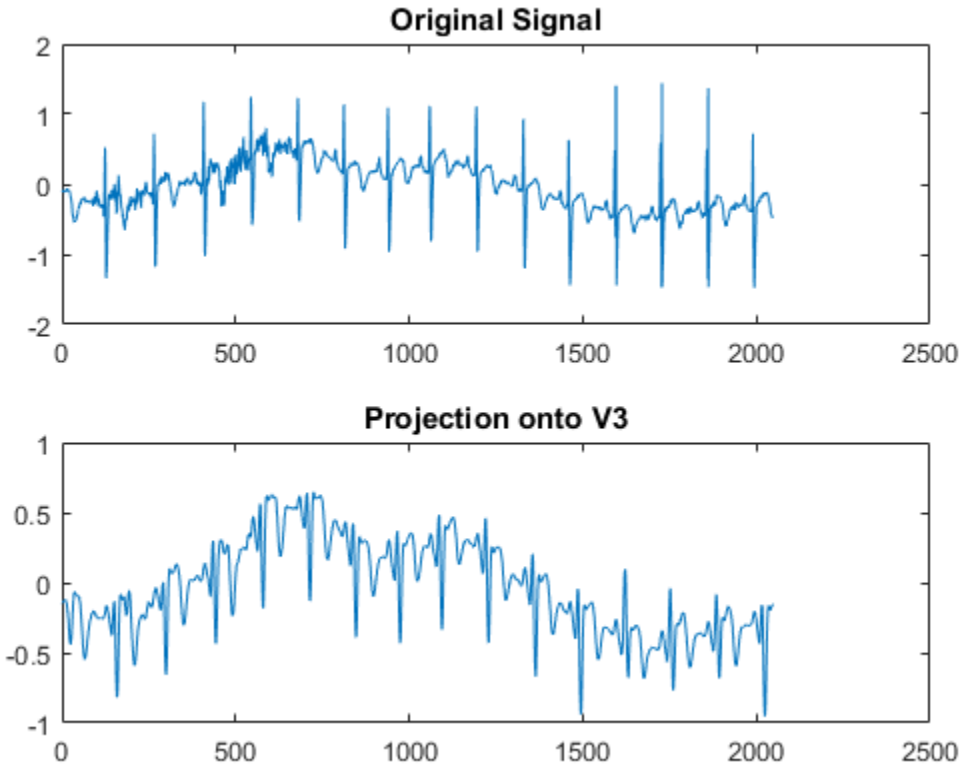
```
wtecg = modwt(wecg);
```

Obtain the projection of the ECG signal onto V_3, the scaling space at level three by using the `imodwt` function.

```
v3proj = imodwt(wtecg,3);
```

Plot the original signal and the projection.

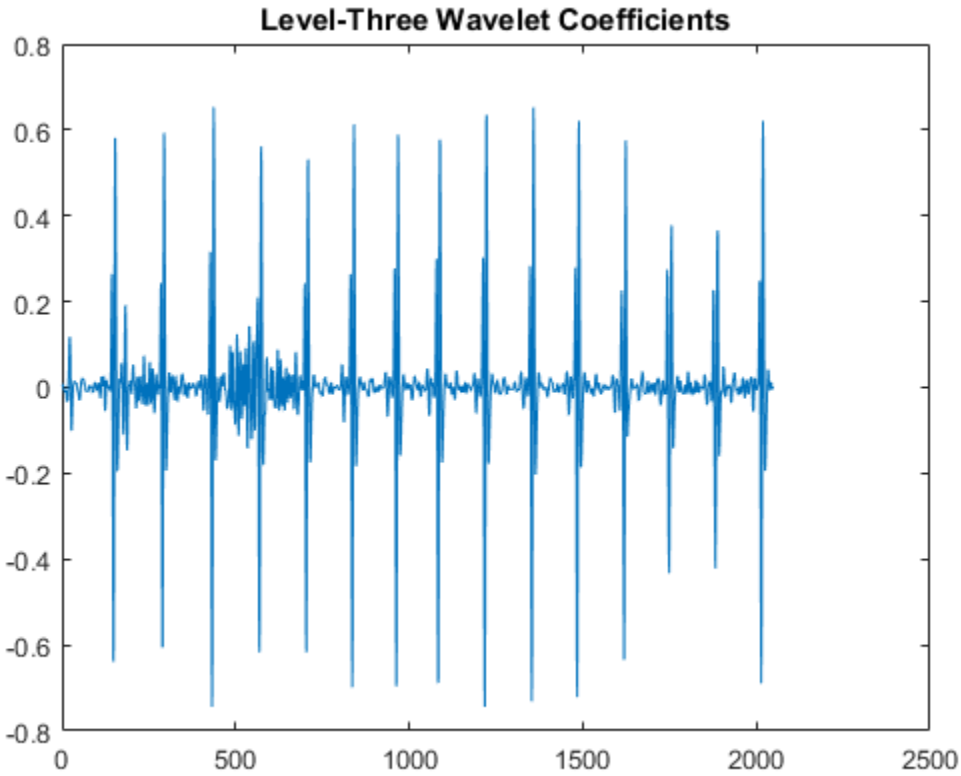
```
subplot(2,1,1)
plot(wecg)
title('Original Signal')
subplot(2,1,2)
plot(v3proj)
title('Projection onto V3')
```



Note that the spikes characteristic of the R waves in the ECG are missing in the V_3 approximation. You can see the missing details by examining the wavelete coefficients at level three.

Plot the level-three wavelet coefficients.

```
figure
plot(wtecg(3,:))
title('Level-Three Wavelet Coefficients')
```



Inverse MODWT with Reflection Boundary

Obtain the inverse MODWT using reflection boundary handling for Southern Oscillation Index data. The sampling period is one day. `imodwt` with the `'reflection'` option assumes that the input matrix, which is the `modwt` output, is twice the length of the original signal length. `imodwt` reflection boundary handling reduces the number of wavelet and scaling coefficients at each scale by half.

```
load soi;
wsoi = modwt(soi,4,'reflection');
xrecsoi = imodwt(wsoi,'reflection');
```

Use the L-infinity norm to show that the difference between the original signal and the reconstruction is extremely small. The largest absolute difference between the original signal and the reconstruction is on the order of 10^{-11}, which demonstrates perfect reconstruction.

```
norm(abs(xrecsoi'-soi),Inf)
```

```
ans =

   1.6435e-11
```

Input Arguments

w — MODWT transform

matrix

MODWT transform, specified as a matrix of size L+1-by-N. `w` is the output of `modwt`, which is the MODWT of an N-point input signal down to level L. By default, `imodwt` assumes that you obtained the MODWT using the `'sym4'` wavelet with periodic boundary handling.

Data Types: `double`

wname — Synthesis wavelet

`'sym4'` (default) | `'db`*L*`'` | `'coif`*L*`'` | `'haar`*L*`'` | `'fk`*L*`'` | `'sym`*L*`'` | string

Synthesis wavelet, specified as a string. The synthesis wavelet must be the same wavelet used in the analysis with `modwt`.

Lo — Scaling filter

even-length real-valued vector

Scaling filter, specified as an even-length real-valued vector. You can specify `Lo` only if you do not specify `wname`. `Lo` must be the same scaling filter used in the analysis with `modwt`.

`Hi` — Wavelet filter

even-length real-valued vector

Wavelet filter, specified as an even-length real-valued vector. You can specify `Hi` only if you do not specify `wname`. `Hi` must be the same wavelet filter used in the analysis with `modwt`.

`lev` — Reconstruction level

0 (default) | nonnegative integer

Reconstruction level, specified as a nonnegative integer between 0 and `size(w,1)-1`. If `lev` is 0 and you do not modify the coefficients, `imodwt` produces a perfect reconstruction of the signal.

Output Arguments

`xrec` — Reconstructed signal

row vector

Reconstructed version of the original signal based on the MODWT and the level of reconstruction, returned as a row vector.

References

[1] Percival, D. B., and A. T. Walden. *Wavelet Methods for Time Series Analysis*. Cambridge, UK: Cambridge University Press, 2000.

See Also

`modwt` | `modwtmra`

Introduced in R2015b

ind2depo

Node index to node depth-position

Syntax

```
[D,P] = ind2depo(ORD,[D P])
```

Description

`ind2depo` is a tree-management utility.

For a tree of order `ORD`, `[D,P] = ind2depo(ORD,N)` computes the depths `D` and the positions `P` (at these depths `D`) for the nodes with indices `N`.

The nodes are numbered from left to right and from top to bottom. The root index is 0.

`N` must be a column vector of integers `(N ≥ 0)`.

Note that `[D,P] = ind2depo(ORD,[D P])`.

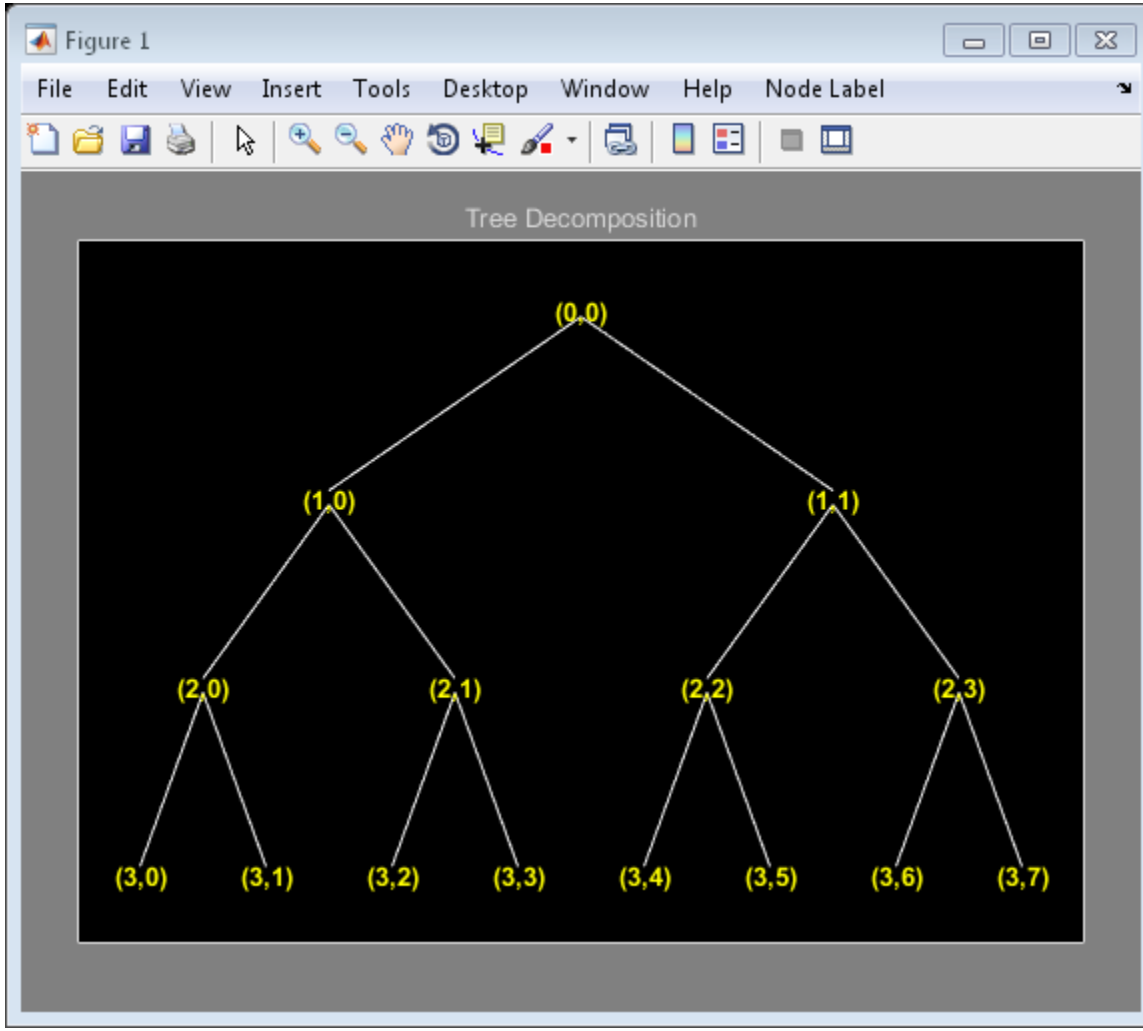
Depth and Position in Wavelet Packet Tree

Create a binary wavelet packet tree with three levels.

```
Ord = 2;
Lev = 3;
T = ntree(Ord,Lev);
```

Plot the binary wavelet packet tree.

```
plot(T)
```



Obtain the indices of the nodes in linear order.

```
idx = allnodes(T);
```

Conver the indices to depth-position format.

```
[depth,pos] = ind2depo(Ord,idx);
table(depth,pos)
```

```
ans =

    depth    pos
    _____    ___

    0        0
    1        0
    1        1
    2        0
    2        1
    2        2
    2        3
    3        0
    3        1
    3        2
    3        3
    3        4
    3        5
    3        6
    3        7
```

See Also

`depo2ind`

Introduced before R2006a

intwave

Integrate wavelet function psi (ψ)

Syntax

```
[INTEG,XVAL] = intwave('wname',PREC)
[INTDEC,XVAL,INTREC] = intwave('wname',PREC)
[INTEG,XVAL] = intwave('wname',PREC)
[INTEG,XVAL] = intwave('wname',PREC,0)
[INTEG,XVAL] = intwave('wname')
[INTEG,XVAL] = intwave('wname',8)
intwave('wname',IN2,IN3), PREC = max(IN2,IN3)
intwave('wname',0)
intwave('wname',8,IN3)
intwave('wname')
intwave('wname',8)
```

Description

`[INTEG,XVAL] = intwave('wname',PREC)` computes the integral, `INTEG`, of the wavelet function ψ (from $-\infty$ to `XVAL` values): $\int_{-\infty}^{x} \psi(y)dy$ for x in `XVAL`.

The function ψ is approximated on the 2^{PREC} points grid `XVAL` where *`PREC`* is a positive integer. '*`wname`*' is a string containing the name of the wavelet ψ (see `wfilters` for more information).

Output argument *`INTEG`* is a real or complex vector depending on the wavelet type.

For biorthogonal wavelets,

`[INTDEC,XVAL,INTREC] = intwave('wname',PREC)` computes the integrals, `INTDEC` and `INTREC`, of the wavelet decomposition function ψ_{dec} and the wavelet reconstruction function ψ_{rec}.

`[INTEG,XVAL] = intwave('wname',PREC)` is equivalent to `[INTEG,XVAL] = intwave('wname',PREC,0)`.

`[INTEG,XVAL] = intwave('wname')` is equivalent to `[INTEG,XVAL] = intwave('wname',8)`.

When used with three arguments `intwave('wname',IN2,IN3), PREC = max(IN2,IN3)` and plots are given.

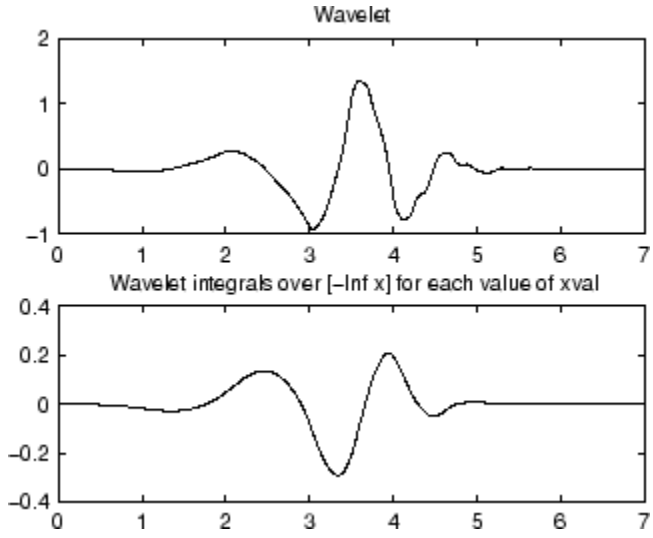
When `IN2` is equal to the special value 0, `intwave('wname',0)` is equivalent to `intwave('wname',8,IN3)`.

`intwave('wname')` is equivalent to `intwave('wname',8)`.

`intwave` is used only for continuous analysis (see `cwt` for more information).

Examples

```
% Set wavelet name.
wname = 'db4';

% Plot wavelet function.
[phi,psi,xval] = wavefun(wname,7);
subplot(211); plot(xval,psi); title('Wavelet');

% Compute and plot wavelet integrals approximations
% on a dyadic grid.
[integ,xval] = intwave(wname,7);
subplot(212); plot(xval,integ);
title(['Wavelet integrals over [-Inf x] ' ...
       'for each value of xval']);
```



More About

Algorithms

First, the wavelet function is approximated on a grid of 2^{PREC} points using `wavefun`. A piecewise constant interpolation is used to compute the integrals using `cumsum`.

See Also

`wavefun`

Introduced before R2006a

isnode

Existing node test

Syntax

```
R = isnode(T,N)
```

Description

`isnode` is a tree-management utility.

`R = isnode(T,N)` returns 1's for nodes *N*, which exist in the tree *T*, and 0's for others.

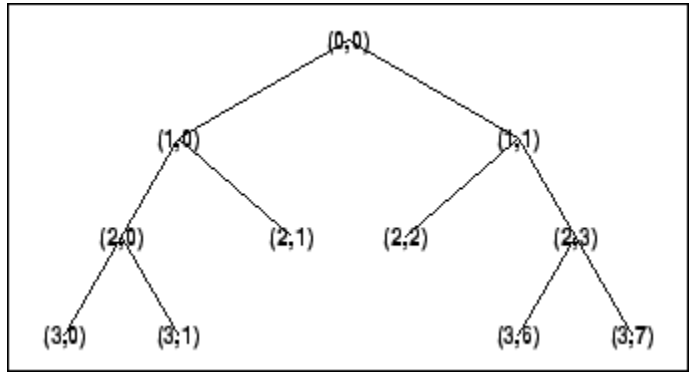
N can be a column vector containing the indices of nodes or a matrix, that contains the depths and positions of nodes.

In the last case, `N(i,1)` is the depth of the `i`-th node and `N(i,2)` is the position of the `i`-th node.

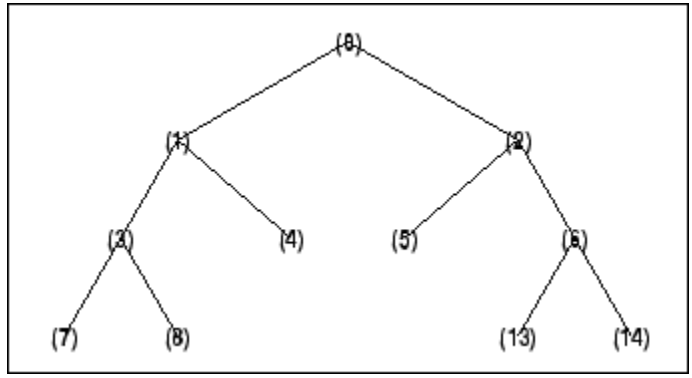
The nodes are numbered from left to right and from top to bottom. The root index is 0.

Examples

```
% Create initial tree.
ord = 2;
t = ntree(ord,3);    % binary tree of depth 3.
t = nodejoin(t,5);
t = nodejoin(t,4);
plot(t)
```




```
% Change Node Label from Depth_Position to Index
% (see the plot function).
```




```
% Check node index.
isnode(t,[1;3;25])

ans =
     1
     1
     0

% Check node Depth_Position.
isnode(t,[1 0;3 1;4 5])

ans =
     1
```

```
1
0
```

See Also

`istnode` | `wtreemgr`

Introduced before R2006a

istnode

Terminal nodes indices test

Syntax

```
R = istnode(T,N)
```

Description

`istnode` is a tree-management utility.

`R = istnode(T,N)` returns ranks (in left to right terminal nodes ordering) for terminal nodes *N* belonging to the tree *T*, and 0's for others.

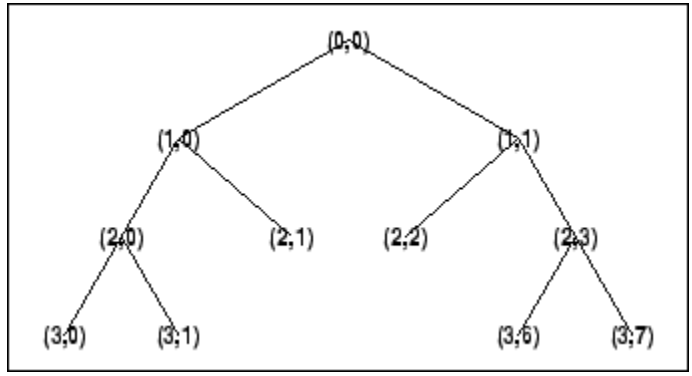
N can be a column vector containing the indices of nodes or a matrix that contains the depths and positions of nodes.

In the last case, `N(i,1)` is the depth of the `i`-th node and `N(i,2)` is the position of the `i`-th node.

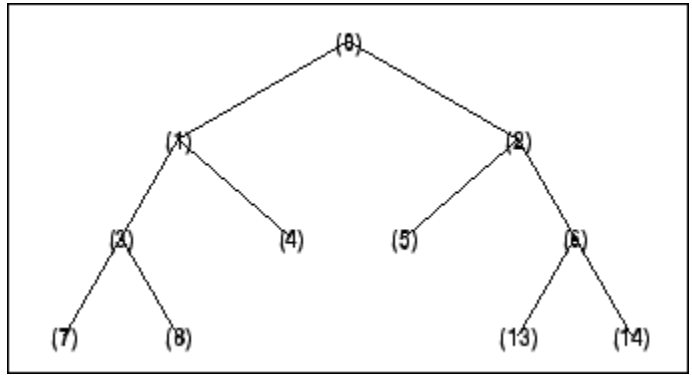
The nodes are numbered from left to right and from top to bottom. The root index is 0.

Examples

```
% Create initial tree.
ord = 2;
t = ntree(ord,3); % binary tree of depth 3.
t = nodejoin(t,5);
t = nodejoin(t,4);
plot(t)
```




```
% Change Node Label from Depth_Position to Inde
% (see the plot function)x.
```




```
% Find terminal nodes and return indices for terminal
% nodes in the tree.
istnode(t,[14])
ans =
    6

istnode(t,[15])
ans =
    0

istnode(t,[1;7;14;25])
ans =
    0
```

```
    1
    6
    0

istnode(t,[1 0;3 1;4 5])
ans =
    0
    2
    0
```

See Also

isnode | wtreemgr

Introduced before R2006a

iswt

Inverse discrete stationary wavelet transform 1-D

Syntax

```
X = iswt(SWC,'wname')
X = iswt(SWA,SWD,'wname')
X = iswt(SWA(end,:),SWD,'wname')
X = iswt(SWC,Lo_R,Hi_R)
X = iswt(SWA,SWD,Lo_R,Hi_R)
X = iswt(SWA(end,:),SWD,Lo_R,Hi_R)
```

Description

`iswt` performs a multilevel 1-D stationary wavelet reconstruction using either a specific orthogonal wavelet (`'wname'`, see `wfilters` for more information) or specific reconstruction filters (`Lo_R` and `Hi_R`).

`X = iswt(SWC,'wname')` or `X = iswt(SWA,SWD,'wname')` or `X = iswt(SWA(end,:),SWD,'wname')` reconstructs the signal `X` based on the multilevel stationary wavelet decomposition structure `SWC` or `[SWA,SWD]` (see `swt` for more information).

`X = iswt(SWC,Lo_R,Hi_R)` or `X = iswt(SWA,SWD,Lo_R,Hi_R)` or `X = iswt(SWA(end,:),SWD,Lo_R,Hi_R)` reconstruct as above, using filters that you specify.

- `Lo_R` is the reconstruction low-pass filter.
- `Hi_R` is the reconstruction high-pass filter.

`Lo_R` and `Hi_R` must be the same length.

Examples

```
% Load original 1D signal.
```

```
load noisbloc; s = noisbloc;

% Perform SWT decomposition at level 3 of s using db1.
swc = swt(s,3,'db1');
% Second usage.
[swa,swd] = swt(s,3,'db1');

% Reconstruct s from the stationary wavelet
% decomposition structure swc.
a0 = iswt(swc,'db1');
% Second usage.
a0bis = iswt(swa,swd,'db1');


% Check for perfect reconstruction.
err = norm(s-a0)
err =
  9.6566e-014

errbis = norm(s-a0bis)
errbis =
  9.6566e-014
```

References

Nason, G.P.; B.W. Silverman (1995), "The stationary wavelet transform and some statistical applications," *Lecture Notes in Statistics*, 103, pp. 281–299.

Coifman, R.R.; Donoho D.L. (1995), "Translation invariant de-noising," *Lecture Notes in Statistics*, 103, pp 125–150.

Pesquet, J.C.; H. Krim, H. Carfatan (1996), "Time-invariant orthonormal wavelet representations," *IEEE Trans. Sign. Proc.*, vol. 44, 8, pp. 1964–1970.

See Also

`idwt` | `swt` | `waverec`

Introduced before R2006a

iswt2

Inverse discrete stationary wavelet transform 2-D

Syntax

```
X = iswt2(SWC,'wname')
X = iswt2(A,H,V,D,wname)
X = iswt2(A(:,:,end),H,V,D,'wname')
X = iswt2(SWC,Lo_R,Hi_R)
X = iswt2(A,H,V,D,Lo_R,Hi_R)
X = iswt2(A(:,:,end),H,V,D,Lo_R,Hi_R)
```

Description

`iswt2` performs a multilevel 2-D stationary wavelet reconstruction using either a specific orthogonal wavelet (`'wname'` see `wfilters` for more information) or specific reconstruction filters (`Lo_R` and `Hi_R`).

`X = iswt2(SWC,'wname')` or `X = iswt2(A,H,V,D,wname)` or `X = iswt2(A(:,:,end),H,V,D,'wname')` reconstructs the signal `X`, based on the multilevel stationary wavelet decomposition structure `SWC` or `[A,H,V,D]` (see `swt2`).

`X = iswt2(SWC,Lo_R,Hi_R)` or `X = iswt2(A,H,V,D,Lo_R,Hi_R)` or `X = iswt2(A(:,:,end),H,V,D,Lo_R,Hi_R)` reconstructs as in the previous syntax, using filters that you specify:

- `Lo_R` is the reconstruction low-pass filter.
- `Hi_R` is the reconstruction high-pass filter.

`Lo_R` and `Hi_R` must be the same length.

Examples

```
% Load original image.
load nbarb1;
```

```
% Perform SWT decomposition
% of X at level 3 using sym4.
swc = swt2(X,3,'sym4');
% Second usage.
[ca,chd,cvd,cdd] = swt2(X,3,'sym4');

% Reconstruct s from the stationary wavelet
% decomposition structure swc.
a0 = iswt2(swc,'sym4');
% Second usage.
a0 = iswt2(ca,chd,cvd,cdd,'sym4');
% Check for perfect reconstruction.
err = max(max(abs(X-a0)))
ans =
  2.3482e-010

errbis = max(max(abs(X-a0bis)))
ans =
  2.3482e-010
```

More About

Tips

If SWC or (cA,cH,cV,cD) are obtained from an indexed image analysis or a truecolor image analysis, then X is an `m`-by-`n` matrix or an `m`-by-`n`-by-3 array, respectively.

For more information on image formats, see the `image` and `imfinfo` reference pages.

References

Nason, G.P.; B.W. Silverman (1995), "The stationary wavelet transform and some statistical applications," *Lecture Notes in Statistics*, 103, pp. 281–299.

Coifman, R.R.; Donoho D.L. (1995), "Translation invariant de-noising," *Lecture Notes in Statistics*, 103, pp. 125–150.

Pesquet, J.C.; H. Krim, H. Carfatan (1996), "Time-invariant orthonormal wavelet representations," *IEEE Trans. Sign. Proc.*, vol. 44, 8, pp. 1964–1970.

See Also

`idwt2` | `swt2` | `waverec2`

Introduced before R2006a

iwsst

Inverse wavelet synchrosqueezed transform

Syntax

```
xrec = iwsst(sst)
xrec = iwsst(sst,f,freqrange)
xrec = iwsst(sst,iridge)
xrec = iwsst( ___ ,wav)
xrec = iwsst( ___ ,iridge,Name,Value)
```

Description

`xrec = iwsst(sst)` inverts the input synchrosqueezed transform, `sst`, and returns the inverse in vector `xrec`. To obtain the `sst` input, use the `wsst` function. The `iwsst` function assumes that you obtain `sst` using the analytic Morlet wavelet.

Note: The wavelet transform does not preserve a nonzero mean. After inverting the synchrosqueezed transform, you must add back the original signal mean.

`xrec = iwsst(sst,f,freqrange)` inverts the synschrosqueezed transform for a specified range of frequencies, `freqrange`, contained in the frequency vector, `f`. The frequency vector, `f`, is the output of `wsst`.

`xrec = iwsst(sst,iridge)` inverts the synchrosqueezed transform along the time-frequency ridges specified by `iridge`, the index column vector. `iridge` is the output of `wsstridge`. The `xrec` output is the same size as `iridge`.

`xrec = iwsst( ___ ,wav)` uses the analytic wavelet specified by `wav` to invert the synchrosqueezed transform. This wavelet must be the same wavelet as used in `wsst`. You can include any of the input arguments from previous syntaxes.

`xrec = iwsst( ___ ,iridge,Name,Value)` returns the inverse synchrosqueezed transform with additional options specified by one or more `Name,Value` pair arguments.

Examples

Inverse Synchrosqueezed Transform of Chirp

Obtain the wavelet synchrosqueezed transform of a quadratic chirp using default values. Then reconstruct the signal using the inverse wavelet synchrosqueezed transform.

```
load quadchirp;
sst = wsst(quadchirp);
xrec = iwsst(sst);
```

Synchrosqueezed and Inverse Synchrosqueezed Transform of Chirp

Obtain the wavelet synchrosqueezed transform of a quadratic chirp sampled at 1000 Hz. Then reconstruct the chirp.

Load the chirp and obtain the synchrosqueezed transform.

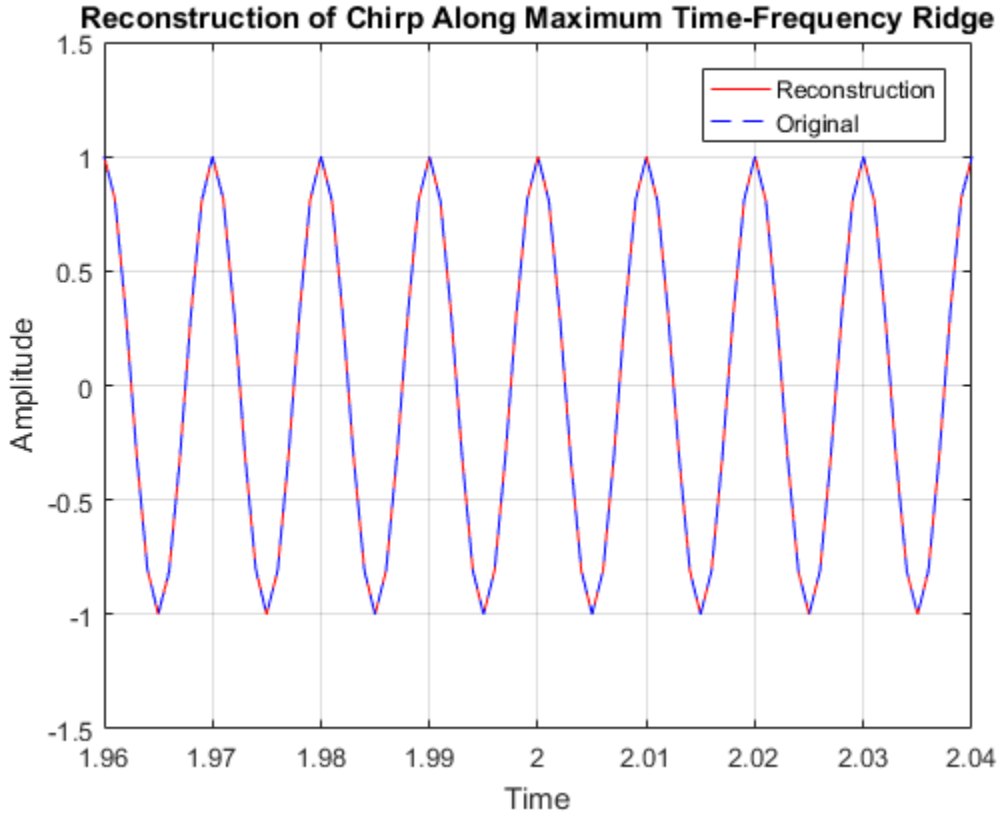
```
load quadchirp;
sstchirp = wsst(quadchirp,'ExtendSignal',true);
```

Extract the maximum energy time-frequency ridge and reconstruct the signal mode along the ridge.

```
[~,iridge] = wsstridge(sstchirp);
xrec = iwsst(sstchirp,iridge);
```

Plot and zoom in on the original and reconstructed signal.

```
plot(tquad,xrec,'r');
hold on;
plot(tquad,quadchirp,'b--');
xlabel('Time'); ylabel('Amplitude');
set(gca,'ylim',[-1.5 1.5]);
legend('Reconstruction','Original');
grid on;
title('Reconstruction of Chirp Along Maximum Time-Frequency Ridge');
xlim([1.96 2.04])
```



Inverse Synchrosqueezed Transform of Range of Frequencies

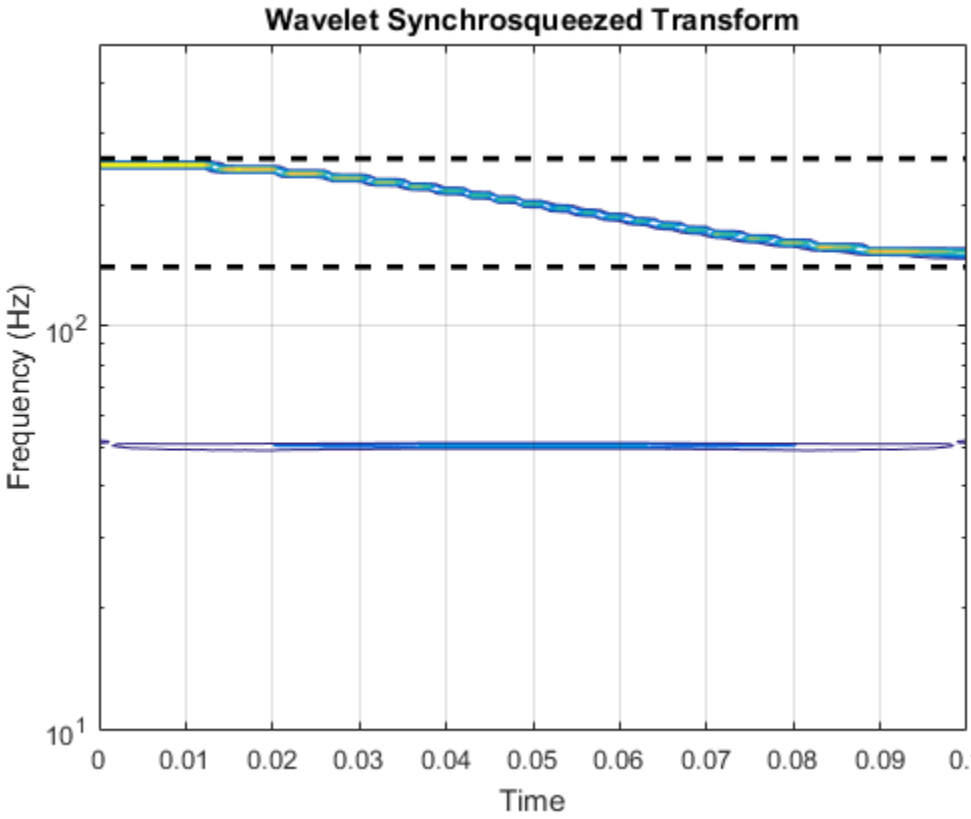
This example shows how to obtain the inverse synchrosqueezed transform for a specified frequency range of a two-component signal. The input is a combination of an amplitude-modulated and frequency-modulated signal and a 50-Hz sine wave.

Create the signal.

```
t = 0:0.001:0.1;
x1 = (2+0.5*cos(2*pi*10*t)).*cos(2*pi*200*t+10*sin(2*pi*5*t));
x2 = cos(2*pi*50*t);
sig = x1+x2;
```

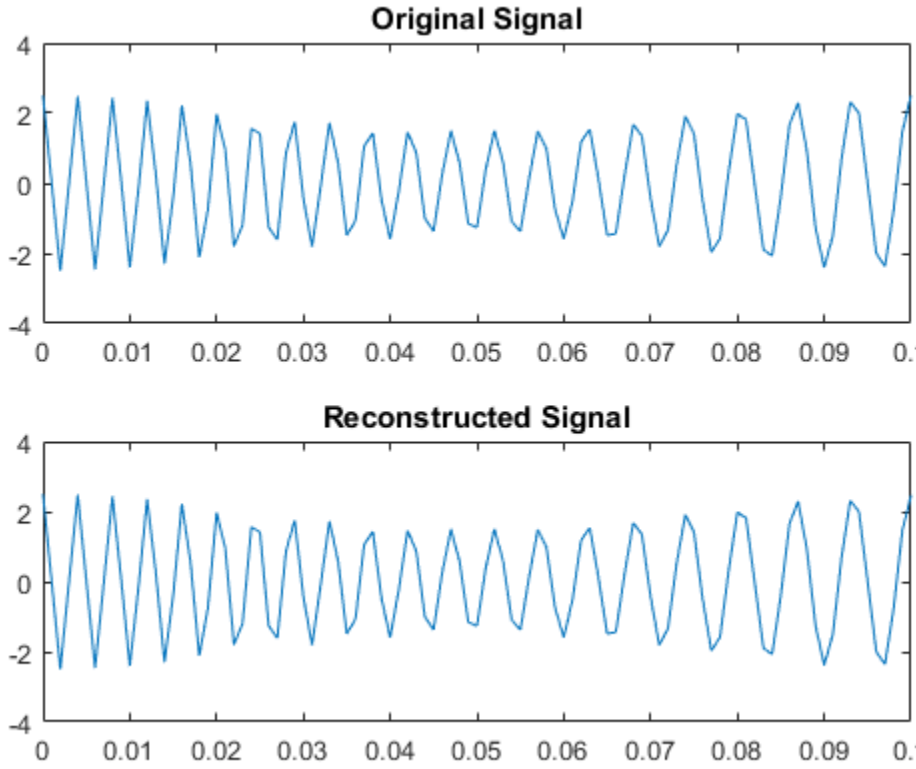
Obtain the wavelet synchrosqueezed transform and plot the result, which shows the two frequency components. Use a log scale on the frequency axis.

```
[sst,f] = wsst(sig,1000,'ExtendSignal',true);
contour(t,f,abs(sst));
AX = gca;
AX.YScale = 'log';
grid on;
title('Wavelet Synchrosqueezed Transform');
ylabel('Frequency (Hz)');
xlabel('Time');
hold on;
plot(t,140*ones(size(t)),'k--','linewidth',2);
plot(t,260*ones(size(t)),'k--','linewidth',2);
```



Obtain the inverse synchrosqueezed transform for frequencies from 140 Hz to 260 Hz. Plot the result.

```
xrec = iwsst(sst,f,[140,260]);
subplot(2,1,1);
plot(t,x1);
title('Original Signal');
subplot(2,1,2);
plot(t,xrec);
title('Reconstructed Signal');
```



Note that the 50-Hz component has been removed. Only the AM-FM signal remains.

Synchrosqueezed and Inverse Synchrosqueezed Transform of Speech Signal

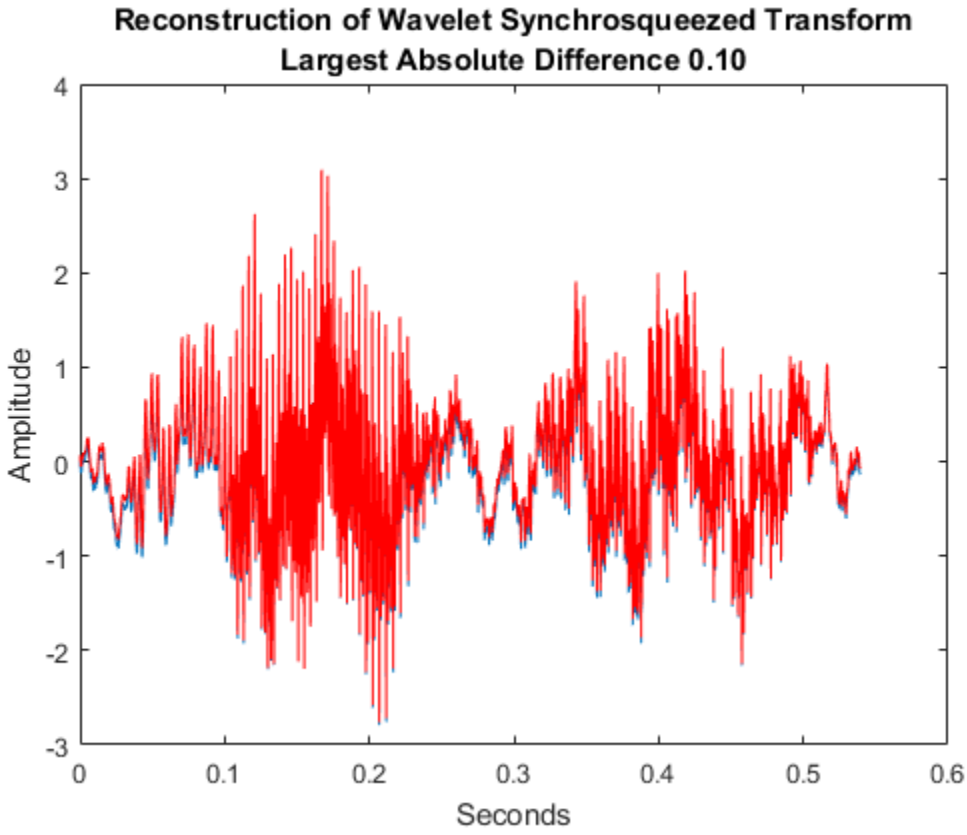
Obtain the wavelet synchrosqueezed transform and inverse synchrosqueezed transform of a speech sample using the bump wavelet.

Load the speech signal and obtain the synchrosqueezed transform and inverse synchrosqueezed transform.

```
load mtlb;
dt = 1/Fs;
t = 0:dt:numel(mtlb)*dt-dt;
Txmtlb = wsst(mtlb,'bump');
xrec = iwsst(Txmtlb,'bump');
```

Obtain the L-infinity norm of the difference between the original waveform and the reconstruction. Plot the results.

```
Linf = norm(abs(mtlb-xrec),Inf);
plot(t,mtlb);
hold on;
xlabel('Seconds'); ylabel('Amplitude');
plot(t,xrec,'r');
title({'Reconstruction of Wavelet Synchrosqueezed Transform'; ...
['Largest Absolute Difference ' num2str(Linf,'%1.2f')]});
```



Synchrosqueezed Transform Using Specified Number of Bins for Chirp

This example shows how to invert the wavelet synchrosqueezed transform using a specified number of frequency bins for a quadratic chirp. The chirp is sampled at 1000 Hz.

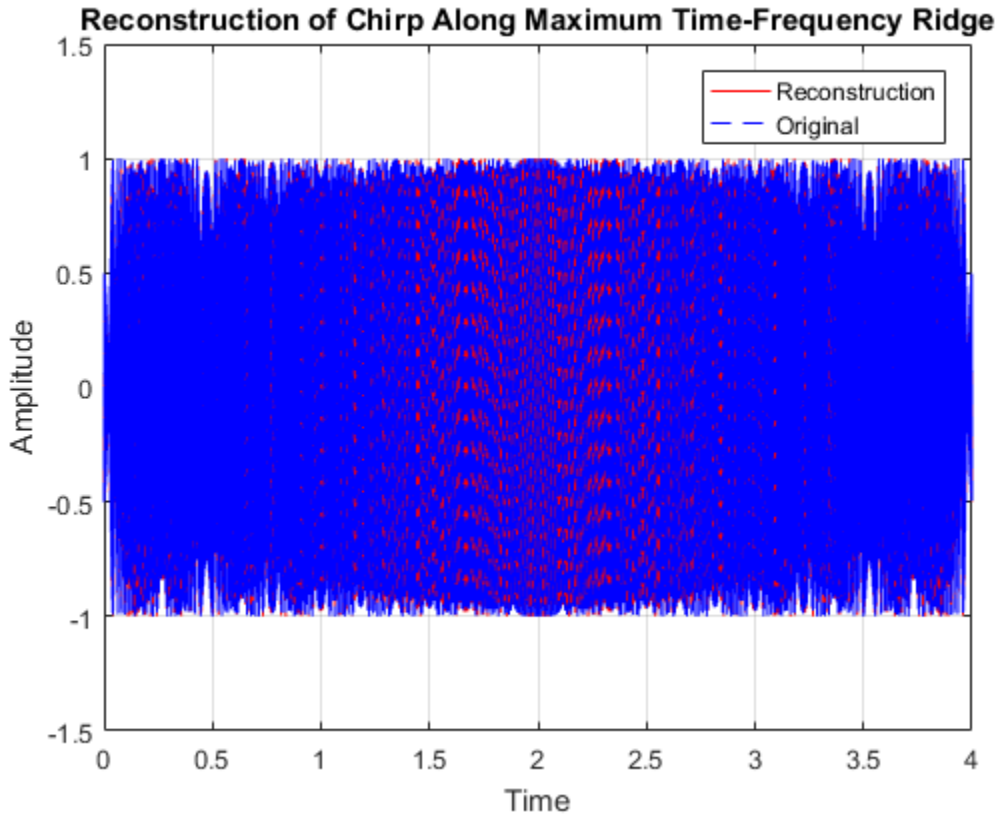
```
load quadchirp;
sstchirp = wsst(quadchirp,'ExtendSignal',true);
```

Extract the maximum energy time-frequency ridge using 10 bins on each side of the the iridge index and reconstruct the signal mode along the ridge.

```
[~,iridge] = wsstridge(sstchirp);
xrec = iwsst(sstchirp,iridge,'NumFrequencyBins',10);
```

Plot the original and reconstructed signal.

```
plot(tquad,xrec,'r');
hold on;
plot(tquad,quadchirp,'b--');
xlabel('Time'); ylabel('Amplitude');
set(gca,'ylim',[-1.5 1.5]);
legend('Reconstruction','Original');
grid on;
title('Reconstruction of Chirp Along Maximum Time-Frequency Ridge');
```



- “Time-Frequency Reassignment and Mode Extraction with Synchrosqueezing”

Input Arguments

`sst` — Synschrosqueezed transform

matrix

Synschrosqueezed transform, specified as a matrix. `sst` is the output from the `wsst` function.

`f` — Synschrosqueezed transform frequencies

vector

Synchrosqueezed transform frequencies corresponding to the rows of the synchrosqueezed transform, specified as a vector. The number of elements in the frequency vector is equal to the number of rows in the `sst` input. If you specify `f`, you must also specify `freqrange`.

`freqrange` — Frequency range

two-element vector

Frequency range for which to return inverse synchrosqueezed transform values, specified as a two-element vector. The values of `freqrange` must be in the range of the values of the frequencies, `f`. The first and second elements of `freqrange` define the start and end of the frequency range, where the frequency values in that range must be positive and strictly increasing. If you specify `freqrange`, you must also specify `f`.

`iridge` — Time-frequency ridge row indices

vector or matrix

Time-frequency ridge row indices of the synchrosqueezed transform specified as a vector or matrix. `iridge` is the output of the `wsstridge` function. If `iridge` is a matrix, `iwsst` inverts the synchrosqueezed transform along the first column of `iridge`. Then, it iteratively reconstructs along subsequent columns of `iridge`. The sizes of `iridge` and the `xrec` output are the same.

`wav` — Analytic wavelet

`'analmorl'` (default) | `'bump'`

Analytic wavelet used to compute the inverse synchrosqueezed transform, specified as `'analmorl'` or `'bump'`. These strings specify the analytic Morlet and bump wavelet, respectively. You must use the same wavelet in the reconstruction that you used to compute the synchrosqueezed transform, `sst`.

Example:

Name-Value Pair Arguments

Specify optional comma-separated pairs of `Name,Value` arguments. `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside single quotes (`' '`). You can specify several name and value pair arguments in any order as `Name1,Value1,...,NameN,ValueN`.

Example: `'NumFrequencyBins',12`

`'NumFrequencyBins'` — Number of frequency bins

16 (default) | positive integer

Number of additional frequency bins to include on either side of each `iridge` index bin, specified as the comma-separated pair consisting of `'NumFrequencyBins'` and a positive integer. If the number of additional bins exceeds the number of frequency bins available at a particular time step, `iwsst` truncates the reconstruction at the first or last frequency bin. The default, `16`, is one half the default number of voices per octave.

You can include this `Name,Value` argument in any position in a syntax. To specify this argument, you also specify `iridge`, which is the output of `wsstridge`. You cannot include a frequency, `f` and frequency range, `freqrange`, if you include the number of frequency bins.

Output Arguments

`xrec` — Inverse synchrosqueezed transform

vector or matrix

Inverse synchrosqueezed transform, returned as a vector or matrix. If you do not specify an `iridge` input, `xrec` is a column vector with the same number of rows as `sst`. If you specify an `iridge` input, `xrec` is the same size as `iridge`.

More About

- “Wavelet Synchrosqueezing”

References

[1] Daubechies, I., I., J. Lu, and H. T. Wu. "Synchrosqueezed Wavelet Transforms: an Empricial Mode Decomposition-like Tool", *Applied and Computational Harmonic Analysis*. Vol. 30(2), pp. 243–261.

[2] Thakur, G., E. Brevdo, N. S. Fučkar, and H. T. Wu. "The Synchrosqueezing algorithm for time-varying spectral analysis: robustness properties and new paleoclimate applications." *Signal Processing*. Vol. 93, pp. 1079–1094.

See Also

`wsst` | `wsstridge`

Introduced in R2016a

laurmat

Laurent matrices constructor

Syntax

```
M = laurmat(V)
```

Description

`M = laurmat(V)` returns the Laurent matrix object `M` associated with *V* which can be a cell array (at most two dimensional) of Laurent polynomials (see `laurpoly`) or an ordinary matrix.

Examples

```
% Define Laurent matrices.
M1 = laurmat(eye(2,2))

       | 1     0  |
       |          |
 M1 = |           |
       |          |
       | 0     1  |

Z  = laurpoly(1,1);
M2 = laurmat({1 Z;0 1})

       | 1     z^(+1)  |
       |               |
 M2 = |                |
       |               |
       | 0       1     |

% Calculus on Laurent polynomials.
P = M1 * M2

      | 1     z^(+1)  |
```

```
     |                 |
 P = |                 |
     |                 |
     | 0       1       |
```

```
d = det(P)

d(z) = 1
```

References

Strang, G.; T. Nguyen (1996), *Wavelets and filter banks*, Wellesley-Cambridge Press.

Sweldens, W. (1998), "The Lifting Scheme: a Construction of Second Generation of Wavelets," *SIAM J. Math. Anal.*, 29 (2), pp. 511–546.

See Also

`laurpoly`

Introduced before R2006a

laurpoly

Laurent polynomials constructor

Syntax

```
P = laurpoly(C,d)
P = laurpoly(C,'dmin',d)
P = laurpoly(C,'dmax',d)
P = laurpoly(C,d)
```

Description

`P = laurpoly(C,d)` returns a Laurent polynomial object. *C* is a vector whose elements are the coefficients of the polynomial `P` and *d* is the highest degree of the monomials of `P`.

If `m` is the length of the vector *C*, *P* represents the following Laurent polynomial:

```
P(z) = C(1)*z^d + C(2)*z^(d-1) + ... + C(m)*z^(d-m+1)
```

`P = laurpoly(C,'dmin',d)` specifies the lowest degree instead of the highest degree of monomials of *P*. The corresponding output *P* represents the following Laurent polynomial:

```
P(z) = C(1)*z^(d+m-1) + ... + C(m-1)*z^(d+1) + C(m)*z^d
```

`P = laurpoly(C,'dmax',d)` is equivalent to `P = laurpoly(C,d)`.

Examples

```
% Define Laurent polynomials.
P = laurpoly([1:3],2);
P = laurpoly([1:3],'dmax',2)

P(z) = + z^(+2) + 2*z^(+1) + 3

P = laurpoly([1:3],'dmin',2)
```

```
P(z) = + z^(+4) + 2*z^(+3) + 3*z^(+2)

% Calculus on Laurent polynomials.
Z = laurpoly(1,1)

Z(z) = z^(+1)

Q = Z*P

Q(z) = + z^(+5) + 2*z^(+4) + 3*z^(+3)

R = Z^1 - Z^-1

R(z) = + z^(+1) - z^(-1)
```

References

Strang, G.; T. Nguyen (1996), *Wavelets and filter banks*, Wellesley-Cambridge Press.

Sweldens, W. (1998), "The Lifting Scheme: a Construction of Second Generation of Wavelets," *SIAM J. Math. Anal.*, 29 (2), pp. 511–546.

See Also

`laurmat`

Introduced before R2006a

leaves

Determine terminal nodes

Syntax

```
N = leaves(T)
[N,K] = leaves(T,'sort')
N = leaves(T,'dp')
[N,K] = leaves(T,'sortdp')
[N,K] = leaves(T,'sdp')
```

Description

`N = leaves(T)` returns the indices of terminal nodes of the tree *T* where `N` is a column vector.

The nodes are ordered from left to right as in tree *T*.

`[N,K] = leaves(T,'s')` or `[N,K] = leaves(T,'sort')` returns sorted indices. `M = N(K)` are the indices reordered as in tree *T*, from left to right.

`N = leaves(T,'dp')` returns a matrix `N`, which contains the depths and positions of terminal nodes.

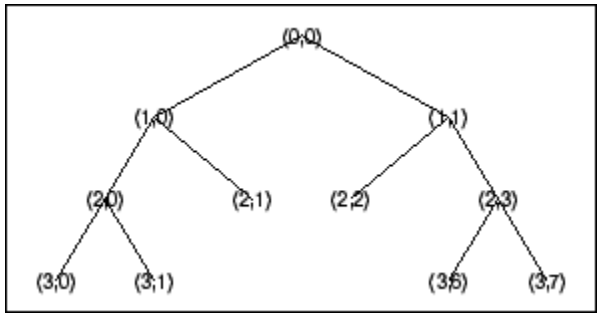
`N(i,1)` is the depth of the `i`-th terminal node, and `N(i,2)` is the position of the `i`-th terminal node.

`[N,K] = leaves(T,'sortdp')` or `[N,K] = leaves(T,'sdp')` returns sorted nodes.

Examples

```
% Create initial tree.
ord = 2;
t = ntree(ord,3);        % binary tree of depth 3.
t=nodejoin(t,5);
t=nodejoin(t,4);
```

```
plot(t)
```




```
% List terminal nodes (index).
tnodes_ind = leaves(t)
tnodes_ind =
     7
     8
     4
     5
    13
    14

% List terminal nodes (sorted on index).
[tnodes_ind,Ind] = leaves(t,'sort')
tnodes_ind =
     4
     5
     7
     8
    13
    14

Ind =
     3
     4
     1
     2
     5
     6

% List terminal nodes (Depth_Position).
tnodes_depo = leaves(t,'dp')
tnodes_depo =
```

```
     3     0
     3     1
     2     1
     2     2
     3     6
     3     7

% List terminal nodes (sorted on Depth_Position).
[tnodes_depo,Ind] = leaves(t,'sortdp')
tnodes_depo =
     2     1
     2     2
     3     0
     3     1
     3     6
     3     7

Ind =
     3
     4
     1
     2
     5
     6
```

See Also

`tnodes` | `noleaves`

Introduced before R2006a

liftfilt

Apply elementary lifting steps on quadruplet of filters

Syntax

```
[LoDN,HiDN,LoRN,HiRN] = liftfilt(LoD,HiD,LoR,HiR,ELS)
liftfilt(LoD,HiD,LoR,HiR,ELS,TYPE,VALUE)
```

Description

`[LoDN,HiDN,LoRN,HiRN] = liftfilt(LoD,HiD,LoR,HiR,ELS)` returns the four filters `LoDN`, `HiDN`, `LoRN`, and `HiRN` obtained by an elementary lifting step (`ELS`) starting from the four filters `LoD`, `HiD`, `LoR`, and `HiR`. The four input filters verify the perfect reconstruction condition.

`ELS` is a structure such that

- `TYPE = ELS.type` contains the type of the elementary lifting step. The valid values for `TYPE` are `'p'` (primal) or `'d'` (dual).
- `VALUE = ELS.value` contains the Laurent polynomial `T` associated with the elementary lifting step (see `laurpoly`). If `VALUE` is a vector, the associated Laurent polynomial `T` is equal to `laurpoly(VALUE,0)`.

In addition, `ELS` may be a scaling step. In that case, `TYPE` is equal to `'s'` (scaling) and `VALUE` is a scalar different from zero.

`liftfilt(LoD,HiD,LoR,HiR,ELS,TYPE,VALUE)` gives the same outputs.

Note If `TYPE = 'p'` , `HiD` and `LoR` are unchanged.
If `TYPE = 'd'` , `LoD` and `HiR` are unchanged.
If `TYPE = 's'`, the four filters are changed.
If `ELS` is an array of elementary lifting steps, `liftfilt(...,ELS)` performs each step successively.

`liftfilt(...,FLAGPLOT)` plots the successive biorthogonal pairs—scaling function and wavelet.

Examples

```
% Get Haar filters.
[LoD,HiD,LoR,HiR] = wfilters('haar');

% Lift the Haar filters.
twoels(1) = struct('type','p','value',...
laurpoly([0.125 -0.125],0));
twoels(2) = struct('type','p','value',...
laurpoly([0.125 -0.125],1));
[LoDN,HiDN,LoRN,HiRN] = liftfilt(LoD,HiD,LoR,HiR,twoels);

% The biorthogonal wavelet bior1.3 is obtained up to
% an unsignificant sign.
[LoDB,HiDB,LoRB,HiRB] = wfilters('bior1.3');
samewavelet = ...
isequal([LoDB,HiDB,LoRB,HiRB],[LoDN,-HiDN,LoRN,HiRN])

samewavelet =

     1
```

See Also

`laurpoly`

Introduced before R2006a

liftwave

Lifting schemes

Syntax

```
LS = liftwave(WNAME)
LS = liftwave(WNAME,'Int2Int')
```

Description

`LS = liftwave(WNAME)` returns the lifting scheme associated with the wavelet specified by `WNAME`. `LS` is a structure, not an integer, and used by `lwt`, `ilwt`, `lwt2`, etc.

`LS = liftwave(WNAME,'Int2Int')` performs an integer to integer wavelet transform. Using `'Int2Int'` produces an `LS` such that when you use `[CA,CD] = lwt(X,LS)` or `Y = lwt(X,LS)` and `X` is a vector of integers, the resulting `CA`, `CD`, and `Y` are vectors of integers. If you omit `'Int2Int'` then `lwt` produces vectors of real numbers.

The valid values for `WNAME` are

WNAME Values	Comments
`'lazy'`	A "lazy" wavelet is a second-generation wavelet and is not a true mathematical wavelet.
`'haar'`	Same as `'db1'`, `'bior1.1'`, and `'cdf1.1'`
`'db1'`, `'db2'`, `'db3'`, `'db4'`, `'db5'`, `'db6'`, `'db7'`, `'db8'`	`'db2'` same as `'sym2'`, `'db3'`, and `'sym4'`
`'sym2'`, `'sym3'`, `'sym4'`, `'sym5'`, `'sym6'`, `'sym7'`, `'sym8'`	
Cohen-Daubechies-Feauveau wavelets `'cdf1.1'`,`'cdf1.3'`,`'cdf1.5'` `'cdf3.1'`,`'cdf3.3'`,`'cdf3.5'` `'cdf5.1'`,`'cdf5.3'`,`'cdf5.5'` `'cdf2.2'`,`'cdf2.4'`,`'cdf2.6'`	`'cdfX.Y'` same as `'biorX.Y'` except for `bior4.4` and `bior5.5`.

WNAME Values	Comments
'cdf4.2','cdf4.4','cdf4.6' 'cdf6.2','cdf6.4','cdf6.6'	
'biorX.Y'	See waveinfo
'rbioX.Y'	Reverse of 'biorX.Y'. See waveinfo
'bs3'	Same as 'cdf4.2'
'rbs3'	Reverse of 'bs3'
'9.7'	Same as 'bior4.4'
'r9.7'	Reverse of '9.7'

For more information about lifting schemes, see lsinfo.

Examples

```
% Start from the db2 wavelet and get the
% corresponding lifting scheme.
lsdb2 = liftwave('db2');

% Visualize the obtained lifting scheme.
displs(lsdb2);

lsdb2 = {...
'd'             [ -1.73205081]              [0]
'p'             [ -0.06698730  0.43301270]  [1]
'd'             [  1.00000000]              [-1]
[  1.93185165]  [  0.51763809]              []
};
```

See Also

laurpoly

Introduced before R2006a

localmax

Identify and chain local maxima

Syntax

```
[lmaxima,indices] = localmax(inputmatrix)
[lmaxima,indices] = localmax(inputmatrix,initrow)
[lmaxima,indices] = localmax(inputmatrix,initrow,regflag)
```

Description

`[lmaxima,indices] = localmax(inputmatrix)` identifies and chains the local maxima in the rows of `inputmatrix`.

`[lmaxima,indices] = localmax(inputmatrix,initrow)` initializes the chaining of local maxima beginning with row `initrow`. If there are no local maxima in `initrow`, all rows in `lmaxima` with indices less than `initrow` consist of only zeros.

`[lmaxima,indices] = localmax(inputmatrix,initrow,regflag)` replaces `initrow` of `inputmatrix` with the level-5 approximation (scaling) coefficients obtained with the `sym4` wavelet.

Input Arguments

`inputmatrix`

`inputmatrix` is a matrix of real or complex numbers. Most often, `inputmatrix` is a matrix of continuous wavelet transform (CWT) coefficients, and you use `localmax` to identify maxima lines. `localmax` operates on the absolute values of `inputmatrix`.

`initrow`

Initialization row for chaining local maxima. The chaining algorithm begins at `initrow` and decrements the row index by 1 until the first row of the matrix is reached. By specifying `initrow`, you can exclude rows from the chaining algorithm.

Default: `size(inputmatrix,1)`

`regflag`

Regularization flag. If you set `regflag` to `true`, the row of `inputmatrix` corresponding to `initrow` is replaced by the level-5 approximation (scaling) coefficients obtained with the `sym4` wavelet.

Default: `true`

Output Arguments

`lmaxima`

Matrix with local maxima chains. `lmaxima` only has nonzero entries at the locations of local maxima in the absolute values of `inputmatrix`. Denote the row index of `lmaxima` by `R`. You can determine the value of `lmaxima` at a local maximum in row `R` as follows:

- If `R>initRow`, the value of `lmaxima` at a local maximum is 1.
- If `R=initRow`, the value of `lmaxima` at a local maximum is the column index in row `R`.
- If `R<initRow`, the value of `lmaxima` at a local maximum in row `R` is the column index of the nearest local maximum in row `R+1`.

  To illustrate this, if `inputmatrix` is:

```
 3     2     5     3
 4     6     3     2
 4     4     7     4
 4     6     2     2
```

  `lmaxima` with `initRow = 4` and `regflag = false` is:

```
     0     0     2     0
     0     3     0     0
     0     0     2     0
     0     2     0     0
```

  `lmaxima` with `initRow = 3` and `regflag = false` is:

```
     0     0     2     0
```

```
     0     3     0     0
     0     0     3     0
     0     1     0     0
```

- If the local maximum in row `R` lies between two local maxima in row `R+1`, the value of the local maximum in row `R` is the higher column index in row `R+1`.

  To illustrate this, if `inputmatrix` is:

```
     0     0     1     0     0     0
     0     1     0     1     0     0
```

  `lmaxima` with `initRow = 2` and `regflag = false` is:

```
     0     0     4     0     0     0
     0     2     0     4     0     0
```

  `lmaxima` with `initRow = 1` and `regflag = false` is:

```
     0     0     3     0     0     0
     0     1     0     1     0     0
```

`indices`

Linear indices of the nonzero values of `lmaxima`. Use `ind2sub` to convert the linear indices to matrix row and column indices.

Examples

Construct a 4-by-4 matrix with local maxima at the following row-column indices: (4,2), (3,3), (2,2), and (1,3). Set `initrow` to 4 and `regflag` to `false`.

```
inputmatrix = ...
[3     2     5     3
 4     6     3     2
 4     4     7     4
 4     6     2     2];
 [lmaxima,indices] = localmax(inputmatrix,4,false);
 lmaxima
```

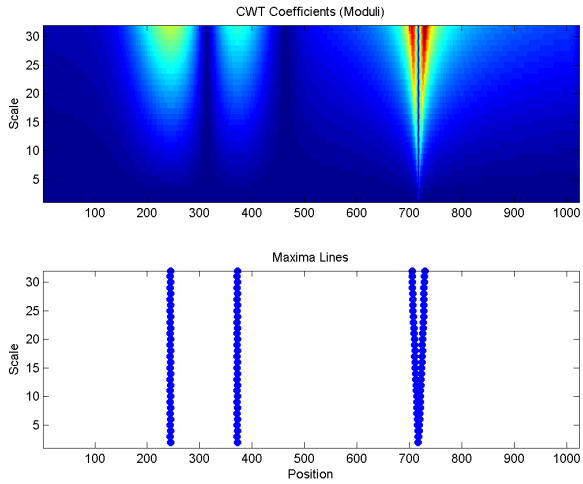
Because `localmax` operates on the absolute values of `inputmatrix`, setting `inputmatrix(4,2) = -inputmatrix(4,2)` produces an identical `lmaxima`.

```
 inputmatrix(4,2) = -inputmatrix(4,2);
```

```
[lmaxima1,indices1] = localmax(inputmatrix,4,false);
isequal(lmaxima,lmaxima1)
```

Determine the local maxima from the CWT of the `cuspamax` signal with the Haar wavelet. Plot the CWT coefficient moduli and the maxima lines.

```
load cuspamax;
x = 1:length(cuspamax);
scales = 1:32;
cfs = cwt(cuspamax,scales,'haar');
[lmaxima,indices] = localmax(cfs,[],false);
[iRow,iCol] = find(lmaxima);
subplot(211);
imagesc(abs(cfs)); axis xy;
axis([1 1024 1 32]);
ylabel('Scale'); title('CWT Coefficients (Moduli)');
subplot(212);
plot(x(iCol),scales(iRow),'marker','o','markerfacecolor',[0 0 1],...
    'linestyle','none');
xlabel('Position'); ylabel('Scale'); title('Maxima Lines');
axis([1 1024 1 32]);
```



Introduced in R2008a

ls2filt

Transform lifting scheme to quadruplet of filters

Syntax

```
[LoD,HiD,LoR,HiR] = ls2filt(LS)
```

Description

`[LoD,HiD,LoR,HiR] = ls2filt(LS)` returns the four filters `LoD`, `HiD`, `LoR`, and `HiR` associated with the lifting scheme `LS`.

Examples

```
% Start from the db2 wavelet and get the
% corresponding lifting scheme.
LS = liftwave('db2')

LS =

    'd'         [   -1.7321]    [ 0]
    'p'         [1x2 double]    [ 1]
    'd'         [         1]    [-1]
    [1.9319]    [    0.5176]      []

% Visualize the obtained lifting scheme.

displs(LS);

LS = {...
'd'             [ -1.73205081]              [0]
'p'             [ -0.06698730  0.43301270]  [1]
'd'             [  1.00000000]              [-1]
[  1.93185165]  [  0.51763809]              []
};

% Get the filters from the lifting scheme.
```

```
[LoD,HiD,LoR,HiR] = ls2filt(LS)

LoD =

   -0.1294    0.2241    0.8365    0.4830

HiD =

   -0.4830    0.8365   -0.2241   -0.1294

LoR =

    0.4830    0.8365    0.2241   -0.1294

HiR =

   -0.1294   -0.2241    0.8365   -0.4830

% Get the db2 filters using wfilters.
% You can check the equality.

[LoDref,HiDref,LoRref,HiRref] = wfilters('db2')

LoDref =

   -0.1294    0.2241    0.8365    0.4830

HiDref =

   -0.4830    0.8365   -0.2241   -0.1294

LoRref =

    0.4830    0.8365    0.2241   -0.1294

HiRref =

   -0.1294   -0.2241    0.8365   -0.4830
```

See Also

filt2ls | lsinfo

Introduced before R2006a

lsinfo

Lifting schemes information

Syntax

```
lsinfo
```

Description

`lsinfo` displays the following information about lifting schemes. A lifting scheme `LS` is a `N` x 3 cell array. The N-1 first rows of the array are elementary lifting steps (`ELS`). The last row gives the normalization of `LS`.

Each `ELS` has this format:

```
{type, coefficients, max_degree}
```

where `type` is `'p'` (primal) or `'d'` (dual), `coefficients` is a vector `C` of real numbers defining the coefficients of a Laurent polynomial `P` described below, and `max_degree` is the highest degree `d` of the monomials of `P`.

The Laurent polynomial `P` is of the form

$P(z) = C(1)*z^d + C(2)*z^(d-1) + ... + C(m)*z^(d-m+1)$

The lifting scheme `LS` is such that for

`k = 1:N-1`, `LS{k,:}` is an `ELS`, where

`LS{k,1}` is the lifting type `'p'` (primal) or `'d'` (dual).

`LS{k,2}` is the corresponding lifting filter.

`LS{k,3}` is the highest degree of the Laurent polynomial corresponding to the filter `LS{k,2}`.

`LS{N,1}` is the primal normalization (real number).

`LS{N,2}` is the dual normalization (real number).

`LS{N,3}` is not used.

Usually, the normalizations are such that `LS{N,1}*LS{N,2} = 1`.

For example, the lifting scheme associated with the wavelet db1 is

```
LS = {...
      'd'          [    -1]    [0]
      'p'          [0.5000]    [0]
      [1.4142]     [0.7071]     []
     }
```

See Also

`displs` | `laurpoly`

Introduced before R2006a

lwt

1-D lifting wavelet transform

Syntax

```
[CA,CD] = lwt(X,W)
X_InPlace = lwt(X,W)
lwt(X,W,LEVEL)
X_InPlace = lwt(X,W,LEVEL,'typeDEC',typeDEC)
[CA,CD] = lwt(X,W,LEVEL,'typeDEC',typeDEC)
```

Description

`lwt` performs a 1-D lifting wavelet decomposition with respect to a particular lifted wavelet that you specify.

`[CA,CD] = lwt(X,W)` computes the approximation coefficients vector `CA` and detail coefficients vector `CD`, obtained by a lifting wavelet decomposition, of the vector `X`. `W` is a lifted wavelet name (see `liftwave`).

`X_InPlace = lwt(X,W)` computes the approximation and detail coefficients. These coefficients are stored in place:

`CA = X_InPlace(1:2:end)` and `CD = X_InPlace(2:2:end)`

`lwt(X,W,LEVEL)` computes the lifting wavelet decomposition at level `LEVEL`.

`X_InPlace = lwt(X,W,LEVEL,'typeDEC',typeDEC)` or `[CA,CD] = lwt(X,W,LEVEL,'typeDEC',typeDEC)` with `typeDEC = 'w'` or `'wp'` computes the wavelet or the wavelet packet decomposition using lifting, at level `LEVEL`.

Instead of a lifted wavelet name, you may use the associated lifting scheme `LS`: `lwt(X,LS,...)` instead of `lwt(X,W,...)`.

For more information about lifting schemes, see `lsinfo`.

Examples

```
% Start from the Haar wavelet and get the
% corresponding lifting scheme.
lshaar = liftwave('haar');

% Add a primal ELS to the lifting scheme.
els = {'p',[-0.125 0.125],0};
lsnew = addlift(lshaar,els);

% Perform LWT at level 1 of a simple signal.
x = 1:8;
[cA,cD] = lwt(x,lsnew)

cA =

    1.9445    4.9497    7.7782   10.6066


cD =

    0.7071    0.7071    0.7071    0.7071

% Perform integer LWT of the same signal.
lshaarInt = liftwave('haar','int2int');
lsnewInt = addlift(lshaarInt,els);
[cAint,cDint] = lwt(x,lsnewInt)

cAint =

     1     3     5     7


cDint =

     1     1     1     1
```

More About

Algorithms

This function uses the polyphase algorithm.

`lwt` reduces to `dwt` with zero-padding extension mode and without extra-coefficients.

References

Strang, G.; T. Nguyen (1996), *Wavelets and filter banks*, Wellesley-Cambridge Press.

Sweldens, W. (1998), "The Lifting Scheme: a Construction of Second Generation of Wavelets," *SIAM J. Math. Anal.*, 29 (2), pp. 511–546.

See Also

`ilwt`

Introduced before R2006a

lwt2

2-D lifting wavelet transform

Syntax

```
[CA,CH,CV,CD] = lwt2(X,W)
X_InPlace = lwt2(X,LS)
lwt2(X,W,LEVEL)
X_InPlace = lwt2(X,W,LEVEL,'typeDEC',typeDEC)
[CA,CH,CV,CD] = LWT2(X,W,LEVEL,'typeDEC',typeDEC)
```

Description

`lwt2` performs a 2-D lifting wavelet decomposition with respect to a particular lifted wavelet that you specify.

`[CA,CH,CV,CD] = lwt2(X,W)` computes the approximation coefficients matrix `CA` and detail coefficients matrices `CH`, `CV`, and `CD`, obtained by a lifting wavelet decomposition, of the matrix `X`. `W` is a lifted wavelet name (see `liftwave`).

`X_InPlace = lwt2(X,LS)` computes the approximation and detail coefficients. These coefficients are stored in place:

- `CA = X_InPlace(1:2:end,1:2:end)`
- `CH = X_InPlace(2:2:end,1:2:end)`
- `CV = X_InPlace(1:2:end,2:2:end)`
- `CD = X_InPlace(2:2:end,2:2:end)`

`lwt2(X,W,LEVEL)` computes the lifting wavelet decomposition at level `LEVEL`.

`X_InPlace = lwt2(X,W,LEVEL,'typeDEC',typeDEC)` or `[CA,CH,CV,CD] = LWT2(X,W,LEVEL,'typeDEC',typeDEC)` with `typeDEC = 'w'` or `'wp'` computes the wavelet or the wavelet packet decomposition using lifting, at level `LEVEL`.

Instead of a lifted wavelet name, you may use the associated lifting scheme `LS`: `lwt2(X,LS,...)` instead of `LWT2(X,W,...)`.

For more information about lifting schemes, see `lsinfo`.

Examples

```
% Start from the Haar wavelet and get the
% corresponding lifting scheme.
lshaar = liftwave('haar');

% Add a primal ELS to the lifting scheme.
els = {'p',[-0.125 0.125],0};
lsnew = addlift(lshaar,els);

% Perform LWT at level 1 of a simple image.
x = reshape(1:16,4,4);
[cA,cH,cV,cD] = lwt2(x,lsnew)

cA =

    5.7500   22.7500
   10.0000   27.0000


cH =

    1.0000    1.0000
    1.0000    1.0000


cV =

    4.0000    4.0000
    4.0000    4.0000


cD =

     0     0
     0     0

% Perform integer LWT of the same image.
lshaarInt = liftwave('haar','int2int');
lsnewInt = addlift(lshaarInt,els);
[cAint,cHint,cVint,cDint] = lwt2(x,lsnewInt)

cAint =
```

```
     3    11
     5    13


cHint =

     1     1
     1     1


cVint =

     4     4
     4     4


cDint =

     0     0
     0     0
```

More About

Tips

When X represents an indexed image, X, as well as the output arrays cA,cH,cV,cD, or `X_InPlace` are `m`-by-`n` matrices. When X represents a truecolor image, it is an `m`-by-`n`-by-3 array, where each `m`-by-`n` matrix represents a red, green, or blue color plane concatenated along the third dimension.

For more information on image formats, see the `image` and `imfinfo` reference pages .

Algorithms

This function implements the polyphase algorithm.

`lwt` reduces to `dwt` with zero-padding extension mode and without extra-coefficients.

References

Strang, G.; T. Nguyen (1996), *Wavelets and filter banks*, Wellesley-Cambridge Press.

Sweldens, W. (1998), "The Lifting Scheme: a Construction of Second Generation of Wavelets," *SIAM J. Math. Anal.*, 29 (2), pp. 511–546.

See Also

`ilwt2`

Introduced before R2006a

lwtcoef

Extract or reconstruct 1-D LWT wavelet coefficients

Syntax

```
Y = lwtcoef(TYPE,XDEC,LS,LEVEL,LEVEXT)
Y = lwtcoef(TYPE,XDEC,W,LEVEL,LEVEXT)
```

Description

`Y = lwtcoef(TYPE,XDEC,LS,LEVEL,LEVEXT)` returns the coefficients or the reconstructed coefficients of level `LEVEXT`, extracted from `XDEC`, the LWT decomposition at level `LEVEL` obtained with the lifting scheme `LS`.

The valid values for `TYPE` are

TYPE Values	Description
`'a'`	Approximations
`'d'`	Details
`'ca'`	Coefficients of approximations
`'cd'`	Coefficients of details

`Y = lwtcoef(TYPE,XDEC,W,LEVEL,LEVEXT)` returns the same output using `W`, which is the name of a lifted wavelet.

Examples

```
% Start from the Haar wavelet and get the
% corresponding lifting scheme.
lshaar = liftwave('haar');

% Add a primal ELS to the lifting scheme.
els = {'p',[-0.125 0.125],0};
lsnew = addlift(lshaar,els);

% Perform LWT at level 2 of a simple signal.
```

```
x = 1:8;
xDec = lwt(x,lsnew,2)

xDec =

    4.3438    0.7071    2.1250    0.7071   13.0313    0.7071
    2.0000    0.7071

% Extract approximation coefficients of level 1.
ca1 = lwtcoef('ca',xDec,lsnew,2,1)

ca1 =

    1.9445    4.9497    7.7782   10.6066

% Reconstruct approximations and details.
a1 = lwtcoef('a',xDec,lsnew,2,1)

a1 =

    1.3750    1.3750    3.5000    3.5000    5.5000    5.5000
    7.5000    7.5000

a2 = lwtcoef('a',xDec,lsnew,2,2)

a2 =

    2.1719    2.1719    2.1719    2.1719    6.5156    6.5156
    6.5156    6.5156

d1 = lwtcoef('d',xDec,lsnew,2,1)

d1 =

   -0.3750    0.6250   -0.5000    0.5000   -0.5000    0.5000
   -0.5000    0.5000

d2 = lwtcoef('d',xDec,lsnew,2,2)

d2 =

   -0.7969   -0.7969    1.3281    1.3281   -1.0156   -1.0156
    0.9844    0.9844
```

```
% Check perfect reconstruction.
err = max(abs(x-a2-d2-d1))

err =

  9.9920e-016
```

See Also

ilwt | lwt

Introduced before R2006a

lwtcoef2

Extract or reconstruct 2-D LWT wavelet coefficients

Syntax

```
Y = lwtcoef2(TYPE,XDEC,LS,LEVEL,LEVEXT)
Y = lwtcoef2(TYPE,XDEC,W,LEVEL,LEVEXT)
```

Description

`Y = lwtcoef2(TYPE,XDEC,LS,LEVEL,LEVEXT)` returns the coefficients or the reconstructed coefficients of level `LEVEXT`, extracted from `XDEC`, the LWT decomposition at level `LEVEL` obtained with the lifting scheme `LS`.

The valid values for `TYPE` are listed in this table.

TYPE Values	Description
`'a'`	Approximations
`'h'`	Horizontal details
`'v'`	Vertical details
`'d'`	Diagonal details
`'ca'`	Coefficients of approximations
`'ch'`	Coefficients of horizontal details
`'cv'`	Coefficients of vertical details
`'cd'`	Coefficients of diagonal details

`Y = lwtcoef2(TYPE,XDEC,W,LEVEL,LEVEXT)` returns the same output using `W`, which is the name of a lifted wavelet.

Examples

```
% Start from the Haar wavelet and get the
% corresponding lifting scheme.
lshaar = liftwave('haar');
```

```
% Add a primal ELS to the lifting scheme.
els = {'p',[-0.125 0.125],0};
lsnew = addlift(lshaar,els);

% Perform LWT at level 2 of a simple image.
x = reshape(1:16,4,4);
xDec = lwt2(x,lsnew,2)

xDec =

   27.4375    4.0000   17.0000    4.0000
    1.0000         0    1.0000         0
    4.2500    4.0000    0.0000    4.0000
    1.0000         0    1.0000         0

% Extract approximation coefficients of level 1.
ca1 = lwtcoef2('ca',xDec,lsnew,2,1)

ca1 =

    5.7500   22.7500
   10.0000   27.0000

% Reconstruct approximations and details.
a1 = lwtcoef2('a',xDec,lsnew,2,1)

a1 =

    2.8750    2.8750   11.3750   11.3750
    2.8750    2.8750   11.3750   11.3750
    5.0000    5.0000   13.5000   13.5000
    5.0000    5.0000   13.5000   13.5000

a2 = lwtcoef2('a',xDec,lsnew,2,2)

a2 =

    6.8594    6.8594    6.8594    6.8594
    6.8594    6.8594    6.8594    6.8594
    6.8594    6.8594    6.8594    6.8594
    6.8594    6.8594    6.8594    6.8594

h1 = lwtcoef2('h',xDec,lsnew,2,1)
```

```
h1 =

   -0.3750   -0.3750   -0.3750   -0.3750
    0.6250    0.6250    0.6250    0.6250
   -0.5000   -0.5000   -0.5000   -0.5000
    0.5000    0.5000    0.5000    0.5000

v1 = lwtcoef2('v',xDec,lsnew,2,1)

v1 =

   -1.5000    2.5000   -2.0000    2.0000
   -1.5000    2.5000   -2.0000    2.0000
   -1.5000    2.5000   -2.0000    2.0000
   -1.5000    2.5000   -2.0000    2.0000

d1 = lwtcoef2('d',xDec,lsnew,2,1)

d1 =

     0     0     0     0
     0     0     0     0
     0     0     0     0
     0     0     0     0

h2 = lwtcoef2('h',xDec,lsnew,2,2)

h2 =

   -0.7969   -0.7969   -0.7969   -0.7969
   -0.7969   -0.7969   -0.7969   -0.7969
    1.3281    1.3281    1.3281    1.3281
    1.3281    1.3281    1.3281    1.3281

v2 = lwtcoef2('v',xDec,lsnew,2,2)

v2 =

   -3.1875   -3.1875    5.3125    5.3125
   -3.1875   -3.1875    5.3125    5.3125
   -3.1875   -3.1875    5.3125    5.3125
   -3.1875   -3.1875    5.3125    5.3125
```

```
d2 = lwtcoef2('d',xDec,lsnew,2,2)

d2 =

  1.0e-015 *

    0.2498    0.2498   -0.4163   -0.4163
    0.2498    0.2498   -0.4163   -0.4163
   -0.4163   -0.4163    0.6939    0.6939
   -0.4163   -0.4163    0.6939    0.6939

% Check perfect reconstruction.
err = max(max(abs(x-a2-h2-v2-d2-h1-v1-d1)))

err =

  3.5527e-015
```

More About

Tips

If XDEC is obtained from an indexed image analysis or a truecolor image analysis, it is an `m`-by-`n` matrix or an `m`-by-`n`-by-3 array, respectively.

For more information on image formats, see the `image` and `imfinfo` reference pages.

See Also

`ilwt2` | `lwt2`

Introduced before R2006a

mdwtcluster

Multisignals 1-D clustering

Syntax

```
S = mdwtcluster(X)
S = mdwtcluster(X,'PropName1',PropVal1,'PropName2',PropVal2,...)
```

Description

`S = mdwtcluster(X)` constructs clusters from a hierarchical cluster tree. The input matrix *X* is decomposed in row direction using the DWT function with the `haar` wavelet and the maximum allowed level.

`S = mdwtcluster(X,'PropName1',PropVal1,'PropName2',PropVal2,...)` allows you to modify some properties. The valid choices for *PropName* are:

Note: `mdwtcluster` requires the Statistics and Machine Learning Toolbox™

`'dirDec'`	`'r'` (row) or `'c'` (column). Default value is `'r'`.
`'level'`	Level of the DWT decomposition. Default value is: `level=fix(log2(size(X,d)))` where `d=1` or `d=2`, depending on the `dirDec` value.
`'wname'`	Wavelet name used for DWT. Default value is `'haar'`.
`'dwtEXTM'`	DWT extension mode (see `dwtmode`).
`'pdist'`	See Statistics and Machine Learning Toolbox `pdist` function. Default value is `'euclidean'`.
`'linkage'`	See Statistics and Machine Learning Toolbox `linkage` function. Default value is `'ward'`.
`'maxclust'`	Number of clusters. Default value is 6. The input variable can be a vector.

`'lst2clu'`	Cell array that contains the list of data to classify. If `N` is the level of decomposition, the allowed name values for the cells are: • `'s'` — Signal • `'aj'` — Approximation at level `j` • `'dj'` — Detail at level `j` • `'caj'` — Coefficients of approximation at level `j` • `'cdj'` — Coefficients of detail at level `j` Default value is `{'s';'ca1';...;'caN'}`.

The output structure S is such that for each partition `j`:

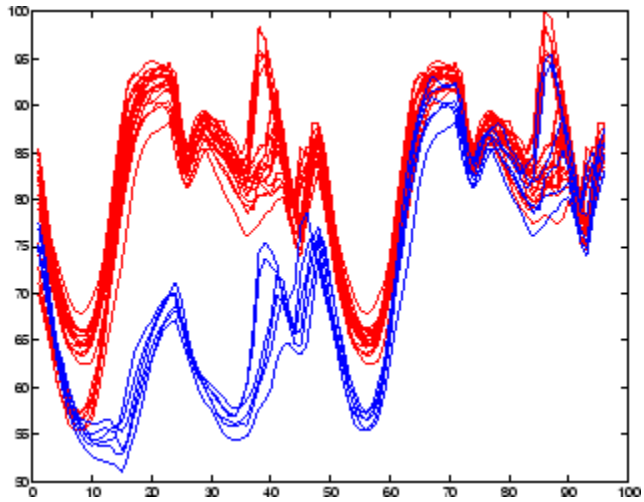
`S.Idx(:,j)`	Contains the cluster numbers obtained from the hierarchical cluster tree (see `cluster` in the Statistics and Machine Learning Toolbox software).
`S.Incons(:,j)`	Contains the inconsistent values of each non-leaf node in the hierarchical cluster tree (see Statistics and Machine Learning Toolbox software function `inconsistent`).
`S.Corr(j)`	Contains the cophenetic correlation coefficients of the partition (see Statistics and Machine Learning Toolbox software function `cophenet`).

Note If `maxclustVal` is a vector, then `IdxCLU` is a multidimensional array such that `IdxCLU(:,j,k)` contains the cluster numbers obtained from the hierarchical cluster tree for `k` clusters.

Examples

```
load elecsig10
lst2clu = {'s','ca1','ca3','ca6'};

% Compute the structure resulting from multisignal clustering
S = mdwtcluster(signals,'maxclust',4,'lst2clu',lst2clu)
```

```
S =

    IdxCLU: [70x4 double]
    Incons: [69x4 double]
      Corr: [0.7920 0.7926 0.7947 0.7631]

% Retrieve indices of clusters
IdxCLU = S.IdxCLU;

% Plot the first cluster
plot(signals(IdxCLU(:,1)==1,:)','r');
hold on;

% Plot the third clustering
plot(signals(IdxCLU(:,1)==3,:)','b')
```




```
% Check the equality of partitions
equalPART = isequal(IdxCLU(:,1),IdxCLU(:,3))

equalPART =

     1

% So we can see that we obtain the same partitions using
```

```
% coefficents of approximation at level 3 instead of original
% signals. Much less information is then used.
```

See Also

mdwtdec | wavedec

Introduced in R2008a

mdwtdec

Multisignal 1-D wavelet decomposition

Syntax

```
DEC = mdwtdec(DIRDEC,X,LEV,WNAME)
DEC = mdwtdec(DIRDEC,X,LEV,LoD,HiD,LoR,HiR)
DEC = mdwtdec(...,'mode',EXTMODE)
```

Description

`DEC = mdwtdec(DIRDEC,X,LEV,WNAME)` returns the wavelet decomposition at level `LEV` of each row (if `DIRDEC = 'r'`) or each column (if `DIRDEC = 'c'`) of matrix *X*, using the wavelet `WNAME`.

The output `DEC` is a structure with the following fields:

`'dirDec'`	Direction indicator: `'r'` (row) or `'c'` (column)
`'level'`	Level of the DWT decomposition
`'wname'`	Wavelet name
`'dwtFilters'`	Structure with four fields `LoD`, `HiD`, `LoR` and `HiR`
`'dwtEXTM'`	DWT extension mode (see `dwtmode`)
`'dwtShift'`	DWT shift parameter (0 or 1)
`'dataSize'`	Size of `X`
`'ca'`	Approximation coefficients at level `LEV`
`'cd'`	Cell array of detail coefficients, from level 1 to level `LEV`

Coefficients `cA` and `cD{k} (for k = 1 to LEV)` are matrices and are stored in rows if `DIRDEC` = `'r'` or in columns if `DIRDEC` = `'c'`.

`DEC = mdwtdec(DIRDEC,X,LEV,LoD,HiD,LoR,HiR)` uses the four filters instead of the wavelet name.

`DEC = mdwtdec(...,'mode',EXTMODE)` computes the wavelet decomposition with the `EXTMODE` extension mode that you specify (see `dwtmode` for the valid extension modes).

Examples

```
% Load original 1D-multisignal.
load thinker

% Perform a decomposition at level 2 using wavelet db2.
dec = mdwtdec('r',X,2,'db2')
dec =
        dirDec: 'r'
         level: 2
         wname: 'db2'
    dwtFilters: [1x1 struct]
       dwtEXTM: 'sym'
      dwtShift: 0
      dataSize: [192 96]
            ca: [192x26 double]
            cd: {[192x49 double]  [192x26 double]}

% Compute the associated filters of db2 wavelet.
[LoD,HiD,LoR,HiR] = wfilters('db2');

% Perform a decomposition at level 2 using filters.
decBIS = mdwtdec('r',X,2,LoD,HiD,LoR,HiR)

decBIS =
        dirDec: 'r'
         level: 2
         wname: ''
    dwtFilters: [1x1 struct]
       dwtEXTM: 'sym'
      dwtShift: 0
      dataSize: [192 96]
            ca: [192x26 double]
            cd: {[192x49 double]  [192x26 double]}
```

References

Daubechies, I. , *Ten lectures on wavelets*, CBMS-NSF conference series in applied mathematics. SIAM Ed., 1992.

Mallat, S., “A theory for multiresolution signal decomposition: the wavelet representation,” *IEEE Pattern Anal. and Machine Intell.*, vol. 11, no. 7, 1989, pp. 674–693.

Meyer, Y. , *Ondelettes et opérateurs*, Tome 1, Hermann Ed. (English translation: *Wavelets and operators*, Cambridge Univ. Press. 1993.)

See Also

`mdwtdec` | `wavedec`

Introduced in R2007a

mdwtrec

Multisignal 1-D wavelet reconstruction

Syntax

```
X = mdwtrec(DEC)
X = mdwtrec(DEC,IDXSIG)
Y = mdwtrec(DEC,TYPE,LEV)
A = mdwtrec(DEC,'a')
A = mdwtrec(DEC,'a',LEVDEC)
D = mdwtrec(DEC,'d')
CA = mdwtrec(DEC,'ca')
CA = mdwtrec(DEC,'ca',LEVDEC)
CD = mdwtrec(DEC,'cd',MODE)
CFS = mdwtrec(DEC,'cfs',MODE)
Y = mdwtrec(...,IDXSIG)
```

Description

`X = mdwtrec(DEC)` returns the original matrix of signals, starting from the wavelet decomposition structure DEC (see `mdwtdec`).

`X = mdwtrec(DEC,IDXSIG)` reconstructs the signals whose indices are given by the vector `IDXSIG`.

`Y = mdwtrec(DEC,TYPE,LEV)` extracts or reconstructs the detail or approximation coefficients at level `LEV` depending on the `TYPE` value. The maximum value for `LEV` is `LEVDEC = DEC.level`.

When TYPE is equal to:

- `'cd'` or `'ca'`, coefficients of level LEV are extracted.
- `'d'` or `'a'`, coefficients of level LEV are reconstructed.
- `'a'` or `'ca'`, LEV must be such that $0 \le LEV \le LEVDEC$.
- `'d'` or `'cd'`, LEV must be such that $1 \le LEV \le LEVDEC$.

`A = mdwtrec(DEC,'a')` is equivalent to `A = mdwtrec(DEC,'a',LEVDEC)`.

`D = mdwtrec(DEC,'d')` returns a matrix containing the sum of all the details, so that `X = A + D`.

`CA = mdwtrec(DEC,'ca')` is equivalent to `CA = mdwtrec(DEC,'ca',LEVDEC)`.

`CD = mdwtrec(DEC,'cd',MODE)` returns a matrix containing all the detail coefficients.

`CFS = mdwtrec(DEC,'cfs',MODE)` returns a matrix containing all the coefficients.

For `MODE = 'descend'` the coefficients are concatened from level `LEVDEC` to level 1 and `MODE = 'descend'` concatenates from level 1 to level `LEVDEC`). The default is `MODE = 'descend'`. The concatenation is made row-wise if `DEC.dirDEC = 'r'` or column-wise if `DEC.dirDEC = 'c'`.

`Y = mdwtrec(...,IDXSIG)` extracts or reconstructs the detail or the approximation coefficients for the signals whose indices are given by the vector `IDXSIG`.

Examples

```
% Load original 1D-multisignal.
load thinker

% Perform a decomposition at level 2 using wavelet db2.
dec = mdwtdec('r',X,2,'db2');

% Reconstruct the original matrix of signals, starting from
% the wavelet decomposition structure dec.
XR = mdwtrec(dec);

% Compute the reconstruction error.
errREC = max(max(abs(X-XR)))

errREC =
  2.1026e-010

% Reconstruct the original signal 31, the corresponding
% approximation at level 2, details at levels 1 and 2.
Y = mdwtrec(dec,31);
A2 = mdwtrec(dec,'a',2,31);
D2 = mdwtrec(dec,'d',2,31);
```

```
D1 = mdwtrec(dec,'d',1,31);

% Compute the reconstruction error for signal 31.
errREC = max(abs(Y-A2-D2-D1))

errREC =
  6.8390e-014
```

References

Daubechies, I., *Ten lectures on wavelets*, CBMS-NSF conference series in applied mathematics. SIAM Ed., 1992.

Mallat, S., "A theory for multiresolution signal decomposition: the wavelet representation," *IEEE Pattern Anal. and Machine Intell.*, vol. 11, no. 7, 1989, pp. 674–693.

Meyer, Y., *Ondelettes et opérateurs*, Tome 1, Hermann Ed. (English translation: *Wavelets and operators*, Cambridge Univ. Press. 1993.)

See Also

`mdwtdec` | `waverec`

Introduced in R2007a

measerr

Approximation quality metrics

Syntax

```
[PSNR,MSE,MAXERR,L2RAT] = measerr(X,XAPP)
[...] = measerr(...,BPS)
```

Description

`[PSNR,MSE,MAXERR,L2RAT] = measerr(X,XAPP)` returns the peak signal-to-noise ratio, `PSNR`, mean square error, `MSE`, maximum squared error, `MAXERR`, and ratio of squared norms, `L2RAT`, for an input signal or image, `X`, and its approximation, `XAPP`.

`[...] = measerr(...,BPS)` uses the bits per sample, `BPS`, to determine the peak signal-to-noise ratio.

Input Arguments

X

`X` is a real-valued signal or image.

XAPP

`XAPP` is a real-valued signal or image approximation with a size equal to that of the input data, `X`.

BPS

`BPS` is the number of bits per sample in the data.

Default: 8

Output Arguments

PSNR

PSNR is the peak signal-to-noise ratio in decibels (dB). The PSNR is only meaningful for data encoded in terms of bits per sample, or bits per pixel. For example, an image with 8 bits per pixel contains integers from 0 to 255.

MSE

The mean square error (MSE) is the squared norm of the difference between the data and the approximation divided by the number of elements.

MAXERR

MAXERR is the maximum absolute squared deviation of the data, X, from the approximation, XAPP.

L2RAT

L2RAT is the ratio of the squared norm of the signal or image approximation, XAPP, to the input signal or image, X.

Examples

Approximate an image and calculate approximation quality metrics.

```
load woman;
Xapp = X;
Xapp(X<=50) = 1;
[psnr,mse,maxerr,L2rat] = measerr(X,Xapp);
figure; colormap(map);
subplot(1,2,1); image(X);
subplot(1,2,2); image(Xapp);
```

Measure approximation quality in an RGB image.

```
X = imread('africasculpt.jpg');
Xapp = X;
Xapp(X<=100) = 1;
[psnr,mse,maxerr,L2rat] = measerr (X,Xapp)
```

```
figure;
subplot(1,2,1); image(X);
subplot(1,2,2); image(Xapp);
```

More About

Peak Signal to Noise Ratio (PSNR)

The following equation defines the PSNR:

$$20\log_{10}(\frac{2^B-1}{\sqrt{MSE}})$$

where MSE represents the mean square error and B represents the bits per sample.

Mean Square Error (MSE)

The mean square error between a signal or image, X, and an approximation, Y, is the squared norm of the difference divided by the number of elements in the signal or image:

$$\frac{||X-Y||^2}{N}$$

References

Huynh-Thu, Q.*Scope of validity of PSNR in image/video quality assessment*, Electronics Letters, 44, 2008, pp. 800–801.

See Also

wden | wdencmp

Introduced in R2010b

mexihat

Mexican hat wavelet

Syntax

```
[PSI,X] = mexihat(LB,UB,N)
```

Description

`[PSI,X] = mexihat(LB,UB,N)` returns values of the Mexican hat wavelet on an `N` point regular grid, `X`, in the interval `[LB,UB]`.

Output arguments are the wavelet function `PSI` computed on the grid `X`.

This wavelet has [-5 5] as effective support. Although [-5 5] is the correct theoretical effective support, a wider effective support, [-8 8], is used in the computation to provide more accurate results.

This function is proportional to the second derivative function of the Gaussian probability density function.

Note: You can use `gauswavf` to obtain a second order derivative of a Gaussian wavelet. If you use the negative of this normalized derivative, the resulting wavelet resembles the Mexican hat wavelet.

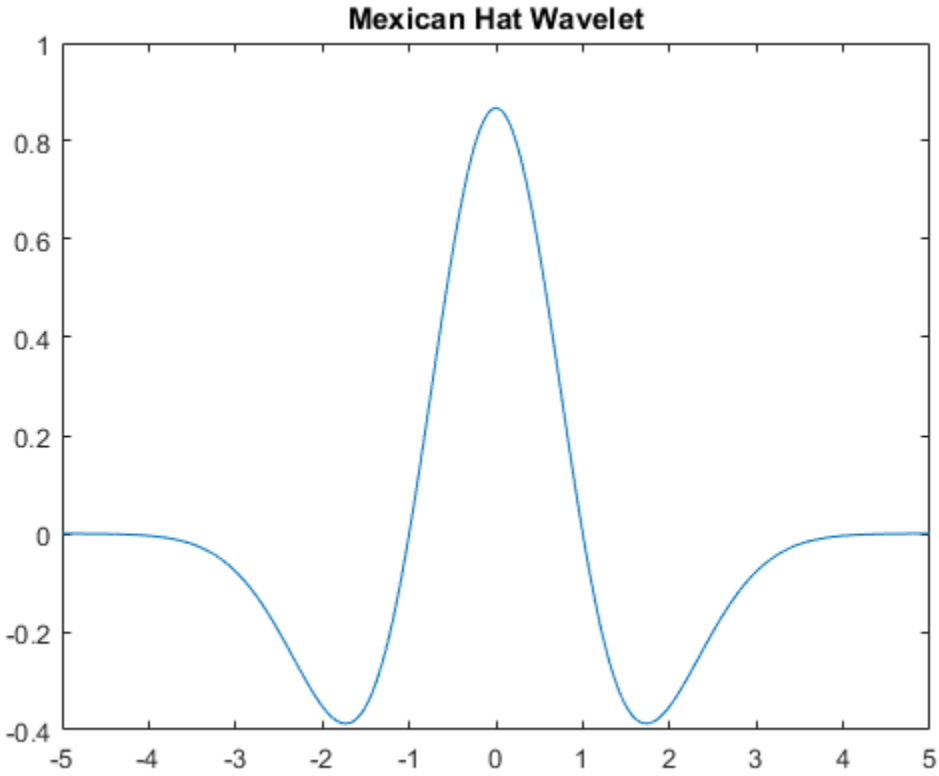
Examples

Mexican Hat Wavelet

Create a Mexican hat wavelet with support on [-5,5]. Use 1,000 sample points. Plot the result.

```
lb = -5;
ub = 5;
N = 1000;
```

```
[psi,xval] = mexihat(lb,ub,N);
plot(xval,psi)
title('Mexican Hat Wavelet');
```



See Also

`waveinfo`

Introduced before R2006a

meyer

Meyer wavelet

Syntax

```
[PHI,PSI,T] = meyer(LB,UB,N)
```

Description

`[PHI,PSI,T] = meyer(LB,UB,N)` returns Meyer scaling and wavelet functions evaluated on an *N* point regular grid in the interval `[LB,UB]`.

N must be a power of two.

Output arguments are the scaling function `PHI` and the wavelet function `PSI` computed on the grid *T*. These functions have [-8 8] as effective support.

If only one function is required, a fourth argument is allowed:

```
[PHI,T] = meyer(LB,UB,N,'phi')
[PSI,T] = meyer(LB,UB,N,'psi')
```

When the fourth argument is used, but not equal to `'phi'` or `'psi'`, outputs are the same as in the main option.

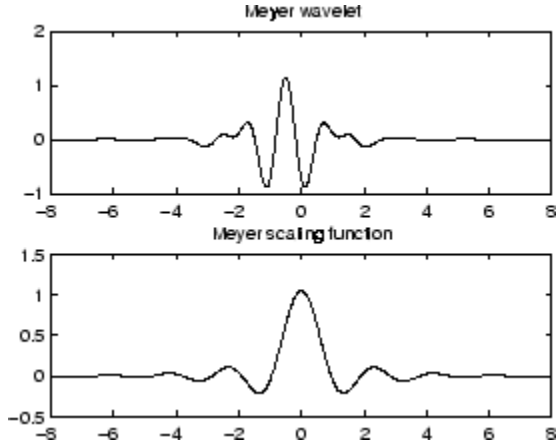
The Meyer wavelet and scaling function are defined in the frequency domain.

By changing the auxiliary function (see `meyeraux` for more information), you get a family of different wavelets.

Examples

```
% Set effective support and grid parameters.
lb = -8; ub = 8; n = 1024;

% Compute and plot Meyer wavelet and scaling functions.
[phi,psi,x] = meyer(lb,ub,n);
```

```
subplot(211), plot(x,psi)
title('Meyer wavelet')
subplot(212), plot(x,phi)
title('Meyer scaling function')
```



More About

Algorithms

Starting from an explicit form of the Fourier transform $\hat{\phi}$ of ϕ, `meyer` computes the values of $\hat{\phi}$ on a regular grid, and then the values of ϕ are computed using `instdfft`, the inverse nonstandard discrete FFT.

The procedure for ψ is along the same lines.

References

Daubechies, I. (1992), *Ten lectures on wavelets*, CBMS-NSF conference series in applied mathematics, SIAM Ed., pp. 117–119, 137, 152.

See Also

`meyeraux` | `wavefun` | `waveinfo`

Introduced before R2006a

meyeraux

Meyer wavelet auxiliary function

Syntax

```
Y = meyeraux(X)
```

Description

`Y = meyeraux(X)` returns values of the auxiliary function used for Meyer wavelet generation evaluated at the elements of the vector or matrix *X*.

The function is

$$35x^4 - 84x^5 + 70x^6 - 20x^7$$

See Also

`meyer`

Introduced before R2006a

modwpt

Maximal overlap discrete wavelet packet transform

Syntax

```
wpt = modwpt(x)
wpt = modwpt(x,wname)
wpt = modwpt(x,lo,hi)
wpt = modwpt(___,lev)

[wpt,packetlevs] = modwpt(___)
[wpt,packetlevs,cfreq] = modwpt(___)
[wpt,packetlevs,cfreq,energy] = modwpt(___)
[wpt,packetlevs,cfreq,energy,relenergy] = modwpt(___)

[___] = modwpt(___,Name,Value)
```

Description

`wpt = modwpt(x)` returns the terminal nodes for the maximal overlap discrete wavelet packet transform (MODWPT) for the 1-D real-valued signal, `x`.

Note: The output of the MODWPT is time-delayed compared to the input signal. Most filters used to obtain the MODWPT have a nonlinear phase response, which makes compensating for the time delay difficult. This is true for all orthogonal scaling and wavelet filters, except the Haar wavelet. It is possible to time-align the coefficients with the signal features, but the result is an approximation, not an exact alignment with the original signal. The MODWPT partitions the energy among the wavelet packets at each level. The sum of the energy over all the packets equals the total energy of the input signal. The output of MODWPT is useful for applications where you want to analyze the energy levels in different packets.

The MODWPT details (`modwptdetails`) are the result of zero-phase filtering of the signal. The features in the MODWPT details align exactly with features in the input signal. For a given level, summing the details for each sample returns the exact original

signal. The output of the MODWPT details is useful for applications that require time-alignment, such as nonparametric regression analysis.

`wpt = modwpt(x,wname)` returns the MODWPT using the orthogonal wavelet filter specified by the string `wname`.

`wpt = modwpt(x,lo,hi)` returns the MODWPT using the orthogonal scaling filter, `lo`, and wavelet filter, `hi`.

`wpt = modwpt(___,lev)` returns the terminal nodes of the wavelet packet tree at positive integer level `lev`.

`[wpt,packetlevs] = modwpt(___)` returns a vector of transform levels corresponding to the rows of `wpt`.

`[wpt,packetlevs,cfreq] = modwpt(___)` returns the center frequencies of the approximate passbands corresponding to the rows of `wpt`.

`[wpt,packetlevs,cfreq,energy] = modwpt(___)` returns the energy (squared L2 norm) of the wavelet packet coefficients for the nodes in `wpt`.

`[wpt,packetlevs,cfreq,energy,relenergy] = modwpt(___)` returns the relative energy for the wavelet packets in `wpt`.

`[___] = modwpt(___,Name,Value)` returns the MODWPT with additional options specified by one or more `Name,Value` pair arguments.

Examples

MODWPT Using Default Wavelet

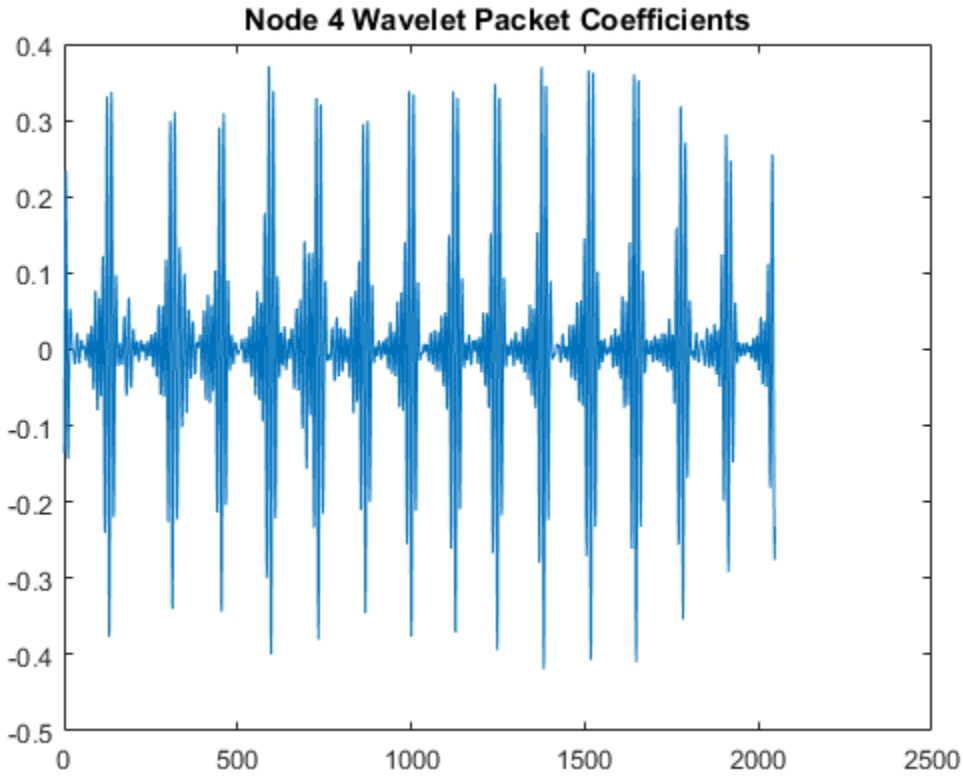
Obtain the MODWPT of an electrocardiogram (ECG) signal using the default length 18 Fejer-Korovkin (`'fk18'`) wavelet.

```
load wecg;
wpt = modwpt(wecg);
```

`wpt` is a 16-by-2048 matrix containing the sequency-ordered wavelet packet coefficients for the wavelet packet transform nodes. In this case, the nodes are at level 4. Each node corresponds to an approximate passband filtering of $[nf_s/2^5, (n+1)f_s/2^5)$, where n =

0,...,15, and f_s is the sampling frequency. Plot the wavelet packet coefficients at node (4,2), which is level 4, node 2.

```
plot(wpt(3,:))
title('Node 4 Wavelet Packet Coefficients')
```



MODWPT Using Daubechies Extremal Phase Wavelet with Two Vanishing Moments

Obtain the MODWPT of Southern Oscillation Index data with the Daubechies extremal phase wavelet with two vanishing moments (`'db2'`).

```
load soi;
wsoi = modwpt(soi,'db2');
```

Verify that the size of the resulting transform contains 16 nodes. Each node is in a separate row.

```
size(wsoi)
```

```
ans =

          16        12998
```

MODWPT Using Scaling and Wavelet Filters

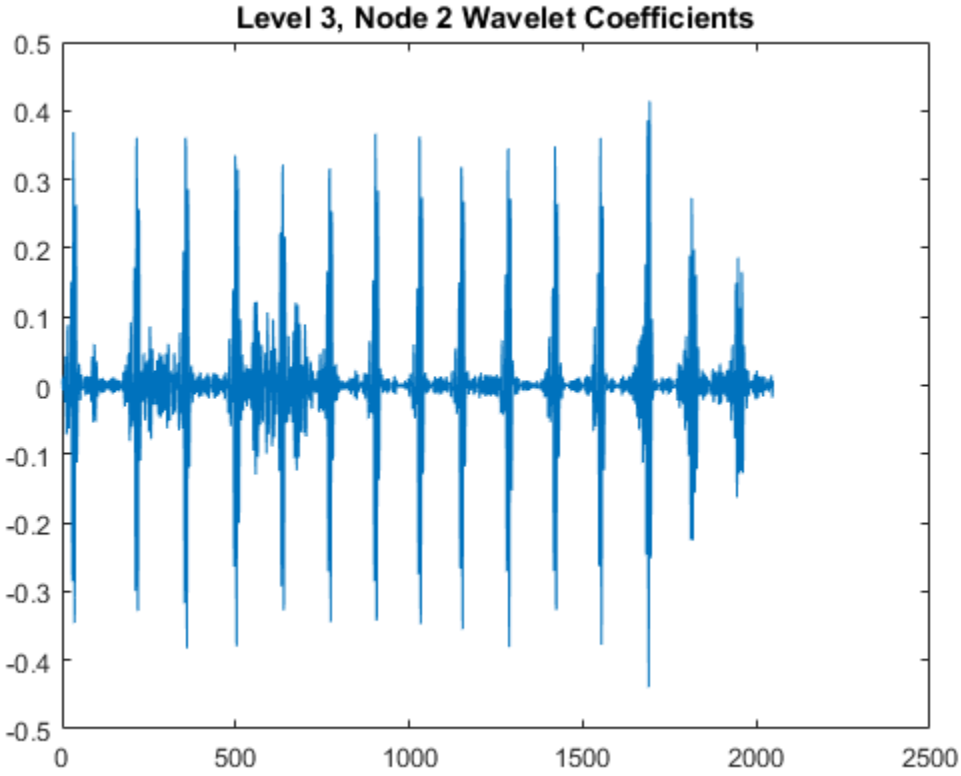
Obtain the MODWPT of an ECG waveform using the Fejer-Korovkin length 18 scaling and wavelet filters.

```
load wecg;
[lo,hi] = wfilters('fk18');
wpt = modwpt(wecg,lo,hi);
```

MODWPT Full Packet Tree and Passband Center Frequencies

Obtain the MODWPT and full wavelet packet tree of an ECG waveform using the default length 18 Fejer-Korovkin (`'fk18'`) wavelet. Extract and plot the node coefficients at level 3, node 2.

```
load wecg;
[wpt,packetlevels,cfreq] = modwpt(wecg,'FullTree',true);
p3 = wpt(packetlevels==3,:);
plot(p3(3,:))
title('Level 3, Node 2 Wavelet Coefficients')
```



Display the center frequencies at level 3.

```
cfreq(packetlevels==3,:)
```

```
ans =

    0.0313
    0.0938
    0.1563
    0.2188
    0.2813
    0.3438
    0.4063
```

```
    0.4688
```

MODWPT Energy and Relative Energy

Obtain and plot the MODWPT energy and relative energy of an ECG waveform.

```
load wecg;
[wpt,~,cfreq,energy,relenergy] = modwpt(wecg);
```

Show that the sum of the MODWPT energies is equal to the sum of the energy in the original signal. The difference between the total MODWPT energy and the signal energy is small enough to be considered insignificant.

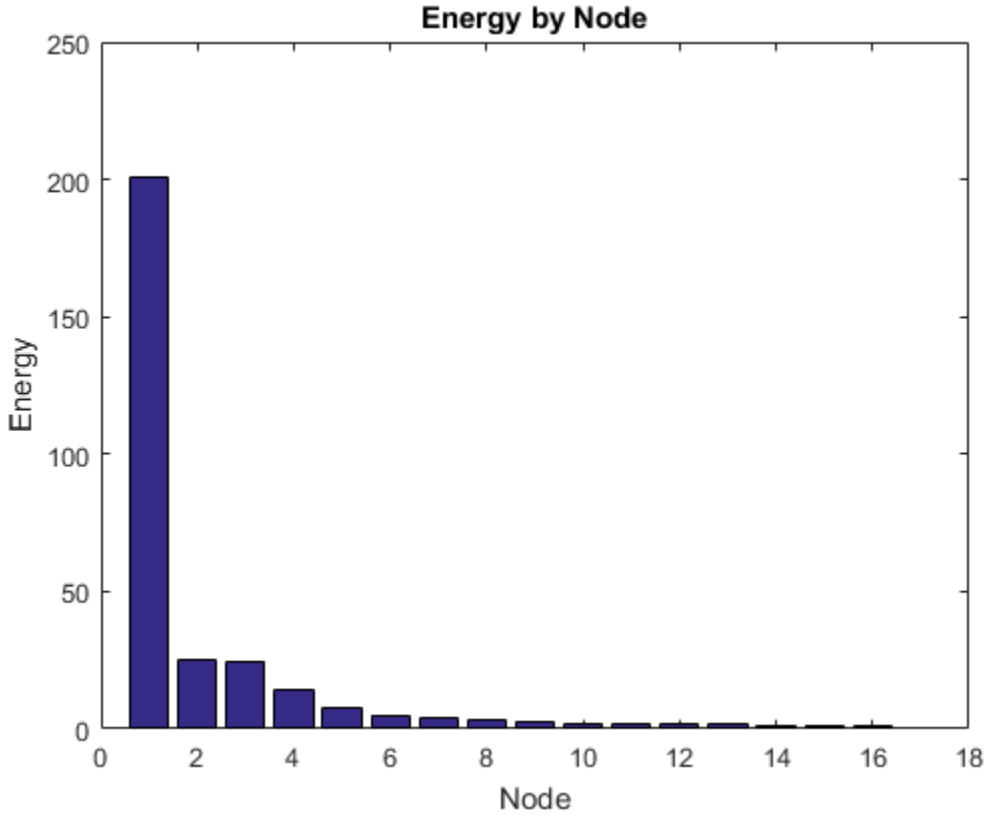
```
sum(energy)-(bandpower(wecg)*length(wecg))
```

```
ans =

   3.6120e-09
```

Plot the MODWPT energy by node.

```
figure;
bar(1:16,energy);
xlabel('Node')
ylabel('Energy')
title('Energy by Node')
sprintf('Total power in passband is %.2f', energy(1))
```

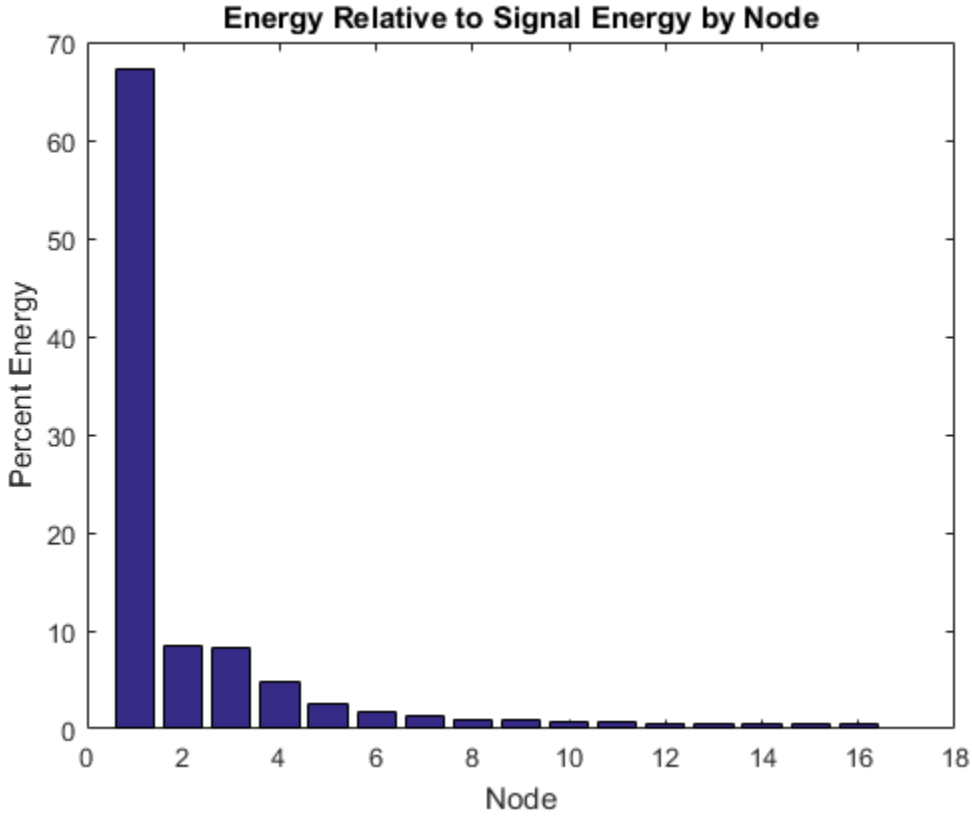
```
ans =

Total power in passband is 200.84
```



Plot the relative energy and show the percentage of signal energy in the first passband [0,5.6250].

```
figure;
bar(1:16,relenergy*100);
xlabel('Node');
ylabel('Percent Energy');
title('Energy Relative to Signal Energy by Node');
sprintf('Percentage of signal power in passband is %.1f',(relenergy(1)*100))
```

```
ans =

Percentage of signal power in passband is 67.3
```



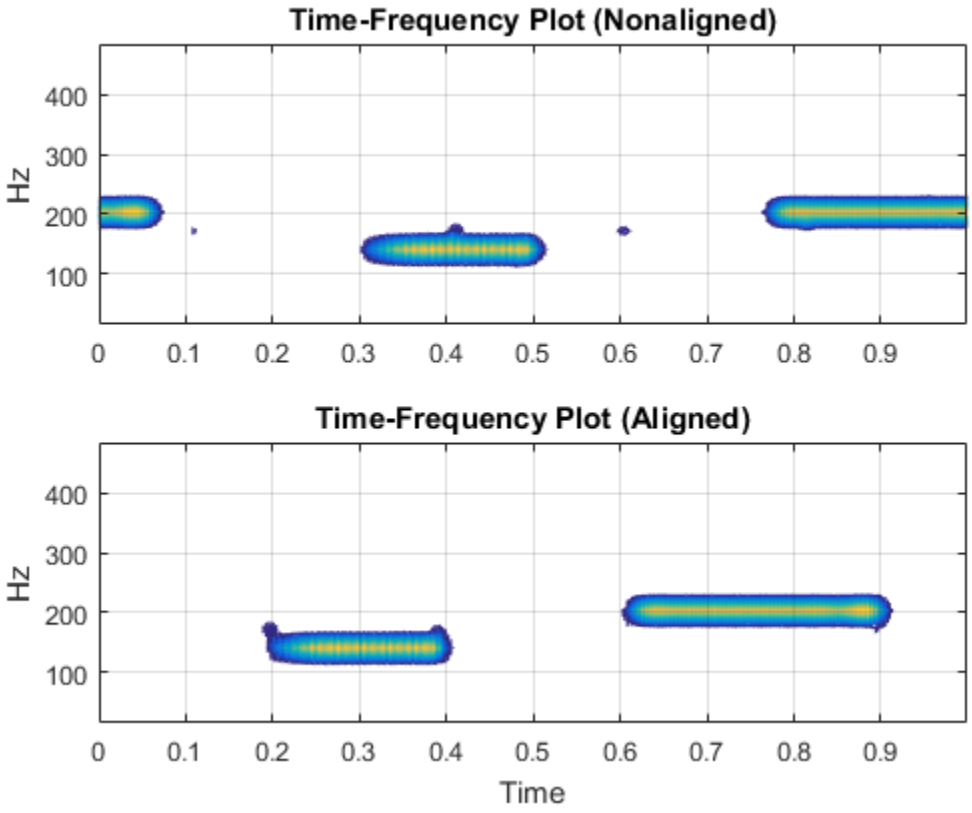
Time-Aligned MODWPT

Obtain the time-aligned MODWPT of two intermittent sine waves in noise. The sine wave frequencies are 150 Hz and 200 Hz. The data is sampled at 1000 Hz.

```
dt = 0.001;
t = 0:dt:1-dt;
x = cos(2*pi*150*t).*(t>=0.2 & t<0.4)+ sin(2*pi*200*t).*(t>0.6 & t<0.9);
y = x+0.05*randn(size(t));
[wpta,~,Falign] = modwpt(x,'TimeAlign',true);
[wptn,~,Fnon] = modwpt(x);
```

Compare the nonaligned and time-aligned time-frequency plots.

```
subplot(2,1,1);
contour(t,Fnon.*(1/dt),abs(wptn).^2);
grid on;
ylabel('Hz');
title('Time-Frequency Plot (Nonaligned)');
subplot(2,1,2)
contour(t,Falign.*(1/dt),abs(wpta).^2);
grid on;
xlabel('Time');
ylabel('Hz');
title('Time-Frequency Plot (Aligned)');
```



Input Arguments

x — Input signal

real-valued vector

Input signal, specified as a real-valued row or column vector. `x` must have at least two elements.

Data Types: `double`

wname — Analyzing wavelet filter

`fk18` (default) | string

Analyzing wavelet filter specified as a string that corresponds to an orthogonal wavelet. If you specify the scaling (`lo`) and wavelet (`hi`) filters, `modwpt` ignores the `wname` input.

Valid orthogonal wavelet families begin with one of the following strings, followed by an integer, *N*, for example, `sym4`. Note, however, that `'haar'` is not followed by an integer.

- `'haar'` — Haar wavelet, which is the same as Daubechies wavelet with one vanishing moment, `'db1'`.
- `'db`*N*`'` — Daubechies wavelet with *N* vanishing moments
- `'sym`*N*`'` — Symlets wavelet with *N* vanishing moments
- `'coif`*N*`'` — Coiflets wavelet with *N* vanishing moments
- `'fk`*N*`'` — Fejer-Korovkin wavelet with *N* coefficients

To check if your wavelet is orthogonal, use `wavemngr('type',wname)` and verify that it returns `1` as the wavelet type. To determine valid values for *N*, use `waveinfo`, for example, `waveinfo('fk')`.

`lo` — Scaling filter

even-length real-valued vector

Scaling filter, specified as an even-length real-valued vector. `lo` must satisfy the conditions necessary to generate an orthogonal scaling function. You cannot specify both the scaling-wavelet filters and the `wname` input.

`hi` — Wavelet filter

even-length real-valued vector

Wavelet filter, specified as an even-length real-valued vector. `hi` must satisfy the conditions necessary to generate an orthogonal wavelet. You cannot specify both the scaling-wavelet filters and the `wname` input.

`lev` — Transform level

positive integer

Transform level, specified as a positive integer less than or equal to `floor(log2(numel(x)))`.

Name-Value Pair Arguments

Specify optional comma-separated pairs of `Name,Value` arguments. `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside single

quotes (' '). You can specify several name and value pair arguments in any order as `Name1,Value1,...,NameN,ValueN`.

Example: `'Fulltree',true` returns the full wavelet packet tree

`'FullTree'` — Full packet tree
`false` (default) | `true`

Option to return the full wavelet packet tree, specified as the comma-separated pair consisting of `'FullTree'` and either `false` or `true`. If you specify `false`, then `modwpt` returns only the terminal (final-level) wavelet packet nodes. If you specify `true`, then `modwpt` returns the full wavelet packet tree down to the specified level.

Example: `'Fulltree',true`

`'TimeAlign'` — Signal time alignment
`false` (default) | `true`

Option to time align wavelet packet coefficients with signal features, specified as the comma-separated pair consisting of `'TimeAlign'` and either `true` to time align or `false` to not align.

The scaling and wavelet filters have a time delay. Circularly shifting the wavelet packet coefficients in all nodes aligns the signal and wavelet coefficients in time. If you want to reconstruct the signal, such as by using `imodwpt`, do not shift the coefficients because time alignment is done during the inversion process.

Example: `'TimeAlign',true`

Output Arguments

`wpt` — Wavelet packet transform
matrix

Wavelet packet tree, returned as a matrix with each row containing the sequency-ordered wavelet packet coefficients. By default, `wpt` contains only the terminal level for the MODWPT. The default terminal level is either level 4 or `floor(log2(numel(x)))`, whichever is smaller. At level 4, `wpt` is a 16-by-`numel(x)` matrix. For the full tree, at level j, `wpt` is a 2^{j+2}-2-by-`numel(x)` matrix, with each row containing the packet

coefficients by level and index. The approximate passband for the nth row of `wpt` at level j is $\left[\frac{n-1}{2^{(j+1)}}, \frac{n}{2^{(j+1)}}\right)$ cycles/sample, where $n = 1,2,...2^j$.

`packetlevs` — Transform levels

vector

Transform levels, returned as a vector. The levels correspond to the rows of `wpt`. If `wpt` contains only the terminal level coefficients, `packetlevs` is a vector of constants equal to the terminal level. If `wpt` contains the full wavelet packet table, `packetlevs` is a vector with 2^j elements for each level, j. To select all the wavelet packet nodes at a particular level, use `packetlevs` with logical indexing.

`cfreq` — Center frequencies of passbands

vector

Center frequencies of the approximate passbands in the `wpt` rows, returned as a vector. The center frequencies are in cycles/sample. To covert the units to cycles/unit time, multiply `cfreq` by the sampling frequency.

`energy` — Energy of the wavelet packet coefficients

vector

Energy of the wavelet packet coefficients for the `wpt` nodes, returned as a vector. The sum of the energies (squared L2 norms) for the wavelet packets at each level equals the energy in the signal.

`relenergy` — Relative energy

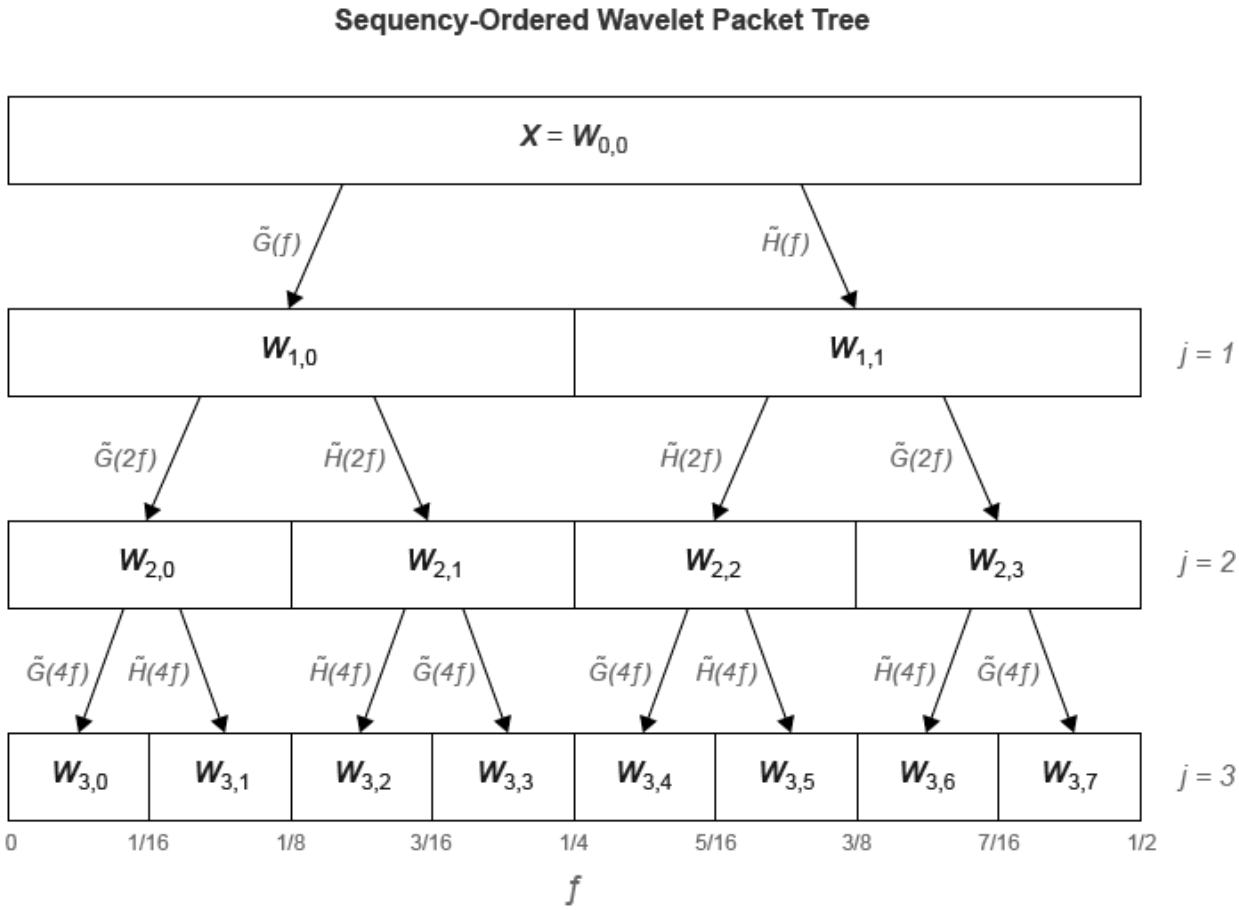
vector

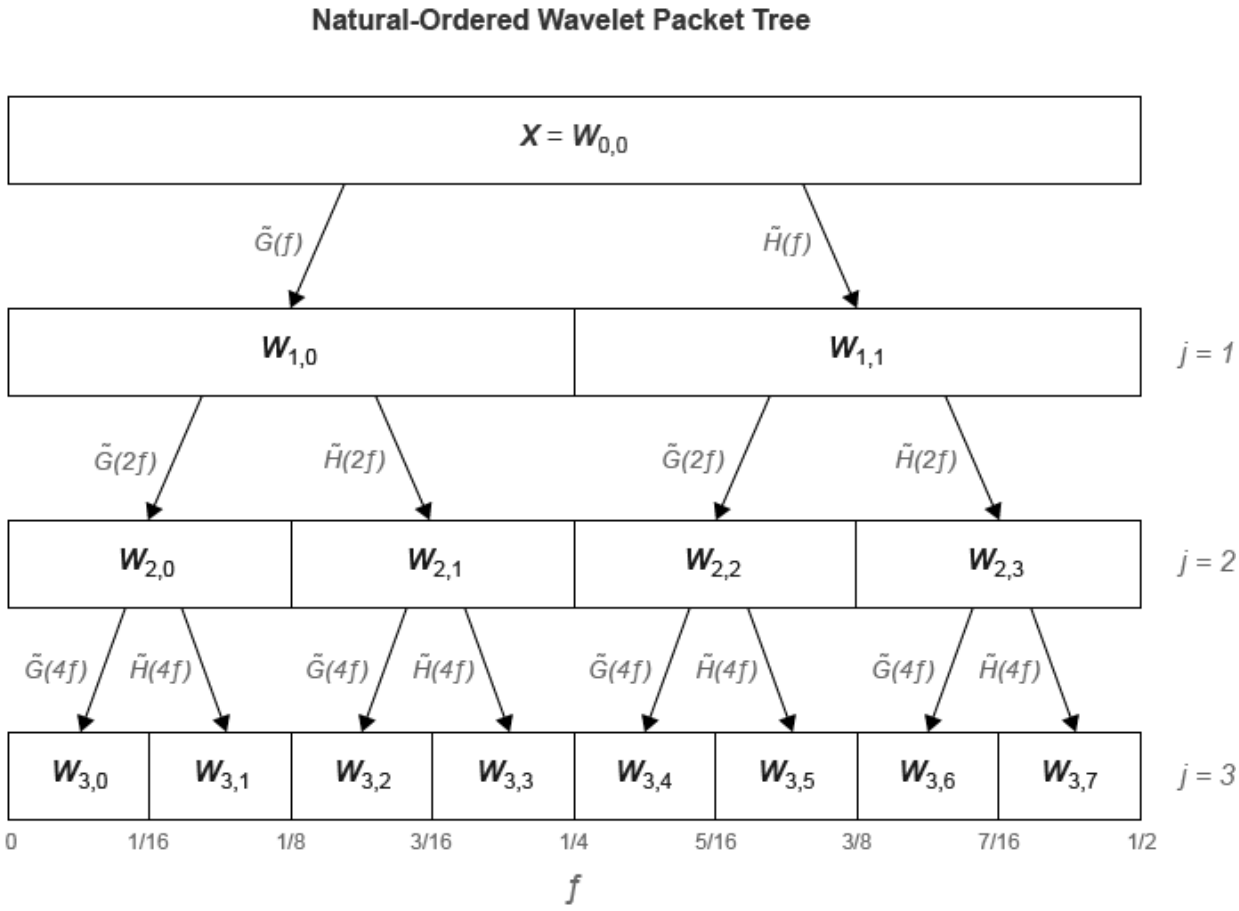
Relative energy for each level, returned as a vector. The relative energy is the proportion of energy in each wavelet packet by level, relative to the total energy of that level. The sum of relative energies in all packets at each level equals 1.

More About

Algorithms

The `modwpt` performs a discrete wavelet packet transform and produces a sequency-ordered wavelet packet tree. Compare the sequency-ordered and normal (Paley)-ordered trees.





References

[1] Walden, A. T., and Cristan, A. C. "The Phase-Corrected Undecimated Discrete Wavelet Packet Transform and the Recurrence of High Latitude Interplanetary Shock Waves." *Statistics Section Technical Report TR-97-03*. London, UK: Dept. of Mathematics, Imperial College of Science, Technology & Medicine, 1997.

See Also

`imodwpt` | `modwptdetails`

Introduced in R2016a

modwptdetails

Maximal overlap discrete wavelet packet transform details

Syntax

```
w = modwptdetails(x)
w = modwptdetails(x,wname)
w = modwptdetails(x,lo,hi)
w = modwptdetails( ___ ,lev)

[w,packetlevs] = modwptdetails( ___ )
[w,packetlevs,cfreq] = modwptdetails( ___ )

[ ___ ] = modwptdetails( ___ ,Name,Value)
```

Description

`w = modwptdetails(x)` returns the maximal overlap discrete wavelet packet transform (MODWPT) details for the 1-D real-valued signal, `x`. The MODWPT details provide zero-phase filtering of the signal. By default, `modwptdetails` returns only the terminal nodes, which are at level 4 or at level `floor(log2(numel(x)))`, whichever is smaller.

Note: To decide whether to use `modwptdetails` or `modwpt`, consider the type of data analysis you need to perform. For applications that require time alignment, such as nonparametric regression analysis, use `modwptdetails`. For applications where you want to analyze the energy levels in different packets, use `modwpt`. For more information, see "Algorithms" on page 1-357

`w = modwptdetails(x,wname)` uses the orthogonal wavelet filter specified by the string `wname`.

`w = modwptdetails(x,lo,hi)` uses the orthogonal scaling filter, `lo`, and wavelet filter, `hi`.

`w = modwptdetails( ___ ,lev)` returns the terminal nodes of the wavelet packet tree at positive integer level `lev`.

`[w,packetlevs] = modwptdetails( ___ )` returns a vector of transform levels corresponding to the rows of `w`.

`[w,packetlevs,cfreq] = modwptdetails( ___ )` returns, `w`, the center frequencies of the approximate passbands corresponding to the MODWPT details in .

`[ ___ ] = modwptdetails( ___ ,Name,Value)` returns the MODWPT with additional options specified by one or more `Name,Value` pair arguments.

Examples

MODWPT Details Using Default Wavelet

Obtain the MODWPT of an electrocardiogram (ECG) signal using the default length 18 Fejer-Korovkin (`'fk18'`) wavelet and the default level, 4.

```
load wecg;
wptdetails = modwptdetails(wecg);
```

Demonstrate that summing the MODWPT details over each sample reconstructs the signal. The largest absolute difference between the original signal and the reconstruction is on the order of 10^{-11}, which demonstrates perfect reconstruction.

```
xrec = sum(wptdetails);
max(abs(wecg-xrec'))
```

```
ans =

   1.7903e-11
```

MODWPT Details for Two Sine Waves

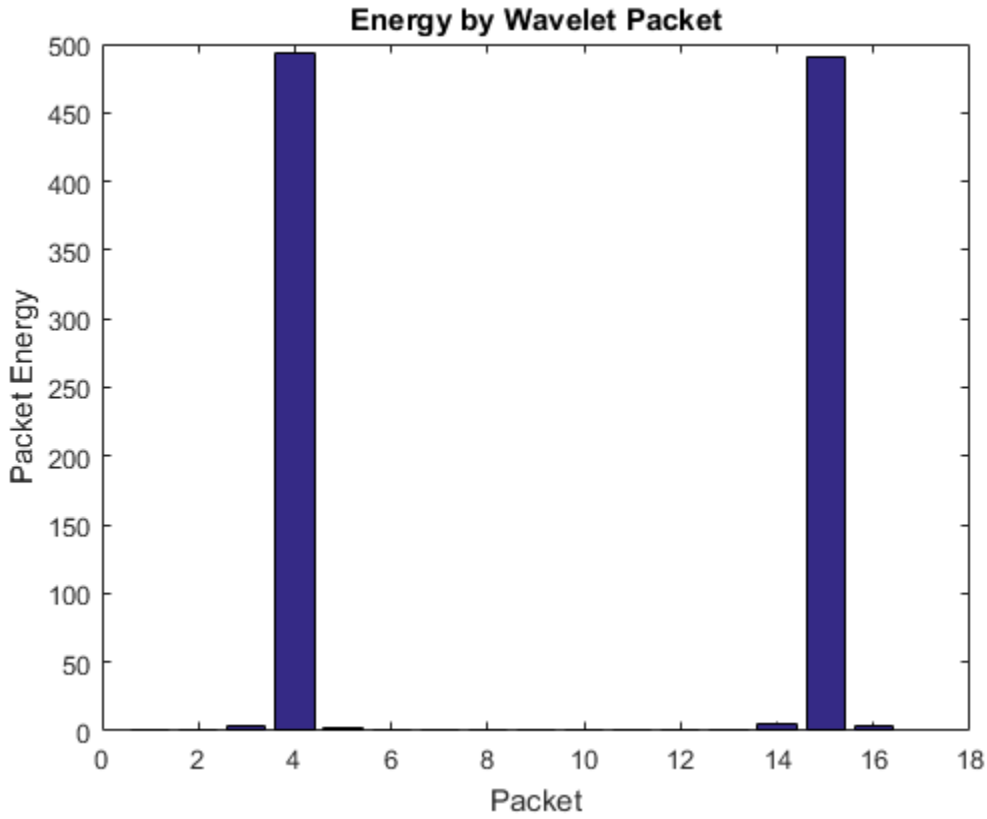
Obtain the MODWPT details for a signal containing 100 Hz and 450 Hz sine waves. Each row of the `modwptdetails` output corresponds to a separate frequency band.

```
dt = 0.001;
fs = 1/dt;
t = 0:dt:1;
x = (sin(2*pi*100*t)+sin(2*pi*450*t));
```

```
[lo,hi] = wfilters('fk22');
wptdetails = modwptdetails(x,lo,hi);
```

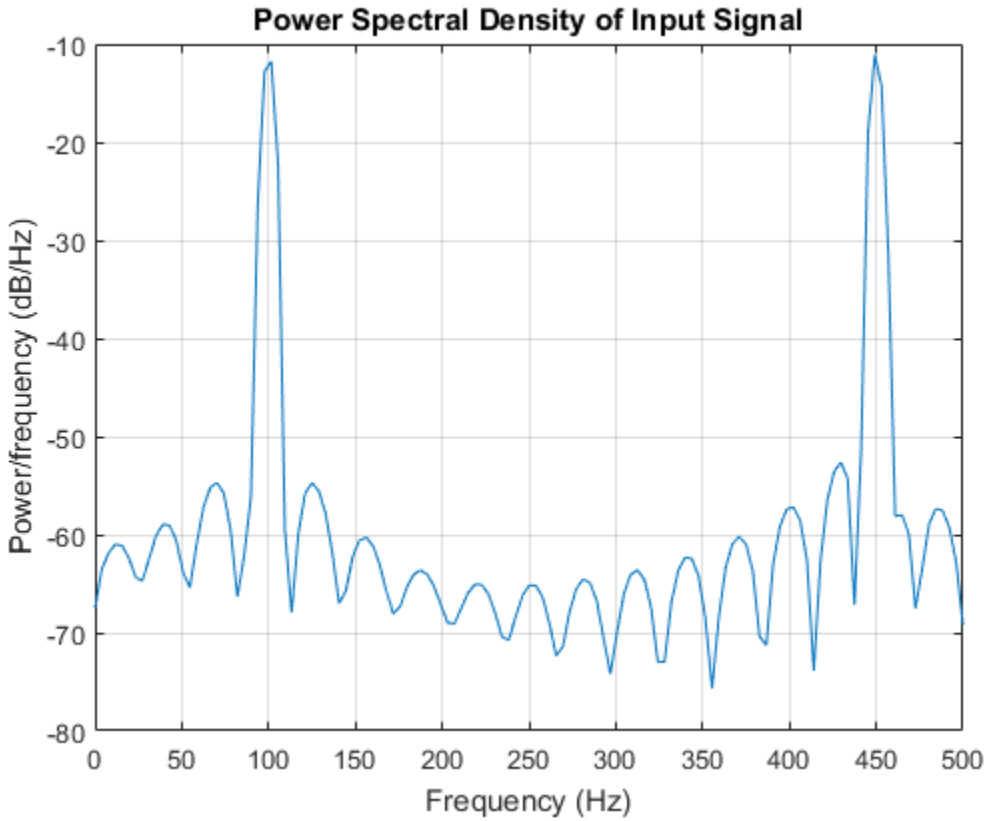
Use `modwpt` to obtain the energy and center frequencies of the signal. Plot the energy in the wavelet packets. The fourth and fifteenth frequency bands contain most of the energy. Other frequency bands have significantly less energy. The frequency ranges of fourth and fifteenth bands are approximately 94-125 Hz and 438-469 Hz, respectively.

```
[wpt,~,cfreqs,energy] = modwpt(x,lo,hi);
figure
bar(1:16,energy);
xlabel('Packet')
ylabel('Packet Energy')
title('Energy by Wavelet Packet')
```



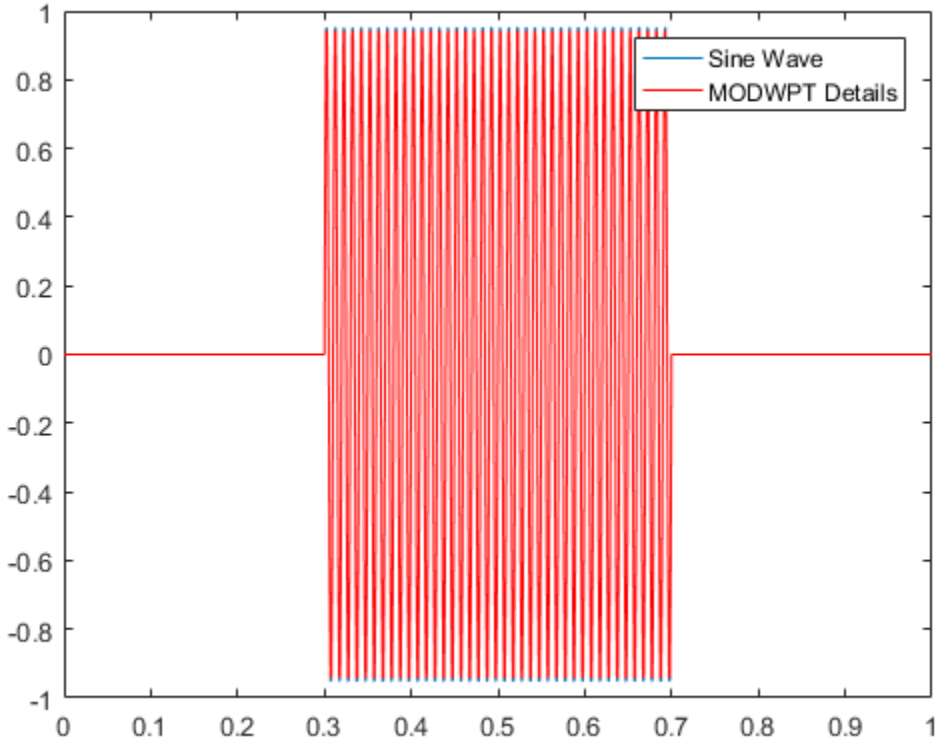
Plot the power spectral density of the input signal.

```
pwelch(x,[],[],[],fs,'onesided');
title('Power Spectral Density of Input Signal')
```



Show that the MODWPT details have zero-phase shift from the 100 Hz input sine.

```
p4 = wptdetails(4,:);
plot(t,sin(2*pi*100*t).*(t>0.3 & t<0.7))
hold on
plot(t,p4.*(t>0.3 & t<0.7),'r')
legend('Sine Wave','MODWPT Details')
```



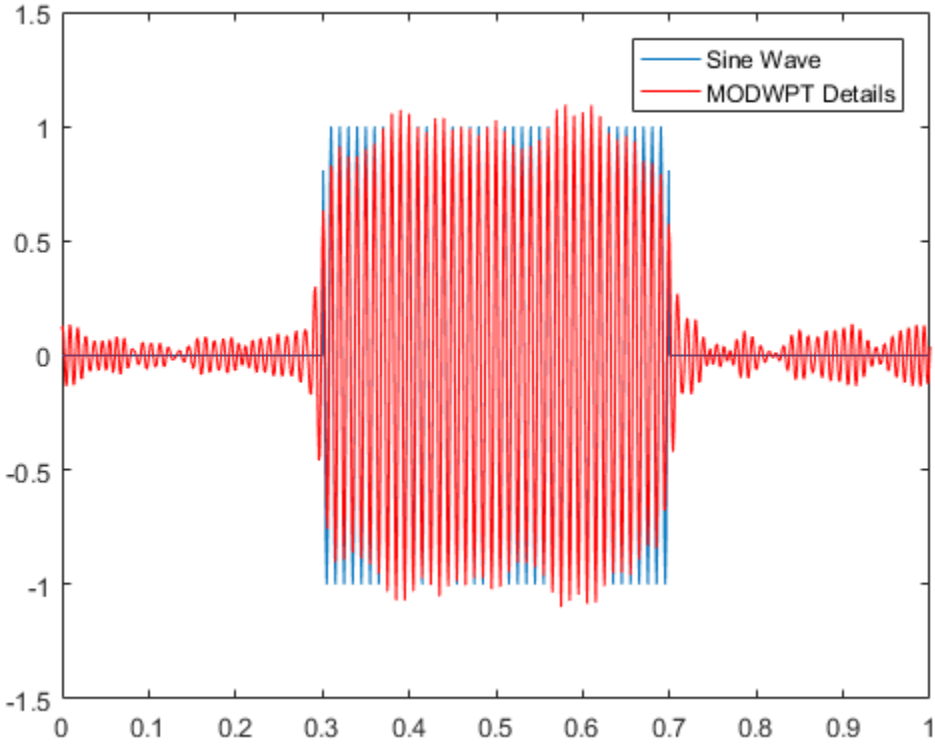
MODWPT Details for Noisy Sine Wave

Obtain the MODWPT details for a 100 Hz time-localized sine wave in noise. The sampling rate is 1000 Hz. Obtain the MODWPT at level 4 using the length 22 Fejer-Korovkin (`'fk22'`) wavelet.

```
dt = 0.001;
t = 0:dt:1;
x = cos(2*pi*100*t).*(t>0.3 & t<0.7)+0.25*randn(size(t));
wptdetails = modwptdetails(x,'fk22');
p4 = wptdetails(4,:);
```

Plot the MODWPT details for level 4, packet number 4. The MODWPT details represent zero-phase filtering of the input signal with an approximate passband of $[3Fs/2^5, 4Fs/2^5)$, where Fs is the sampling frequency.

```
plot(t,cos(2*pi*100*t).*(t>0.3 & t<0.7));
hold on
plot(t,p4,'r')
legend('Sine Wave','MODWPT Details')
```



MODWPT Details Using Scaling and Wavelet Filters

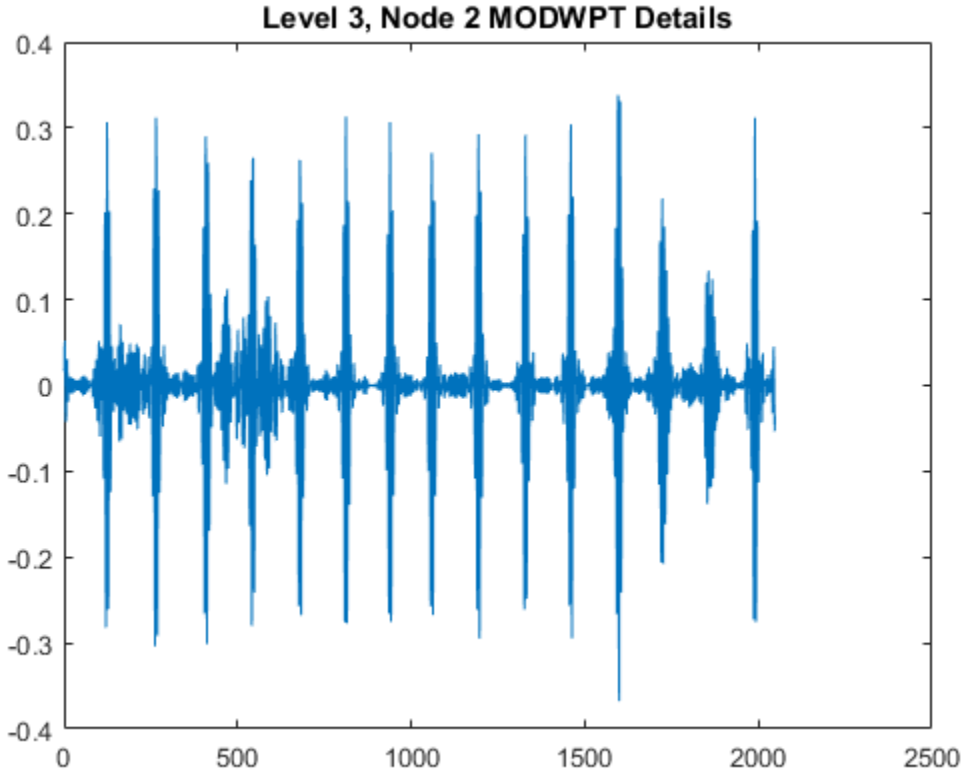
Obtain the MODWPT details of an ECG waveform using the length 18 Fejer-Korovkin scaling and wavelet filters.

```
load wecg;
[lo,hi] = wfilters('fk18');
wpt = modwptdetails(wecg,lo,hi);
```

MODWPT Details for Full Packet Tree

Obtain the MODWPT details for the full wavelet packet tree of an ECG waveform. Use the default length 18 Fejer-Korovkin (`'fk18'`) wavelet. Extract and plot the node coefficients at level 3, node 2.

```
load wecg;
[w,packetlevels] = modwptdetails(wecg,'FullTree',true);
p3 = w(packetlevels==3,:);
plot(p3(3,:))
title('Level 3, Node 2 MODWPT Details')
```



Input Arguments

x — Input signal

real-valued vector

Input signal, specified as a real-valued row or column vector. `x` must have at least two elements.

Data Types: `double`

wname — Analyzing wavelet filter

`fk18` (default) | string

Analyzing wavelet filter, specified as a string that corresponds to an orthogonal wavelet.

Valid orthogonal wavelet families begin with one of the following strings, followed by an integer, *N*. For example, `sym4`.

- `'haar`*N*`'` — Haar wavelet with *N* vanishing moments
- `'db`*N*`'` — Daubechies wavelet with *N* vanishing moments
- `'sym`*N*`'` — Symlets wavelet with *N* vanishing moments
- `'coif`*N*`'` — Coiflets wavelet with *N* vanishing moments
- `'fk`*N*`'` — Fejer-Korovkin wavelet with *N* coefficients

To check if your wavelet is orthogonal, use `wavemngr('type',wname)` and verify that it returns `1` as the wavelet type. To determine valid values for *N*, use `waveinfo`. For example, `waveinfo('fk')`.

`lo` — Scaling filter

even-length real-valued vector

Scaling filter, specified as an even-length real-valued vector. `lo` must satisfy the conditions necessary to generate an orthogonal scaling function. You can specify the `lo` and `hi` scaling-wavelet filter pair only if you do not specify `wname`.

`hi` — Wavelet filter

even-length real-valued vector

Wavelet filter, specified as an even-length real-valued vector. `hi` must satisfy the conditions necessary to generate an orthogonal wavelet. You can specify the `lo` and `hi` scaling-wavelet filter pair only if you do not specify `wname`.

`lev` — Transform level

positive integer

Transform level, specified as a positive integer less than or equal to `floor(log2(numel(x)))`.

Name-Value Pair Arguments

Specify optional comma-separated pairs of `Name,Value` arguments. `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside single

quotes (' '). You can specify several name and value pair arguments in any order as `Name1,Value1,...,NameN,ValueN`.

Example: `'Fulltree',true` returns the full wavelet packet tree

`'FullTree'` — Option to return full packet tree details

`false` (default) | `true`

Option to return full wavelet packet tree details, specified as the comma-separated pair consisting of `'FullTree'` and either `false` or `true`. If you specify `false`, then `modwptdetails` returns details about only the terminal (final-level) wavelet packet nodes. If you specify `true`, then `modwptdetails` returns details about the full wavelet packet tree down to the default or specified level.

For the full wavelet packet tree, `w` is a 2^{j+1}-2-by-`numel(x)` matrix. Each level, j, has 2^j wavelet packet details.

Output Arguments

`w` — Wavelet packet tree details

matrix

Wavelet packet tree details, returned as a matrix with each row containing the sequency-ordered wavelet packet details for the terminal nodes. The terminal nodes are at level 4 or at level `floor(log2(numel(x)))`, whichever is smaller. The MODWPT details are zero-phase-filtered projections of the signal onto the subspaces corresponding to the wavelet packet nodes. The sum of the MODWPT details over each sample reconstructs the original signal.

For the default terminal nodes, w is a 2^j-by-`numel(x)` matrix. For the full packet table, at level j, `w` is a 2^{j+1}-2-by-`numel(x)` matrix of sequency-ordered wavelet packet coefficients by level and index. The approximate passband for the nth row of `w` at level j is $\left[\frac{n-1}{2^{(j+1)}},\frac{n}{2^{(j+1)}}\right)$ cycles per sample, where $n = 1,2,...,2^j$.

`packetlevs` — Transform levels

vector

Transform levels, returned as a vector. The levels correspond to the rows of `w`. If `w` contains only the terminal level coefficients, `packetlevs` is a vector of constants equal

to the terminal level. If `w` contains the full wavelet packet tree of details, `packetlevs` is a vector with 2^{j-1} elements for each level, j. To select all the MODWPT details at a particular level, use `packetlevs` with logical indexing.

`cfreq` — Center frequencies of passbands

vector

Center frequencies of the approximate passbands in the `w` rows, returned as a vector. The center frequencies are in cycles per sample. To covert the units to cycles per unit time, multiply `cfreq` by the sampling frequency.

More About

Algorithms

The MODWPT details (`modwptdetails`) are the result of zero-phase filtering of the signal. The features in the MODWPT details align exactly with features in the input signal. For a given level, summing the details for each sample returns the exact original signal.

The output of the MODWPT (`modwpt`) is time delayed compared to the input signal. Most filters used to obtain the MODWPT have a nonlinear phase response, which makes compensating for the time delay difficult. All orthogonal scaling and wavelet filters have this response, except the Haar wavelet. It is possible to time align the coefficients with the signal features, but the result is an approximation, not an exact alignment with the original signal. The MODWPT partitions the energy among the wavelet packets at each level. The sum of the energy over all the packets equals the total energy of the input signal.

References

[1] Percival, D. B., and A. T. Walden. *Wavelet Methods for Time Series Analysis*. Cambridge, UK: Cambridge University Press, 2000.

[2] Walden, A. T., and A. C. Cristan. "The Phase-Corrected Undecimated Discrete Wavelet Packet Transform and the Recurrence of High Latitude Interplanetary Shock Waves." *Statistics Section Technical Report TR-97-03*. London, UK: Dept. of Mathematics, Imperial College of Science, Technology & Medicine, 1997.

See Also

`imodwpt` | `modwpt`

Introduced in R2016a

modwt

Maximal overlap discrete wavelet transform

Syntax

```
w = modwt(x)
w = modwt(x,wname)
w = modwt(x,Lo,Hi)
w = modwt(___,lev)
w = modwt(___,'reflection')
```

Description

`w = modwt(x)` returns the maximal overlap discrete wavelet transform (MODWT) of the 1-D real-valued signal, `x`.

`w = modwt(x,wname)` uses the orthogonal wavelet, `wname`, for the MODWT.

`w = modwt(x,Lo,Hi)` uses the scaling filter, `Lo`, and wavelet filter, `Hi`, to compute the MODWT. These filters must satisfy the conditions for an orthogonal wavelet. You cannot specify `Lo` and `Hi` if you specify `wname`.

`w = modwt(___,lev)` computes the MODWT down to the specified level, `lev`, using any of the arguments from previous syntaxes.

`w = modwt(___,'reflection')` computes the MODWT using reflection boundary handling. Other inputs can be any of the arguments from previous syntaxes. Before computing the wavelet transform, `modwt` extends the signal symmetrically at the right boundary to twice the signal length, `[x flip(x)]`. The number of wavelet and scaling coefficients that `modwt` returns is equal to twice the length of the input signal. By default, the signal is extended periodically.

Examples

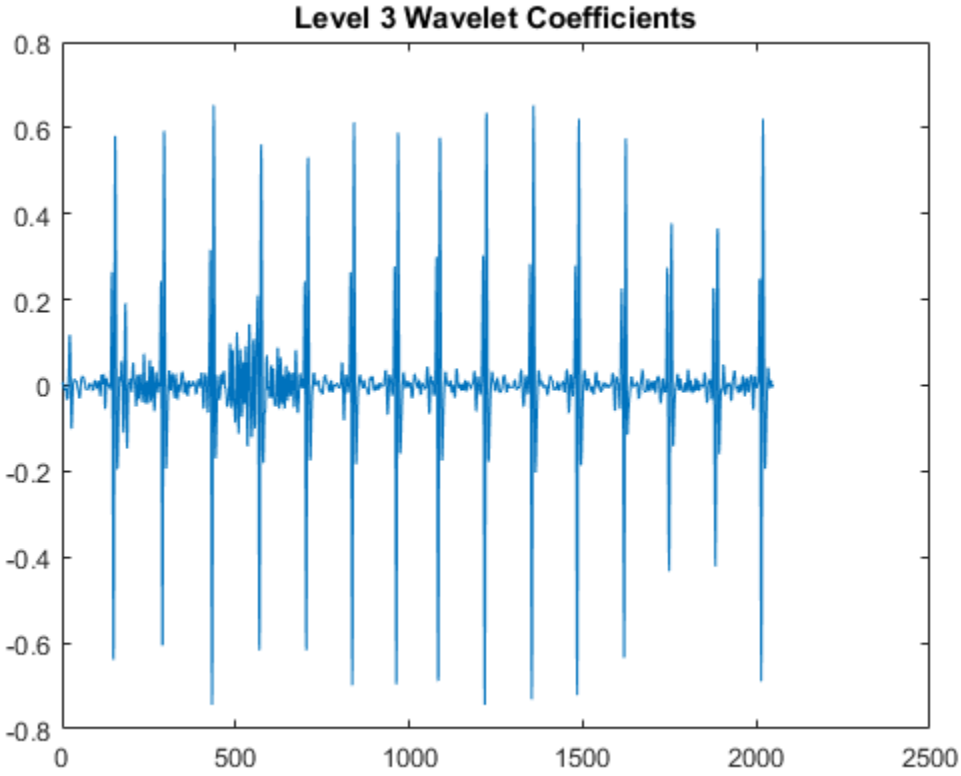
MODWT Using Default Wavelet

Obtain the MODWT of an electrocardiogram (ECG) signal using the default `sym4` wavelet down to the maximum level.

```
load wecg;
wtecg = modwt(wecg);
```

wtecg is 12-by-2048 matrix. The first eleven rows are the wavelet coefficients for scales 2^1 to 2^{11}. The final row contains the scaling coefficients at scale 2^{11}. Plot the detail (wavelet) coefficients for scale 2^3.

```
plot(wtecg(3,:))
title('Level 3 Wavelet Coefficients')
```



MODWT Using Daubechies Extremal Phase Wavelet with Two Vanishing Moments

Obtain the MODWT of Southern Oscillation Index data with the `'db2'` wavelet down to the maximum level.

```
load soi;
wsoi = modwt(soi,'db2');
```

MODWT Using Scaling and Wavelet Filters

Obtain the MODWT of the Deutsche Mark - U.S. Dollar exchange rate data using the Fejer-Korovkin length 8 scaling and wavelet filters.

```
load DM_USD;
```

```
[Lo,Hi] = wfilters('fk8');
wdm = modwt(DM_USD,Lo,Hi);
```

MODWT to a Specified Level

Obtain the MODWT of an ECG signal down to scale 2^4, which corresponds to level four. Use the default `'sym4'` wavelet.

```
load wecg;
wtecg = modwt(wecg,4);
```

`wtecg` is a 5-by-2048 matrix. The row size is L+1, where, in this case, the level (L) is 4. The column size matched the number of input samples.

MODWT with Reflection Boundary

Obtain the MODWT of an ECG signal using reflection boundary handling. Use the default `'sym4'` wavelet and obtain the transform down to level 4.

```
load wecg;
wtecg = modwt(wecg,4,'reflection');
```

`wtecg` has 4096 columns, which is twice the length of the input signal, `wecg`.

- "Wavelet Analysis of Financial Data"

Input Arguments

x — Input signal

real-valued vector

Input signal, specified as a row or column vector. `x` must have at least two elements.

By default, `modwt` computes the wavelet transform down to level `floor(log2(length(x)))` using the Daubechies least-asymmetric wavelet with four vanishing moments (`'sym4'`) and periodic boundary handling.

Data Types: `double`

wname — Analyzing wavelet

`'sym4'` (default) | string

Analyzing wavelet, specified as a string that corresponds to an orthogonal wavelet. Valid orthogonal wavelet families begin with one of the following strings, followed by an integer, *N*, which indicates the number of vanishing moments, for example, `sym`*N*. For `'fk'`, *N* is the number of coefficients.

- `'haar'` — Haar wavelet
- `'db'` — Daubechies wavelet
- `'sym'` — Symlets wavelet
- `'coif'` — Coiflets wavelet
- `'fk'` — Fejer-Korovkin wavelet

To check if your wavelet is orthogonal, use `wavemngr('type',wname)` and verify that it returns `1` as the wavelet type. To determine valid values for *N*, use `waveinfo`, for example, `waveinfo('db')`.

`Lo` — Scaling filter

even-length real-valued vector

Scaling filter, specified as an even-length real-valued vector. `Lo` must satisfy the conditions necessary to generate an orthogonal scaling function. You can specify `Lo` only if you do not specify `wname`.

`Hi` — Wavelet filter

even-length real-valued vector

Wavelet filter, specified as an even-length real-valued vector. `Hi` must satisfy the conditions necessary to generate an orthogonal wavelet. You can specify `Hi` only if you do not specify `wname`.

`lev` — Transform level

positive integer

Transform level, specified as a positive integer less than or equal to `floor(log2(length(x)))`.

Output Arguments

`w` — Wavelet transform

matrix

Wavelet transform , returned as an L+1-by-N matrix containing wavelet coefficients and final-level scaling coefficients. L is the level of the MODWT. N is equal to the input signal length unless you specify `'reflection'` boundary handling, in which case N is twice the length of the input signal. The kth row of `w` contains the wavelet coefficients for scale 2^k (wavelet scale $2^{(k-1)}$). The final, (L+1)th, row of `w` contains the scaling coefficients for scale 2^L.

More About

Algorithms

The standard algorithm for the MODWT implements the circular convolution directly in the time domain. This implementation of the MODWT performs the circular convolution in the Fourier domain. The wavelet and scaling filter coefficients at level j are computed by taking the inverse discrete Fourier transform (DFT) of a product of DFTs. The DFTs in the product are the signal's DFT and the DFT of the j^{th} level wavelet or scaling filter.

Let H_k and G_k denote the length N DFTs of the MODWT wavelet and scaling filters, respectively. Let j denote the level and N denote the sample size.

The j^{th} level wavelet filter is defined by

$$\frac{1}{N}\sum_{k=0}^{N-1} H_{j,k} e^{i2\pi nk/N}$$

where

$$H_{j,k} = H_{2^{j-1}k \bmod N} \prod_{m=0}^{j-2} G_{2^m k \bmod N}$$

The j^{th} level scaling filter is

$$\frac{1}{N}\sum_{k=0}^{N-1} G_{j,k} e^{i2\pi nk/N}$$

where

$$G_{j,k} = \prod_{m=0}^{j-1} G_{2^m k \bmod N}$$

References

[1] Percival, D. B., and A. T. Walden. *Wavelet Methods for Time Series Analysis*. Cambridge, UK: Cambridge University Press, 2000.

[2] Percival, D. B., and H. O. Mofjeld. "Analysis of subtidal coastal sea level fluctuations using wavelets."*Journal of the American Statistical Association*. Vol. 92, pp 868–880.

See Also

`imodwt` | `modwtcorr` | `modwtmra` | `modwtvar` | `modwtxcorr`

Introduced in R2015b

modwtcorr

Multiscale correlation using the maximal overlap discrete wavelet transform

Syntax

```
wcorr = modwtcorr(w1,w2)
wcorr = modwtcorr(w1,w2,wav)

[wcorr,wcorrci] = modwtcorr( ___ )
[wcorr,wcorrci] = modwtcorr( ___ ,conflevel)
[wcorr,wcorrci,pval] = modwtcorr( ___ )
[wcorr,wcorrci,pval,nj] = modwtcorr( ___ )

wcorrtable = modwtcorr( ___ ,'table')

[ ___ ] = modwt( ___ ,'reflection')

modwtcorr( ___ )
```

Description

`wcorr = modwtcorr(w1,w2)` returns the wavelet correlation by scale for the maximal overlap discrete wavelet transforms (MODWTs) specified in `w1` and `w2`. `wcorr` is an *M*-by-1 vector of correlation coefficients, where *M* is the number of levels with nonboundary wavelet coefficients. If the final level has enough nonboundary coefficients, `modwtcorr` returns the scaling correlation in the final row of `wcorr`.

`wcorr = modwtcorr(w1,w2,wav)` uses the wavelet `wav` to determine the number of boundary coefficients by level.

`[wcorr,wcorrci] = modwtcorr( ___ )` returns in `wcorrci` the lower and upper 95% confidence bounds for the correlation coefficients of `wcorr`, using any arguments from the previous syntaxes.

`[wcorr,wcorrci] = modwtcorr( ___ ,conflevel)` uses `conflevel` for the coverage probability of the confidence interval. `conflevel` is a real scalar strictly greater than 0 and less than 1. If `conflevel` is unspecified or specified as empty, the coverage probability defaults to 0.95.

`[wcorr,wcorrci,pval] = modwtcorr( ___ )` returns the p-values for the null hypothesis test that the correlation coefficient in `wcorr` is equal to zero. `pval` is an *M*-by-2 matrix, where *M* is the number of levels with nonboundary wavelet coefficients. T

`[wcorr,wcorrci,pval,nj] = modwtcorr( ___ )` returns the number of nonboundary coefficients used in the computation of the correlation estimates by level, `nj`.

`wcorrtable = modwtcorr( ___ ,'table')` returns an *M*-by-6 table with the correlation, confidence bounds, p-value, and adjusted p-value. The table also lists the number of nonboundary coefficients by level. The row names of the table `wcorrtable` designate the type and level of each estimate. For example, `D1` designates that the row corresponds to a wavelet or detail estimate at level 1 and `S6` designates that the row corresponds to the scaling estimate at level 6. The scaling correlation is only computed for the final level of the MODWT and only when there are nonboundary scaling coefficients. You can specify the `'table'` flag anywhere after the input transforms `w1` and `w2`. You must enter the entire string `'table'`. If you specify `'table'`, `modwtcorr` only outputs one argument.

`[ ___ ] = modwt( ___ ,'reflection')` reduces the number of wavelet and scaling coefficients at each scale by half before computing the correlation. Use this option only when you obtain the MODWT of `w1` and `w2` were obtained using the `'reflection'` boundary condition. You must enter the entire string `'reflection'`. If you added a wavelet named 'reflection' using the wavelet manager, you must rename that wavelet prior to using this option.

`modwtcorr` supports only unbiased estimates of the wavelet correlation. For these estimates, the algorithm must removed the extra coefficients obtained using the `'reflection'` boundary condition. Specifying the `'reflection'` option in `modwtcorr` is identical to first obtaining the MODWT of `w1` and `w2` using the default `'periodic'` boundary handling and then computing the wavelet correlation estimates.

`modwtcorr( ___ )` with no output arguments plots the wavelet correlations by scale with lower and upper confidence bounds. By default, the coverage probability is 0.95. Scales with NaNs for the confidence bounds and the scaling correlation are excluded.

Examples

Correlation by Scale

Find the correlation by scale for monthly DM-USD exhange rate returns from 1970 to 1998. The return data are log transformed. Use the Daubechies wavelet with two

vanishing moments ('db2') to obtain the MODWT down to level 6. Then, obtain the correlation data.

```
load DM_USD;
load JY_USD;
wdm = modwt(DM_USD,'db2',6);
wjy = modwt(JY_USD,'db2',6);
wcorr = modwtcorr(wdm,wjy,'db2')
```

```
wcorr =

    0.5854
    0.5748
    0.6264
    0.4948
    0.3787
    0.9072
    0.7976
```

`wcorr` contains seven elements. The first six elements are the correlation coefficients for the wavelet (detail) levels one to six. The final element is the correlation for the scaling (lowpass) level six.

Multiscale Correlation

Obtain the MODWT of the Southern Oscillation Index and Truk Island daily pressure data sets. Tabulate the correlation between the two data sets by level.

```
load soi;
load truk;
wsoi = modwt(soi);
wtruk = modwt(truk);
wcorr = modwtcorr(wsoi,wtruk)
```

```
wcorr =

    0.1749
    0.2936
    0.0914
    0.0883
    0.2667
```

```
    0.0894
   -0.0415
    0.4825
    0.4394
    0.7433
```

Show that the number of nonboundary coefficients, in this case, is less than the maximal length of the input. The MODWT is computed down to level thirteen, which is the maximal level for the length of the input. Level thirteen contains thirteen wavelet coefficient vectors and one scaling coefficient vector.

```
size(wsoi,1)
```

```
ans =

    14
```

The multiscale correlations are computed only down to level ten because the levels after than do not contain nonboundary coefficients. For unbiased estimates, you must use nonboundary coefficients only.

```
numel(wcorr)
```

```
ans =

    10
```

Confidence Intervals for Correlation

Obtain the MODWT of the monthly US-DM and US-JPY exchange return data from 1970 to 1998. The return data are log transformed. Use the Daubechies wavelet with two degrees of freedom ('db2') and obtain wavelet and obtain the MODWT of each series down to level six. Obtain the correlation estimates by scale and the 95% confidence intervals.

```
load DM_USD
load JY_USD
wdm = modwt(DM_USD,'db2',6);
wjy = modwt(JY_USD,'db2',6);
[wcorr,wcorrci] = modwtcorr(wdm,wjy,'db6');
```

```
[wcorr wcorrci]


ans =

    0.5855    0.4780    0.6756
    0.5753    0.4139    0.7017
    0.6257    0.4007    0.7796
    0.5578    0.1661    0.7974
    0.7202    0.1657    0.9287
```

The width of the confidence interval increases as you go down in level.

Confidence Intervals with 0.99 Coverage Probability

Specify the coverage probability for the confidence intervals. Obtain the 99% confidence intervals for the US-DM and US-JY exchange returns.

```
load DM_USD;
load JY_USD;
wdm = modwt(DM_USD,'db2',6);
wjy = modwt(JY_USD,'db2',6);
[wcorr,wcorrci] = modwtcorr(wdm,wjy,'db2',0.99);
[wcorr wcorrci]


ans =

    0.5854    0.4407    0.7005
    0.5748    0.3557    0.7340
    0.6264    0.3169    0.8153
    0.4948   -0.0646    0.8176
    0.3787   -0.5191    0.8792
    0.9072   -0.3006    0.9975
    0.7976   -0.6227    0.9941
```

P-values for Correlation

Return *p*-values for the test of zero correlation by scale. Obtain the MODWT of the DM-USD and JY-USD exchange return data down to level six using the Daubechies wavelet with two degrees of freedom ('db2') wavelet. Compute the correlation by scale and return the *p*-values.

```
load DM_USD;
load JY_USD;
wdm = modwt(DM_USD,'db2',6);
wjy = modwt(JY_USD,'db2',6);
[wcorr,wcorrci,pval] = modwtcorr(wdm,wjy,'db2');
format longe
pval
format
```

```
pval =

     2.694174887029476e-17     4.889927419958497e-16
     7.125460513474001e-09     6.466355415977655e-08
     7.012389783536512e-06     4.242495819039590e-05
     2.258540027996919e-02     1.024812537703602e-01
     2.805930327935258e-01     7.275376493146417e-01
     3.348079529469826e-02     1.215352869197547e-01
     1.059217509938026e-01     3.204132967562528e-01
```

The first column contains the *p*-value and the second column contains the adjusted *p*-value based on the false discovery rate.

Multiscale Correlation in Tabular Form

Output results from `modwtcorr` in tabular form. Obtain the MODWT of the DM-USD and JY-USD exchange returns down to level six using the Daubechies wavelet with two degrees of freedom ('db2'). Output the results in a table.

```
load DM_USD;
load JY_USD;
wdm = modwt(DM_USD,'db2',6);
wjy = modwt(JY_USD,'db2',6);
corrtable = modwtcorr(wdm,wjy,'db2','table')
```

```
corrtable =

          NJ      Lower       Rho       Upper       Pvalue      AdjustedPvalue
          ___    ________    _______    _______    __________    ______________

    D1    344     0.47797    0.58542    0.67561    2.6942e-17    4.8899e-16
    D2    338     0.41329    0.57483    0.70129    7.1255e-09    6.4664e-08
```

```
    D3     326      0.40163     0.62641     0.78001     7.0124e-06     4.2425e-05
    D4     302     0.080255      0.4948     0.76342       0.022585        0.10248
    D5     254     -0.32954     0.37865     0.81417        0.28059        0.72754
    D6     158      0.12469     0.90716     0.99393       0.033481        0.12154
    S6     158     -0.28573     0.79761     0.98601        0.10592        0.32041
```

Correlation with Reflection Boundary Conditions

Obtain multiscale correlation estimates when using `'reflection'` boundary handling. Obtain the MODWT of the Southern Oscillation Index and Truk Islands pressure data sets using `'reflection'` boundary handling for both data sets.

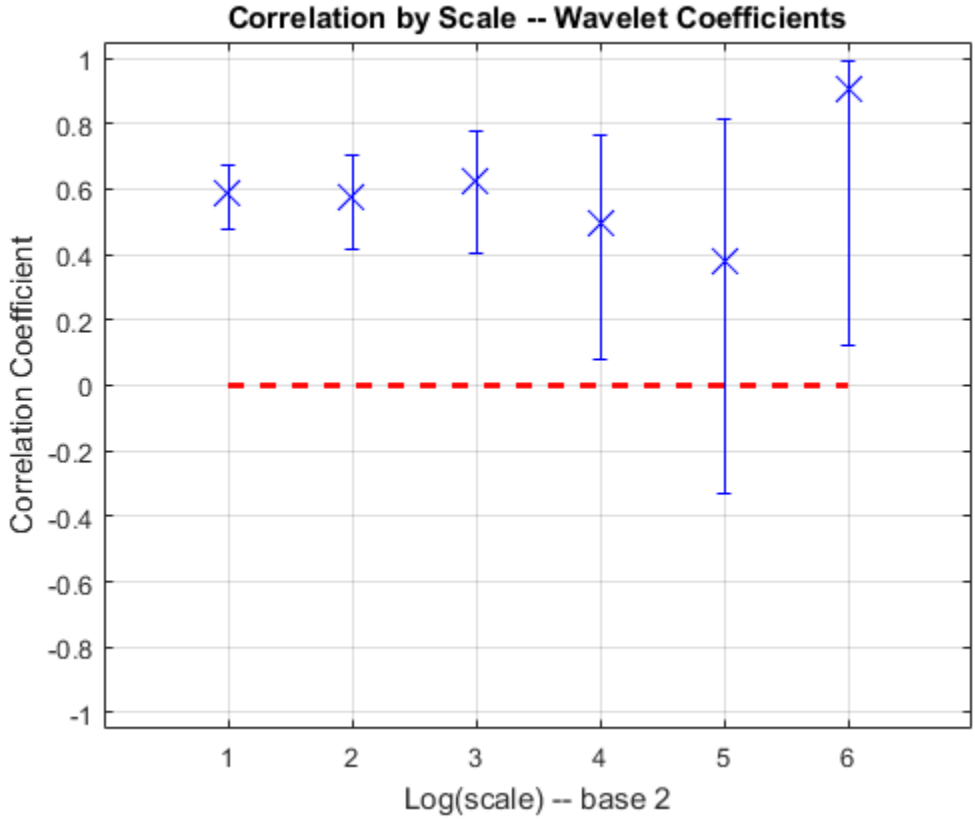
```
load soi
load truk
wsoi = modwt(soi,'fk4',6,'reflection');
wtruk = modwt(truk,'fk4',6,'reflection');
corrtable = modwtcorr(wsoi,wtruk,'fk4',0.95,'reflection','table')
```

```
corrtable =

          NJ        Lower        Rho        Upper        Pvalue       AdjustedPvalue
    ________    _________    _______    _______    ___________    ______________

    D1    12995      0.16942    0.19294    0.21624    1.5466e-55     2.8071e-54
    D2    12989      0.21426    0.24683    0.27885    2.7037e-46     2.4536e-45
    D3    12977     0.057885    0.10623    0.15407     1.789e-05      6.494e-05
    D4    12953     0.048034    0.11645    0.18378    0.00088579     0.0026795
    D5    12905      0.13281     0.2272     0.3175    3.7566e-06     1.7046e-05
    D6    12809    -0.019835     0.1182    0.25181      0.093044        0.24125
    S6    12809      0.26664    0.39003    0.50084    8.8066e-09      5.328e-08
```

Plot Correlation with Confidence Intervals

Plot the multiscale correlation of the DM-USD and JY-USD exchange returns down to level six. Use modwtcorr with no output arguments.

```
load DM_USD;
load JY_USD;
wdm = modwt(DM_USD,'db2',6);
wjy = modwt(JY_USD,'db2',6);
modwtcorr(wdm,wjy,'db2')
```



- "Wavelet Cross-Correlation for Lead-Lag Analysis"
- "Wavelet Analysis of Financial Data"

Input Arguments

w1 — MODWT transform of signal 1

matrix

MODWT transform of signal 1, specified as a matrix. `w1` is the output of `modwt`. It must be the same size and must have been obtained using the same analyzing wavelet.

Data Types: `double`

`w2` — MODWT transform of signal 2

matrix

MODWT transform of signal 2, specified as a matrix. `w2` is the output of `modwt`. It must be the same size and must have been obtained using the same analyzing wavelet.

`wav` — Wavelet

`'sym4'` (default) | string | positive even scalar

Wavelet, specified as a string indicating a valid wavelet string or as a positive even scalar indicating the length of the wavelet and scaling filters. `wav` must be the same wavelet and length used to obtain the MODWTs of `w1` and `w2`. For a list of valid wavelets, see `modwt`. If unspecified or specified as an empty, `[]`, `wav` defaults to the symlets wavelet with four degrees of freedom,`'sym4'`.

`conflevel` — Confidence level

0.95 (default) | positive scalar less than 1

Confidence level, specified as a positive scalar less than 1. `conflevel` determines the coverage probability of the confidence intervals in `wcorrci` and in the table, if you specify `'table'` as an input. If unspecified, or if specified as empty, `[]`, `conflevel` defaults to 0.95.

Output Arguments

`wcorr` — Correlation coefficients by scale

vector

Correlation coefficients by scale, returned as a vector. `wcorr` is an *M*-by-1 vector of correlation coefficients, where *M* is the number of levels with nonboundary wavelet coefficients. `modwtcorr` returns correlation estimates only where there are nonboundary coefficients. This condition is satisfied when the transform level is not greater than `floor(log2(N/(L-1)+1))`, where *N* is the length of the original signal and *L* is the filter length. If the final level has enough nonboundary coefficients, `modwtcorr` returns the scaling correlation in the final row of `wcorr`. By default, `modwtcorr` uses the symlet wavelet with four degrees of freedom `'sym4'` to determine the boundary coefficients.

`wcorrci` — Confidence intervals by scale

matrix

Confidence intervals by scale, returned as a matrix. The matrix is of size M-by-2 , where M is the number of levels with nonboundary wavelet coefficients. The first column contains the lower confidence bound and the second column contains the upper confidence bound. The `conflevel` determines the coverage probability.

Confidence bounds are computed using Fisher's Z-transformation. The standard error of Fisher's Z statistic is the square root of $(N - 3)$. In this case, N is the equivalent number of coefficients in the critically sampled discrete wavelet transform (DWT), `floor(size(w1,2)/2^LEV)`, where `LEV` is the level of the wavelet transform. `modwtcorr` returns NaNs for the confidence bounds when N^3 is less than or equal to zero.

`pval` — *P*-values for null hypothesis test

matrix

P-values for null hypothesis test, returned as a matrix. `pval` is an M-by-2 matrix.

- The first column of `pval` is the p-value computed using the standard t-statistic test for a correlation coefficient of zero.
- The second column of `pval` contains the adjusted p-value using the false discovery procedure of Benjamini & Yekutieli under arbitrary dependence assumptions.

The degrees of freedom, $(N - 2)$, for the t-statistic are determined by the equivalent number of coefficients in the critically sampled DWT, `floor(size(w1,2)/2^LEV)`, where `LEV` is the levels of the wavelet transform. `modwtcorr` returns NaNs when N^3 is less than or equal to zero.

`nj` — Number of nonboundary coefficients

vector

Number of nonboundary coefficients by scale, returned as a vector.

`wcorrtable` — Correlation table

table

Correlation table, returned as a MATLAB table. The table contains six variables:

- **NJ** — Number of nonboundary coefficients by level.

- **Lower** — Lower confidence bound for the coverage probability specified by `conflevel`.
- **Rho** — Correlation coefficient.
- **Upper** — Upper confidence bound for the coverage probability specified by `conflevel`.
- **Pvalue** — P-value for hypothesis test. The null hypothesis is that the correlation coefficient is equal to zero.
- **AdjustedPvalue** — P-value adjusted for multiple comparisons. The p-values are adjusted using false discovery rate under dependency assumptions.

References

[1] Percival, D. B., and Walden, A. T. *Wavelet Methods for Time Series Analysis*. Cambridge, U.K: Cambridge University Press, 2000.

[2] Whitcher, B., P. Guttorp, and D. B. Percival. "Wavelet analysis of covariance with application to atmospheric time series." *Journal of Geophysical Research*, Vol. 105, 2000, pp. 14941–14962.

See Also

`imodwt` | `modwt` | `modwtmra` | `modwtvar` | `modwtxcorr`

Introduced in R2015b

modwtmra

Multiresolution analysis based on MODWT

Syntax

```
mra = modwtmra(w)
mra = modwtmra(w,wname)
mra = modwtmra(w,Lo,Hi)
mra = modwtmra(___,'reflection')
```

Description

`mra = modwtmra(w)` returns the multiresolution analysis (MRA) of the maximal overlap discrete wavelet transform (MODWT) matrix, `w`. The MODWT matrix, `w`, is the output of the `modwt` function.

`mra = modwtmra(w,wname)` constructs the MRA using the wavelet corresponding to `wname`. The `wname` wavelet must be the same wavelet used to obtain the MODWT.

`mra = modwtmra(w,Lo,Hi)` constructs the MRA using the scaling filter `Lo` and wavelet filter `Hi`. The `Lo` and `Hi` filters must be the same filters used to obtain the MODWT.

`mra = modwtmra(___,'reflection')` uses the reflection boundary condition in the construction of the MRA using any of the arguments from previous syntaxes. If you specify `'reflection'`, `modwtmra` assumes that the length of the original signal is one half the number of columns in the input coefficient matrix.

Examples

Perfect Reconstruction with the MODWTMRA

Obtain the MODWTMRA of a simple time-series signal and demonstrate perfect reconstruction.

Create a time-series signal

```
t = 1:10;
x =sin(2*pi*200*t);
```

Obtain the MODWT and the MODWTMRA and sum the MODWTMRA rows.

```
m = modwt(x);
mra = modwtmra(m);
xrec = sum(mra);
```

Use the maximum of the absolute values to show that the difference between the original signal and the reconstruction is extremely small. The largest absolute value is on the order of 10^{-25}, which demonstrates perfect reconstruction.

```
max(abs(x-xrec))
```

```
ans =

   5.5738e-25
```

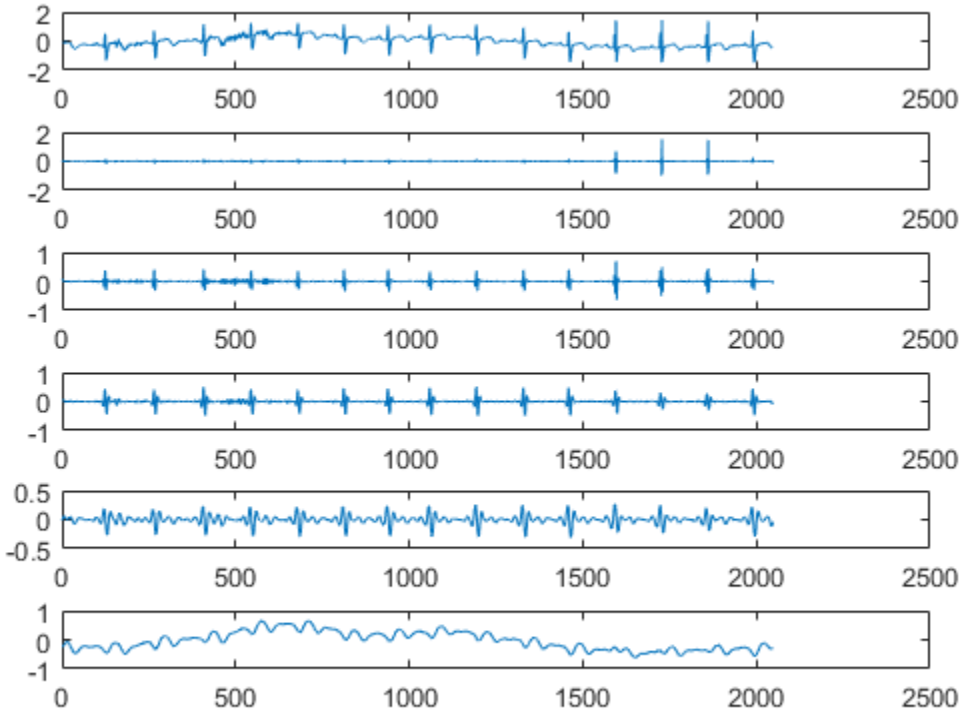
MRA Using Non-Default Wavelet

Construct an MRA of an ECG signal down to level four using the 'db2' wavelet.

```
load wecg;
lev = 4;
wtecg = modwt(wecg,'db2',lev);
mra = modwtmra(wtecg,'db2');
```

Plot the ECG waveform and the MRA.

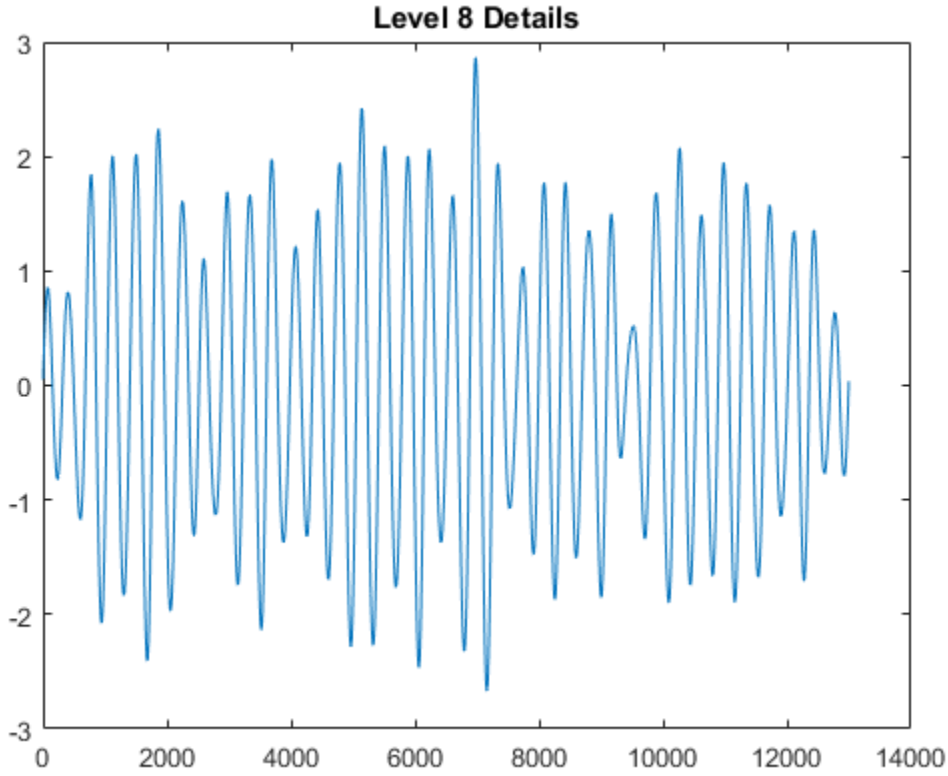
```
subplot(6,1,1)
plot(wecg)
    for kk = 2:lev+2
        subplot(6,1,kk)
        plot(mra(kk-1,:))
    end
```



MRA Using the Default Wavelet

Construct a multiresolution analysis for the Southern Oscillation Index data. The sampling period is one day. Plot the level eight details corresponding to a scale of 2^8 days. The details at this scale capture oscillations on a scale of approximately one year.

```
load soi
wtsoi = modwt(soi);
mrasoi = modwtmra(wtsoi);
plot(mrasoi(8,:))
title('Level 8 Details')
```



MRA Using Minimum Bandwidth Scaling and Wavelet Filters

Obtain the MRA for the Deutsch Mark - U.S. Dollar exchange rate data using the minimum bandwidth scaling and wavelet filters with four coefficients.

```
load DM_USD;
Lo = [0.4801755, 0.8372545, 0.2269312, -0.1301477];
Hi = qmf(Lo);
wdm = modwt(DM_USD,Lo,Hi);
mra = modwtmra(wdm,Lo,Hi);
```

MRA Using Reflection Boundary

Obtain the MRA for an ECG signal using `'reflection'` boundary handling.

```
load wecg;
wtecg = modwt(wecg,'reflection');
mra = modwtmra(wtecg,'reflection');
```

Show that the number of columns in the MRA is equal to the number of elements in the original signal.

```
isequal(size(mra,2),numel(wecg))
```

```
ans =

     1
```

Input Arguments

w — MODWT transform

matrix

Maximal overlap discrete wavelet transform, specified as a matrix. `w` is the output of `modwt`.

The input `w` is an L+1-by-N matrix containing the MODWT of an N-point input signal down to level L. By default, `modwtmra` assumes that you obtained the MODWT using the symlet wavelet with four vanishing moments, `'sym4'`, and using periodic boundary handling.

Example:

Data Types: `double`

wname — Synthesis wavelet

`'sym4'` (default) | string

Synthesis wavelet, specified as a string. The synthesis wavelet must be the same wavelet used to obtain the MODWT with the `modwt` function.

Lo — Scaling filter

even-length real-valued vector

Scaling filter, specified as an even-length real-valued vector. You can specify `Lo` only if you do not specify `wname`. `Lo` must be the same scaling filter used to obtain the MODWT with the `modwt` function.

`Hi` — Wavelet filter

even-length real-valued vector

Wavelet filter, specified as an even-length real-valued vector. You can specify `Hi` only if you do not specify `wname`. `Hi` must be the same wavelet filter used to obtain the MODWT with the `modwt` function.

Output Arguments

`mra` — Multiresolution analysis

matrix

Multiresolution analysis, returned as a matrix. By default, the `mra` is the same size as the input transform matrix `w`. If you specify reflection boundary handling, then `mra` has one half the number of columns as the input matrix `w`.

The output `mra` is an L+1-by-N matrix. The k^{th} row of `mra` contains the details for the k^{th} level. The $(L+1)^{\text{th}}$ row of `mra` contains the L^{th} level smooth.

References

[1] Percival, D. B., and Walden, A. T. *Wavelet Methods for Time Series Analysis*. Cambridge, U.K: Cambridge University Press, 2000.

[2] Whitcher, B., P. Guttorp, and D. B. Percival. "Wavelet analysis of covariance with application to atmospheric time series." *Journal of Geophysical Research*, Vol. 105, 2000, pp. 14941–14962.

See Also

`imodwt` | `modwt`

Introduced in R2015b

modwtvar

Multiscale variance of maximal overlap discrete wavelet transform

Syntax

```
wvar = modwtvar(w)
wvar = modwtvar(w,wname)
[wvar,wvarci] = modwtvar( ___ )

[ ___ ] = modwtvar(w,wname, ___ ,conflevel)
[ ___ ] = modwtvar(w,wname, ___ ,Name,Value,)
[wvar,wvarci,nj] = modwtvar(w,wname, ___ )

wvartable = modwtvar(w,wname,'table')

modwtvar( ___ )
```

Description

`wvar = modwtvar(w)` returns unbiased estimates of the wavelet variance by scale for the maximal overlap discrete wavelet transform (MODWT). The default wavelet type is `sym4`.

`wvar = modwtvar(w,wname)` uses the wavelet `wname` to determine the number of boundary coefficients by level for unbiased estimates.

`[wvar,wvarci] = modwtvar( ___ )` returns the 95% confidence intervals for the variance estimates by scale.

`[ ___ ] = modwtvar(w,wname, ___ ,conflevel)` uses `conflevel` for the coverage probability of the confidence interval.

`[ ___ ] = modwtvar(w,wname, ___ ,Name,Value,)` returns wavelet variance with additional options specified by one or more `Name,Value` pair arguments.

`[wvar,wvarci,nj] = modwtvar(w,wname, ___ )` returns the number of coefficients used to form the variance and confidence intervals by level.

`wvartable = modwtvar(w,wname,'table')`, where `'table'` returns a MATLAB table, `wvartable`, containing the number of MODWT coefficients by level, the confidence boundaries, and the variance estimates. You can place `'table'` anywhere after input `w`, except between the name and value of another `Name,Value` pair.

`modwtvar(___)` with no output arguments plots the wavelet variances by scale with lower and upper confidence bounds. The scaling variance is not included in the plot because the scaling variance can be much larger than the wavelet variances.

Examples

Wavelet Variance Using Default Wavelet

Obtain the MODWT of the Southern Oscillation Index data using the default symlets wavelet with 4 vanishing moements. Compute the unbiased estimates of the wavelet variance by scale.

```
load soi
wsoi = modwt(soi);
wvar = modwtvar(wsoi)
```

```
wvar =

    0.3568
    0.9026
    1.1576
    1.0952
    0.9678
    0.5478
    0.6353
    1.9570
    0.8398
    0.8247
```

Wavelet Variance Using Specified Wavelet

Obtain the MODWT of the Southern Oscillation Index data using the Daubechies wavelet with 2 vanishing moments ('db2'). Compute the unbiased estimates of the wavelet variance by scale.

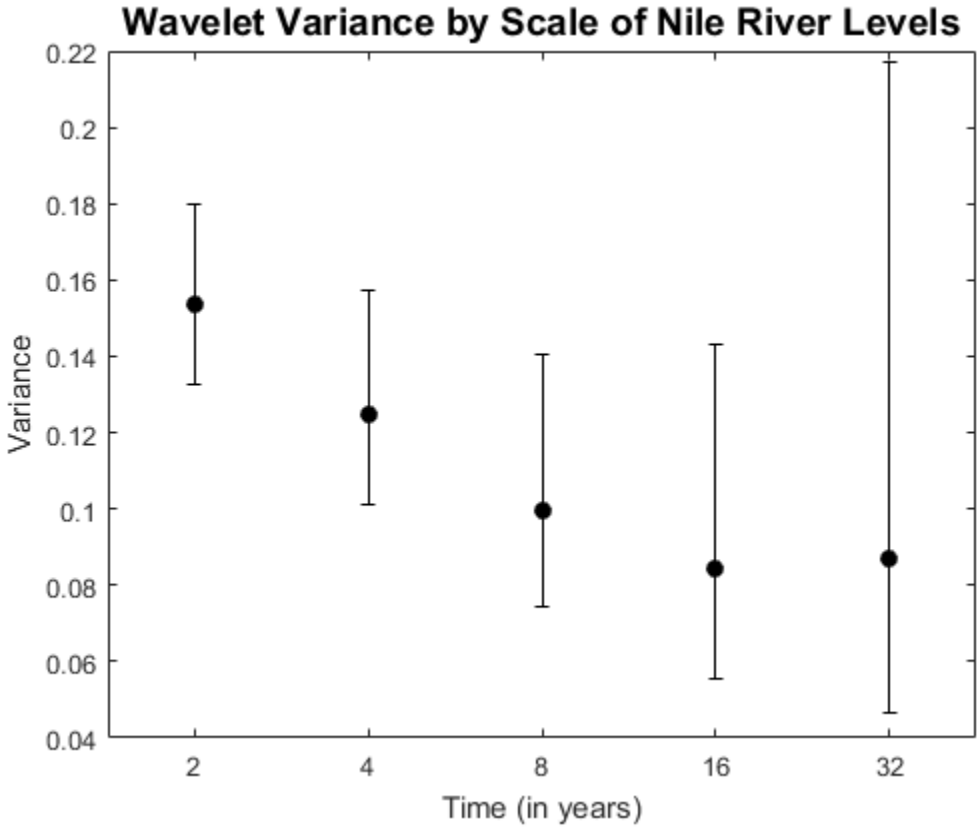
```
load soi
wsoi = modwt(soi,'db2');
wvar = modwtvar(wsoi,'db2')
```

```
wvar =

    0.4296
    0.9204
    1.1370
    1.0847
    0.9255
    0.5932
    0.7630
    1.6672
    0.8048
    0.7555
    0.2095
    0.0377
```

Variance Estimates and Confidence Intervals Using MODWTVAR

Obtain the MODWT of the Nile River minimum level data using the Fejer- Korovkin wavelet with eight coefficients down to level five. Use `modwtvar` to obtain and plot the variance estimates and 95% confidence intervals.

```
load nileriverminima;
wtnile = modwt(nileriverminima,'fk8',5);
[wnilevar,wvarci] = modwtvar(wtnile,'fk8');

errlower = (wnilevar-wvarci(:,1));
errupper = (wvarci(:,2)-wnilevar);
errorbar(1:5,wnilevar(1:5),errlower(1:5),...
    errupper(1:5),'ko','markerfacecolor','k')
hold on
title('Wavelet Variance by Scale of Nile River Levels','fontsize',14);
ylabel('Variance');
xlabel('Time (in years)');
ax = gca;
ax.XTick = [1:5];
ax.XTickLabel = {'2','4','8','16','32'};
hold off
```



Wavelet Confidence Intervals

Show how different confidence level values affect the width of the confidence intervals. An increased confidence level value increases the confidence interval width.

Obtain the MODWT of the Southern Oscillation Index data using the Fejer-Korovkin wavelet with eight coefficients.

```
load soi;
wsoi = modwt(soi,'fk8');
```

Obtain the width of the.90, .95, and .99 confidence intervals for each level.

```
[~,wvarci90] = modwtvar(wsoi,'fk8',0.90);
```

```
w90 = wvarci90(:,2)-wvarci90(:,1);
[~,wvarci95] = modwtvar(wsoi,'fk8',0.95);
w95 = wvarci95(:,2)-wvarci95(:,1);
[~,wvarci99] = modwtvar(wsoi,'fk8',0.99);
w99 = wvarci99(:,2)-wvarci99(:,1);
```

Compare the three columns. The first column shows the .90 confidence level values, the second the .95 values, and the third the .99 values. Each row is the width of the interval at each wavelet scale. You can see that the width of the confidence interval increases with larger confidence level values.

```
[w90,w95,w99]
```

```
ans =

    0.0195    0.0233    0.0306
    0.0739    0.0880    0.1158
    0.1347    0.1606    0.2113
    0.1798    0.2145    0.2826
    0.2304    0.2751    0.3634
    0.1825    0.2184    0.2900
    0.2858    0.3435    0.4613
    1.5445    1.8757    2.5837
    1.0625    1.3262    1.9551
    2.8460    3.9883    7.8724
```

Compare Chi2Eta2 and Gaussian Confidence Intervals

Specify non-default confidence methods using name-value pairs to compare the width of their confidence levels. Note that for Gaussian confidence level intervals, it is possible to obtain negative lower confidence bounds.

Obtain the MODWT of the Southern Oscillation Index data using the Fejer-Korovkin wavelet with eight coefficients.

```
load soi;
wsoi = modwt(soi,'fk8');
```

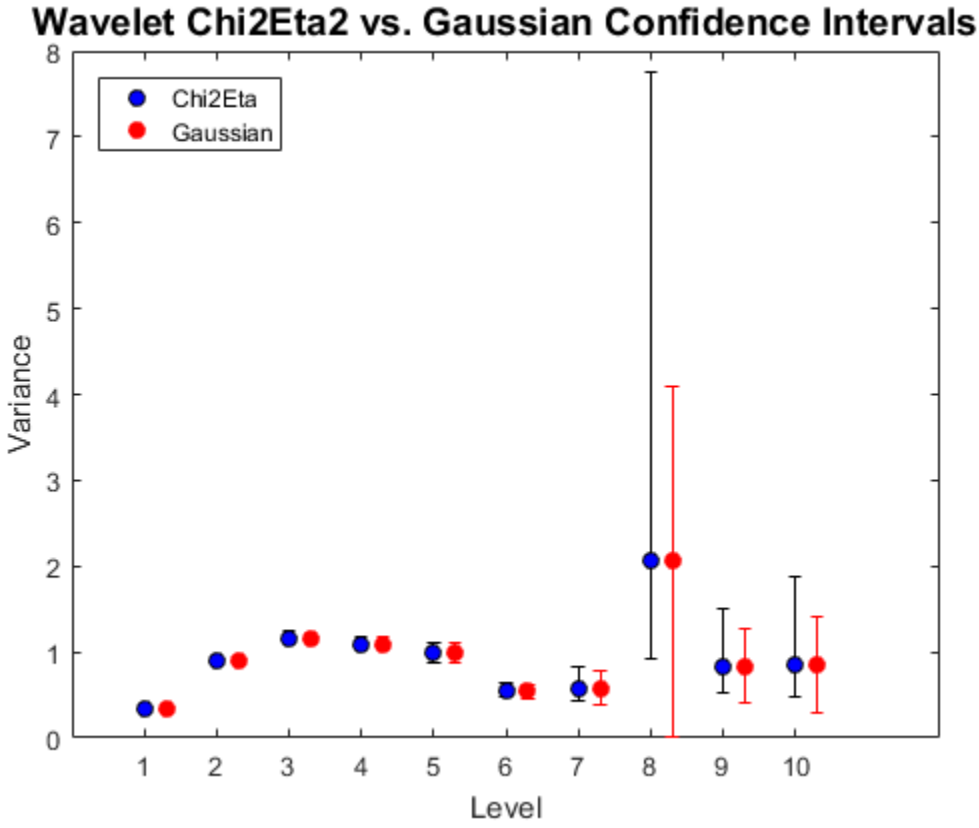
Use the Chi2Eta and Gaussian confidence methods to obtain the variances and confidence interval bounds for each method.

```
[wvar_c,wvarci_c] = modwtvar(wsoi,'fk8',[],'ConfidenceMethod','chi2eta1');
```

```
[wvar_g,wvarci_g] = modwtvar(wsoi,'fk8',[],'ConfidenceMethod','gaussian');
```

Compute the upper and lower errors for each confidence interval and plot the results. Note that the Gaussian intervals are slightly shifted to enable better visualization.

```
errlower_c = wvar_c-wvarci_c(:,1);
errupper_c = wvarci_c(:,2)-wvar_c;

errlower_g = wvar_g(:,1)-wvarci_g(:,1);
errupper_g = wvarci_g(:,2)-wvar_g;

errorbar(1:10,wvar_c(1:10),errlower_c(1:10),...
    errupper_c(1:10),'ko','markerfacecolor','b')
hold on;
xoffset = (1.3:10.3);
errorbar(xoffset,wvar_g(1:10),errlower_g(1:10),...
    errupper_g(1:10),'ro','markerfacecolor','r')

title('Wavelet Chi2Eta2 vs. Gaussian Confidence Intervals','fontsize',14);
ylabel('Variance');
xlabel('Level')
ax = gca;
ax.XTick = [1:10];
legend('Chi2Eta','Gaussian','Location','northwest');
hold off
```



Compare Number of Coefficients for Unbiased and Biased Variance Estimates

Compare the number of coefficients for unbiased and biased wavelet variance estimates. For the unbiased (default) estimates, the number of nonboundary coefficients decreases by scale. For biased estimates, the number of coefficients matches the number of input rows and is constant for every scale.

Obtain the MODWT of the Southern Oscillation Index data using the Fejer-Korovkin wavelet with eight coefficients. Compute the unbiased and biased estimates of the wavelet variance down to level ten. The number of coefficients used in the unbiased estimates decrease by scale.

```
load soi
```

```
wsoi = modwt(soi,'fk8');
[wvar_unb,wvarci_unb,nj_unb] = modwtvar(wsoi,'fk8');
[wvar_b,wvarci_b,nj_b] = modwtvar(wsoi,'fk8',[],'EstimatorType','biased');
[nj_unb(1:10),nj_b(1:10)]
```

```
ans =

       12991       12998
       12977       12998
       12949       12998
       12893       12998
       12781       12998
       12557       12998
       12109       12998
       11213       12998
        9421       12998
        5837       12998
```

Table of Wavelet Variance Estimates Using Gaussian Confidence Intervals

Compute the MODWT of the Southern Oscillation Index data using the Fejer- Korovkin wavelet with eight coefficients. Compute a variance table for the data.

```
load soi;
wsoi = modwt(soi,'fk8');
[wvartable] = modwtvar(wsoi,'fk8',0.90,'ConfidenceMethod','Gaussian',...
    'table')
```

```
wvartable =

            NJ       Lower      Variance      Upper
          _____    ________    _________    _______

    D1    12991      0.3291     0.33848     0.34786
    D2    12977     0.87172      0.9034     0.93508
    D3    12949      1.1041      1.1628      1.2216
    D4    12893      1.0204      1.0933      1.1662
    D5    12781      0.8833     0.98255      1.0818
    D6    12557     0.47178     0.54152     0.61125
    D7    12109     0.41916     0.57934     0.73951
    D8    11213     0.33639       2.055      3.7736
    D9     9421      0.4752     0.83369      1.1922
```

```
    D10      5837     0.37485     0.84386       1.3129
```

The resulting table contains the number of nonboundary coefficients, the lower and upper conficence level bounds, and the variance estimate for each level.

- "Wavelet Analysis of Financial Data"

Input Arguments

w — MODWT transform matrix

matrix

MODWT transform, specified as a matrix. `w` is the output of `modwt`.

Data Types: `double`

wname — Wavelet

`'sym4'` (default) | string | positive even scalar

Wavelet, specified as a string corresponding to a valid wavelet, or as a positive even scalar indicating the length of the wavelet and scaling filters. The wavelet filter length must match the length used in the MODWT of the input.

If you use `Name,Value` pairs arguments or the `'table'` syntax and you did not specify a `wname` , you must use `[]` as the second argument.

conflevel — Confidence level

0.95 (default) | real scalar greater than 0 and less than 1

Confidence level, specified as a real scalar value greater than 0 and less than 1. The confidence level determines the coverage probability of the confidence intervals. The confidence levels are also shown in `wvartable`, if you specify `'table'` as an input.

Name-Value Pair Arguments

Specify optional comma-separated pairs of `Name,Value` arguments. `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside single quotes (`' '`). You can specify several name and value pair arguments in any order as `Name1,Value1,...,NameN,ValueN`.

Example: `'EstimatorType','biased'` specifies a biased estimator.

`'EstimatorType'` — Estimator
`'unbiased'` (default) | `'biased'`

Type of estimator used for variance estimates and confidence bounds, specified as the comma-separated pair consisting of `'EstimatorType'` and one of these values.

- 'unbiased' — Unbiased estimator, which identifies and removes boundary coefficients prior to computing the variance estimates and confidence bounds. Unbiased estimates are used more frequently for wavelet variance computations.
- 'biased' — Biased estimator, which uses all coefficients to compute the variance estimates and confidence bounds.

`'ConfidenceMethod'` — Confidence method
`'chi2eta3'` (default) | `'chi2eta1'` | `'gaussian'`

Confidence method used to compute the confidence intervals, specified as the comma-separated pair consisting of `'ConfidenceMethod'` and one of these values:

`'chi2eta3'`	Chi-square probability density method three, which determines the degrees of freedom.[1].
`'chi2eta1'`	Chi-square probability density method one, which determines the degrees of freedom [1].
`'gaussian'`	Gaussian method [1] . This method can result in negative lower bounds.

See “Algorithm” on page 1-394 for information on each of these confidence methods.

`'Boundary'` — Boundary condition
`'periodic'` (default) | `'reflection'`

Boundary condition used to compute the variance estimates and confidence bounds, specified as the comma-separated pair consisting of `'Boundary'` and one of these values:

`'periodic'`	Periodic boundary handling, which does not change the original signal before computing the MODWT. If `modwt` uses periodic boundary handling, you must specify `'Boundary','periodic'` for `modwtvar` to obtain a correct estimate.

`'reflection'`	Reflection boundary handling. If the MODWT uses reflection boundary handling, you must also specify `'Boundary','reflection'` for `modwtvar` to obtain a correct unbiased estimate. The MODWT, with reflection boundary handling, extends the original signal symmetrically at the right boundary to twice the signal length. The MODWTVAR algorithm has to know about this extended signal to calculate the correct unbiased estimate. For biased estimators, all the coefficients are used to form the variance estimates and confidence intervals regardless of the boundary handling.

Output Arguments

`wvar` — Wavelet variance estimates

matrix

Wavelet variance estimates, returned as vector. The number of elements in `wvar` depends on the number of scales in the input matrix and, for unbiased estimates, on the wavelet length. For the unbiased case, `modwtvar` returns estimates only where nonboundary coefficients exist. This condition is satisfied when the transform level is not greater than `floor(log2(N/(L-1)+1))`, where N is the input signal length and L is the length of the wavelet filter. The number of biased estimates equals the input signal length. If the final level has sufficient nonboundary coefficients, `modwtvar` returns the scaling variance in the final element of `wvar`.

`wvarci` — Confidence intervals for each variance estimate

matrix

Confidence bounds, expressed as upper and lower confidence bounds, for the variance estimates in `wvar`, returned as a matrix. The default is 95% confidence bounds, but you can use a different value using the `conflevel` input argument. The confidence bounds matrix is M-by-2, where M is the number of levels. For unbiased estimates, the number of levels is limited by the number of nonboundary coefficients. For biased

estimates, all levels are used. The first column of the confidence interval matrix contains the lower confidence bound and the second column contains the upper confidence bound. By default, `modwtvar` calculates the confidence intervals using the chi-square probability density, with the equivalent degrees of freedom estimated using the `'Chi2Eta3'` confidence method.

`nj` — Number of coefficients by level

vector

Number of nonboundary coefficients by scale, returned as a vector. For unbiased estimates, `nj` is the number of nonboundary coefficients and decreases by level. For biased estimates, `nj` is a vector of constants equal to the number of columns in the input matrix.

`wvartable` — Variance table

table

Variance table, returned as a MATLAB table. The four variables in the table are:

- **NJ** — Number of MODWT coefficients by level. For biased estimates, NJ is the number of coefficients in the MODWT. For unbiased estimates, NJ is the number of nonboundary coefficients.
- **Lower** — Lower confidence bound for the variance estimate.
- **Variance** — Variance estimate by level.
- **Upper** — Upper confidence bound for the variance estimate.

The row names of `wvartable` indicate the type and level of each estimate. For example, D1 indicates that the row corresponds to a wavelet or detail estimate at level 1. S6 indicates that the row corresponds to the scaling estimate at level 6. The scaling variance is computed for the final level of the MODWT. For unbiased estimates, `modwtvar` computes the scaling variance only when nonboundary scaling coefficients exist.

More About

Algorithms

The following expressions define the variance and confidence methods used in the MODWTVAR. The variables are

- N_j — Number of coefficients at level j

- v^2 — Variance
- j — Level
- $W_{j,t}$ — Wavelet coefficients

The variance estimate is

$$\hat{v}_j^2 = \frac{1}{N_j} \sum_{t=0}^{N_j-1} W_{j,t}^2$$

The degrees of freedom for the Chi2Eta1 (`chi2eta1`) method are defined as

$$\eta_1 = \frac{N_j \hat{v}_j^4}{\hat{A}_j}$$

where

$$\hat{A}_j = \frac{1}{2} \int_{-1/2}^{1/2} \left[\hat{S}_j^{(p)}(f) \right]^2 df$$

In this equation, $\hat{S}_j^{(p)}$ is the spectral density function estimate of the wavelet coefficients at level j.

The chi-square statistic is

$$\frac{\eta_1 N_j \hat{v}_j^2}{v_j^2} \sim \mathrm{X}_{\eta_1}^2$$

The degrees of freedom for the Chi2Eta3 (`chi2eta3`) method are defined as

$$\eta_3 = \max\left(\frac{N_j}{2^j}, 1 \right)$$

The chi-square statistic is

$$\frac{\eta_3 N_j \hat{v}_j^2}{v_j^2} \sim \mathrm{X}^2_{\eta_3}$$

For the Gaussian method, the statistic

$$\frac{N_j^{1/2}\left((v_j^2 - v_j^2)\right)}{\left(2A_j\right)^{1/2}}$$

is distributed as `N(0,1)`. The variable $\hat{A}_j$ is as described for `chi2eta1`.

References

[1] Percival, D. B., and A. T. Walden. *Wavelet Methods for Time Series Analysis*. Cambridge, UK: Cambridge University Press, 2000.

[2] Percival, D. B., D. Mondal, "A Wavelet Variance Primer." *Handbook of Statistics, Volume. 300, Time Series Analysis: Methods and Applications*, (T. S. Rao, S. S. Rao, and C. R. Rao, eds.). Oxford, UK: Elsevier, 2012, pp. 623–658.

[3] Cornish, C. R., C. S. Bretherton, and D. B. Percival. "Maximal Overlap Wavelet Statistical Analysis with Application to Atmospheric Turbulence." *Boundary-Layer Meteorology*. Vol. 119, Number 2, 2005, pp. 339–374.

See Also

`imodwt` | `modwt` | `modwtcorr` | `modwtmra` | `modwtxcorr`

Introduced in R2015b

modwtxcorr

Wavelet cross-correlation sequence estimates using the maximal overlap discrete wavelet transform (MODWT)

Syntax

```
xcseq = modwtxcorr(w1,w2)
xcseq = modwtxcorr(w1,w2,wav)

[xcseq,xcseqci] = modwtxcorr( ___ )
[xcseq,xcseqci] = modwtxcorr(w1,w2,wav,conflevel)
[xcseq,xcseqci,lags] = modwtxcorr( ___ )

[ ___ ] = modwtxcorr( ___ ,'reflection')
```

Description

`xcseq = modwtxcorr(w1,w2)` returns the wavelet cross-correlation sequence estimates for the maximal overlap discrete wavelet transform (MODWT) transforms specified in `w1` and `w2`. `xcseq` is a cell array of vectors where the elements in each cell correspond to cross-correlation sequence estimates. If there are enough nonboundary coefficients at the final level, `modwtxcorr` returns the scaling cross-correlation sequence estimate in the final cell of `xcseq`.

`xcseq = modwtxcorr(w1,w2,wav)` uses the wavelet `wav` to determine the number of boundary coefficients by level.

`[xcseq,xcseqci] = modwtxcorr( ___ )` returns in `xcseqci` the 95% confidence intervals for the cross-correlation sequence estimates in `xcseq`, using any arguments from the previous syntaxes.

`[xcseq,xcseqci] = modwtxcorr(w1,w2,wav,conflevel)` uses `conflevel` for the coverage probability of the confidence interval. `conflevel` is a real scalar strictly greater than 0 and less than 1. If `conflevel` is unspecified or specified as empty, the coverage probability defaults to 0.95.

`[xcseq,xcseqci,lags] = modwtxcorr( ___ )` returns the lags for the wavelet cross-correlation sequence estimates in a cell array of column vectors.

`[ ___ ] = modwtxcorr( ___ ,'reflection')` reduces the number of wavelet and scaling coefficients at each scale by half before computing the cross-correlation sequences. Specifying the `'reflection'` option in `modwtxcorr` is equivalent to first obtaining the MODWT of `w1 w2` with `'periodic'` boundary handling and then computing the wavelet cross-correlation sequence estimates. Use this option only when you obtain the MODWT of `w1` and `w2` using the `'reflection'` boundary condition. You must enter the entire string `'reflection'`. If you added a wavelet named `'reflection'` using the wavelet manager, you must rename that wavelet prior to using this option. `'reflection'` may be placed in any position in the input argument list after the input transforms `w1 w2`.

Examples

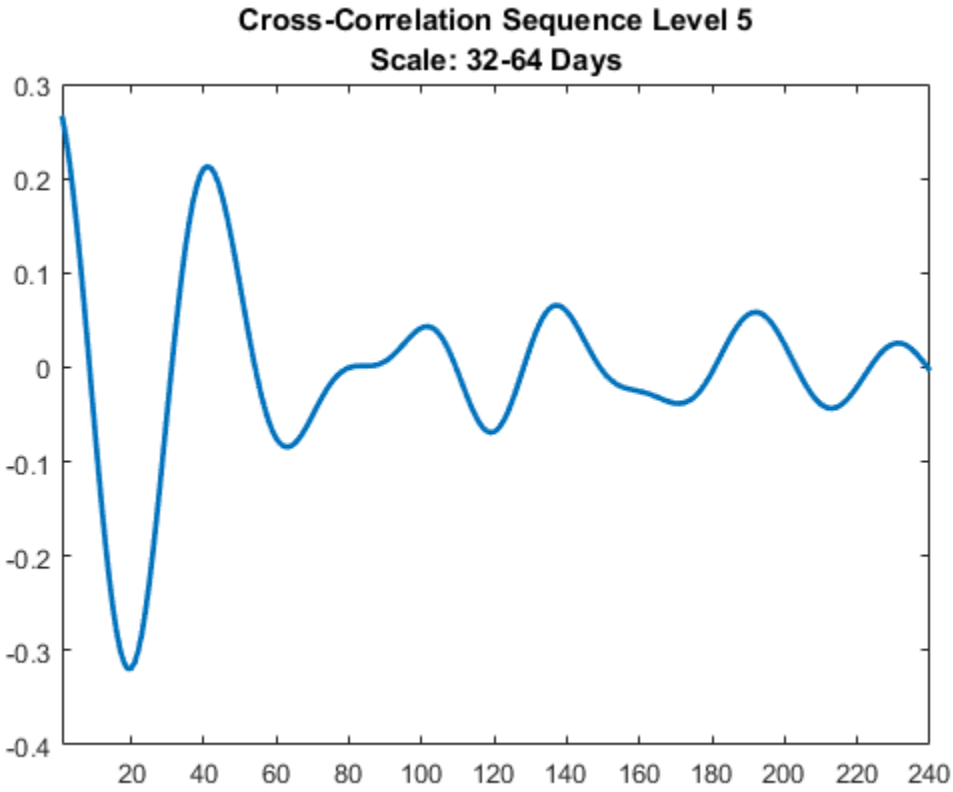
Cross-Correlation Sequence

Obtain the MODWT of the Southern Oscillation Index and Truk Islands pressure data. The sampling period is one day.

```
load soi
load truk
wsoi = modwt(soi);
wtruk = modwt(truk);
```

Compute the wavelet cross-correlation sequences. Examine the level-five cross-correlation sequence corresponding to a scale of 32-64 days. Determine the index corresponding to a lag of zero and plot out to 240 lags.

```
xcseq = modwtxcorr(wsoi,wtruk);
zerolag = floor(numel(xcseq{5})/2)+1;
plot(xcseq{5}(zerolag:zerolag+240),'linewidth',2)
set(gca,'xlim',[1 240]);
title({'Cross-Correlation Sequence Level 5'; 'Scale: 32-64 Days'});
hold off
```



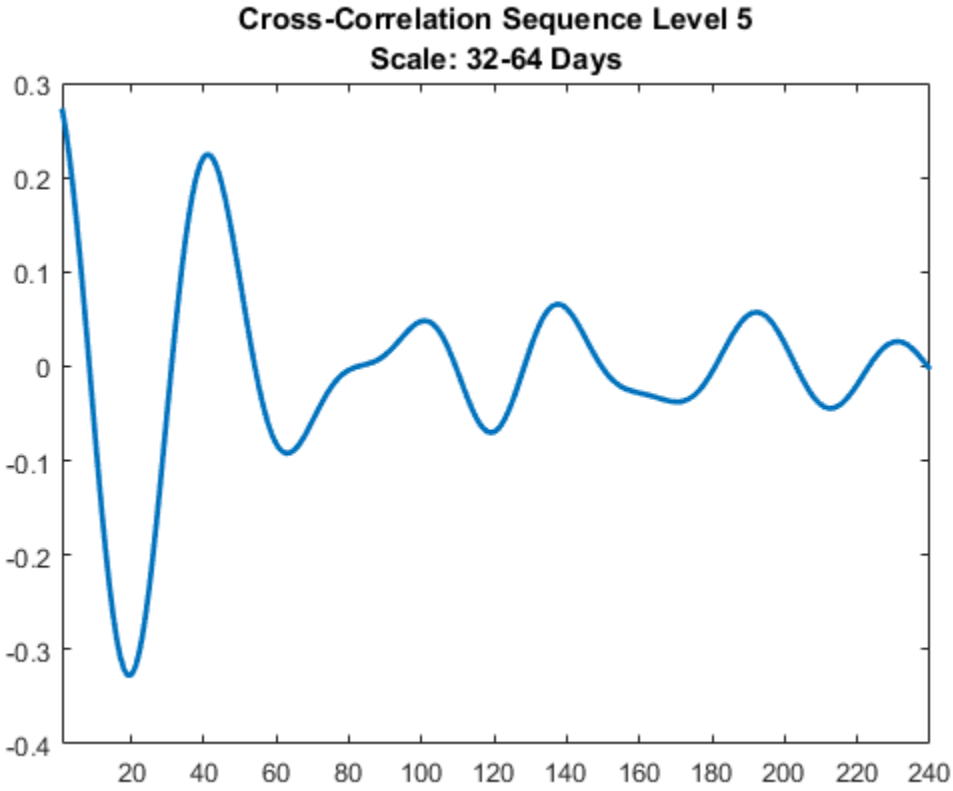
Cross-Correlation Sequence with Fejer-Korovkin Wavelet

Obtain the MODWT of the Southern Oscillation Index and Truk Islands pressure data using the Fejer-Korovkin wavelet filter with 8 coefficients. The sampling period of the data is one day.

```
load soi
load truk
wsoi = modwt(soi,'fk8');
wtruk = modwt(truk,'fk8');
```

Compute the wavelet cross-correlation sequences. Examine the level-five cross-correlation sequence corresponding to a scale of 32-64 days. Determine the index corresponding to a lag of zero and plot out to 240 lags.

```
xcseq = modwtxcorr(wsoi,wtruk,'fk8');
zerolag = floor(numel(xcseq{5})/2)+1;
plot(xcseq{5}(zerolag:zerolag+240),'linewidth',2)
set(gca,'xlim',[1 240]);
title({'Cross-Correlation Sequence Level 5'; 'Scale: 32-64 Days'});
hold off
```

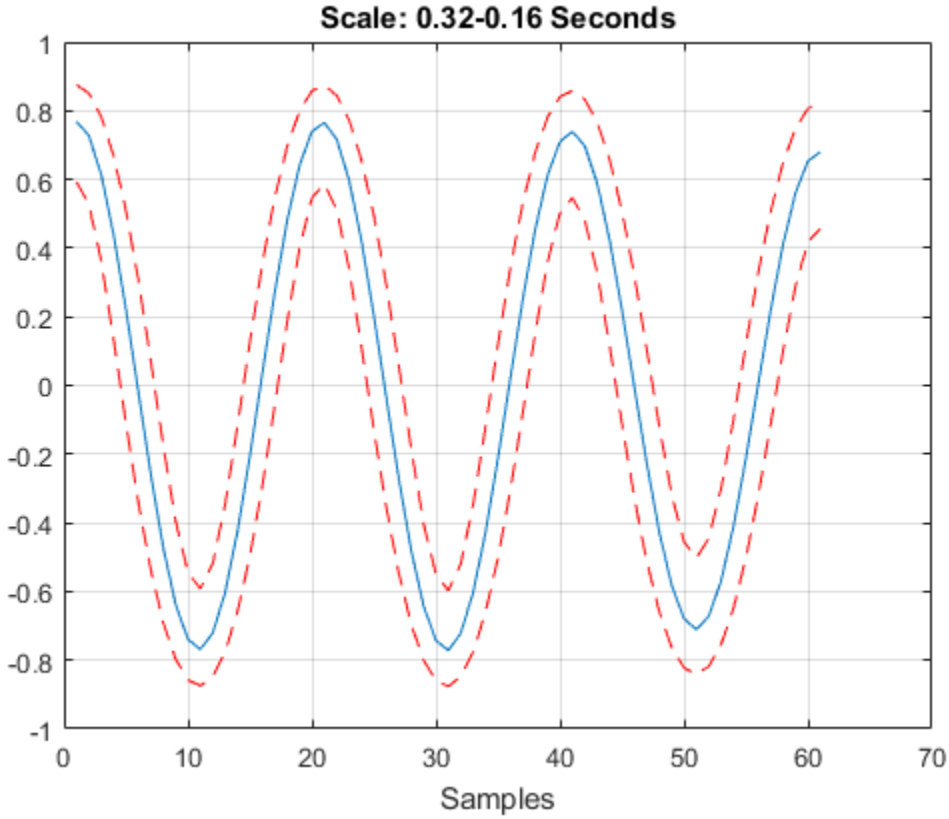


Cross-Correlation Confidence Intervals by Scale

Plot the wavelet cross-correlation with 95% confidence intervals at scale 4 for two 5-Hz sine wave signals with additive noise.

```
dt = 0.01;
t = 0:dt:6;
x = cos(2*pi*5*t)+1.5*randn(size(t));
```

```
y = cos(2*pi*5*t-pi)+2*randn(size(t));
wx = modwt(x,'fk14',5);
wy = modwt(y,'fk14',5);
modwtcorr(wx,wy,'fk14')
j = 4;
[xcseq,xcseqci] = modwtxcorr(wx,wy,'fk14');
zerolag = floor(numel(xcseq{j})/2)+1;
lagidx = zerolag-30:zerolag+30;
plot(xcseq{j}(lagidx));
hold on;
grid
plot(xcseqci{j}(lagidx,:),'r--');
xlabel('Samples');
title('Scale: 0.32-0.16 Seconds');
```



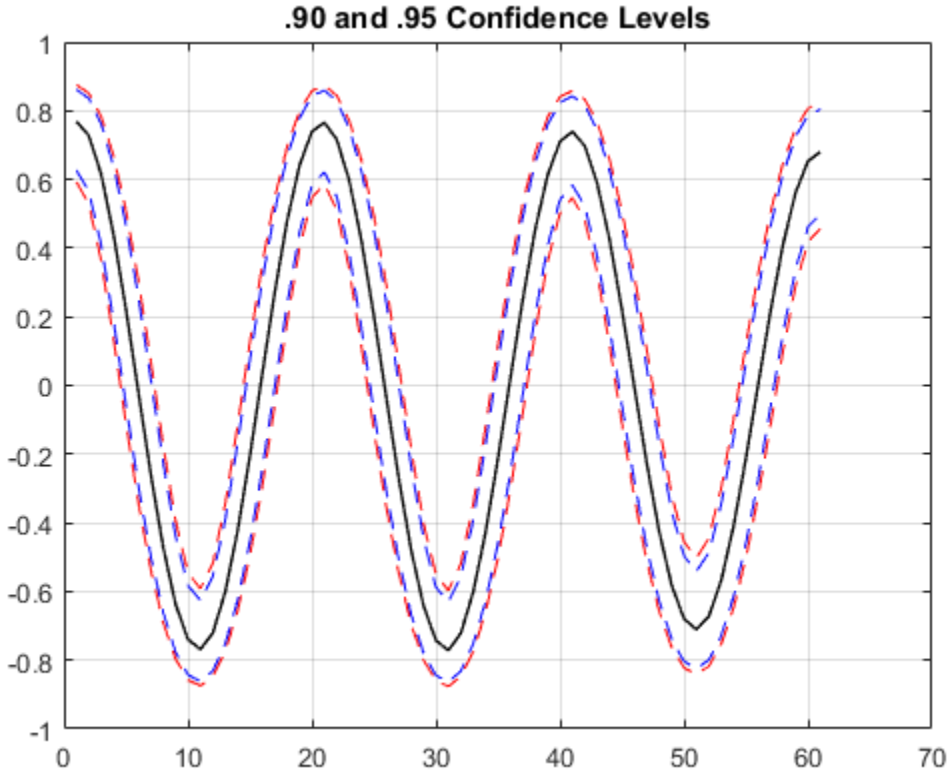
Cross-Correlation .90 and .95 Confidence Intervals Comparison

Compare the .90 and .95 (default) confidence intervals for the wavelet cross-correlation at level four.

Obtain the MODWT for two noisy sine waves using the Fejer-Korovkin with 14 coefficients, and specify the level to use.

```
dt = 0.01;
t = 0:dt:6;
x = cos(2*pi*5*t)+1.5*randn(size(t));
y = cos(2*pi*5*t-pi)+2*randn(size(t));
wx = modwt(x,'fk14',4);
wy = modwt(y,'fk14',4);
```

```
lev = 4;

[xcseq,xcseqci] = modwtxcorr(wx,wy,'fk14');
[xcseq90,xcseqci90] = modwtxcorr(wx,wy,'fk14',0.90);

zerolag = floor(numel(xcseq{lev})/2)+1;
zerolag90 = floor(numel(xcseq90{lev})/2)+1;

lagidx = zerolag-30:zerolag+30;
lagidx90 = zerolag90-30:zerolag90+30;

plot(xcseqci{lev}(lagidx,:),'--r');
hold on
plot(xcseqci90{lev}(lagidx90,:),'--b');
plot(xcseq{lev}(lagidx),'-k','LineWidth',1);
grid
title('.90 and .95 Confidence Levels')
```



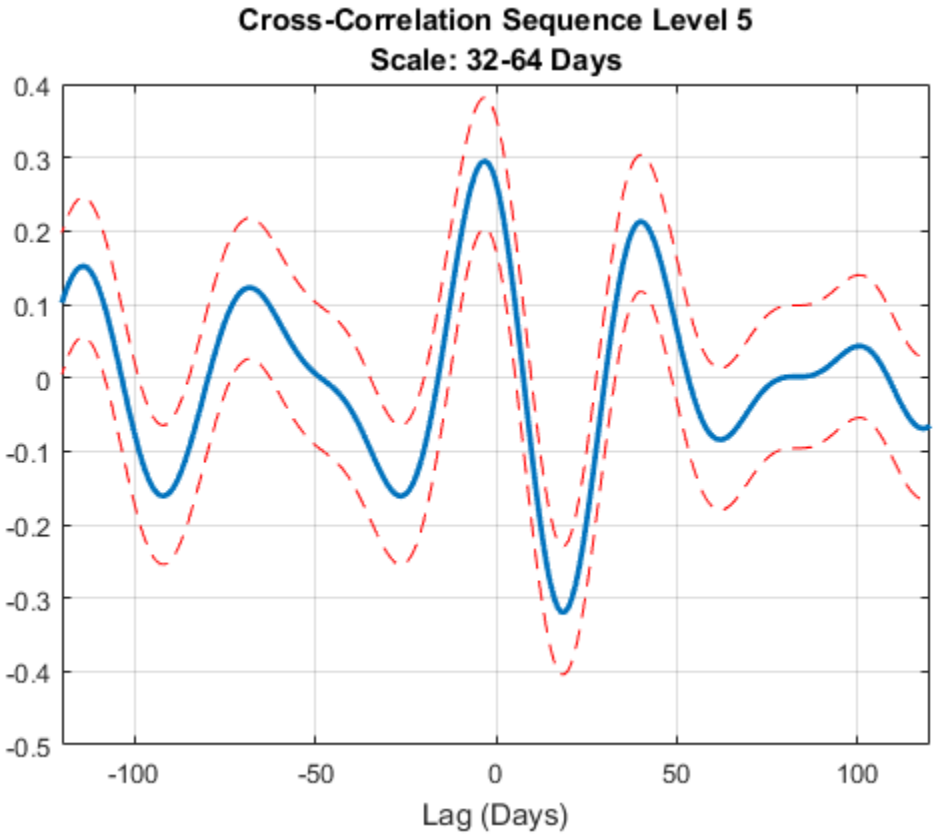
Notice that the .95 confidence interval width (in red) is larger than the .90 confidence interval width (in blue).

Plot Cross-Correlation Sequences by Lag

Plot the cross-correlation sequence estimate for the Southern Oscillation Index and Truk Island pressure data. Estimate 95% confidence intervals for scale of 2^5 days.

```
load soi
load truk
wsoi = modwt(soi);
wtruk = modwt(truk);
[xcseq,xcseqci,lags] = modwtxcorr(wsoi,wtruk);
plot(lags{5},xcseq{5},'linewidth',2)
```

```
hold on
plot(lags{5},xcseqci{5},'r--')
set(gca,'xlim',[-120 120]);
xlabel('Lag (Days)');
grid
title({'Cross-Correlation Sequence Level 5'; 'Scale: 32-64 Days'});
hold off
```



Cross-Correlation with Reflection Boundary

Obtain the MODWT of 36 years of Southern Oscillation Index and Truk Islands pressure data with both periodic and reflection boundary conditions. The `modwt` with the `'reflection'` option extends the input signal symmetrically at the right boundary. The input signal is then twice its original length. MODWTXCORR with the reflection

boundary handling reduces the number of wavelet and scaling coefficients at each half before computing the cross-correlation sequences. The size of the cross-correlation sequences is the same as acquiring the MODWT with the default periodic boundary condition.

```
load soi
load truk
```

Obtain the MODWT with the default periodic boundary condition.

```
wsoi_default = modwt(soi);
wtruk_default = modwt(truk);
```

Obtain the MODWT with the reflection boundary condition.

```
wsoi_reflect = modwt(soi,'reflection');
wtruk_reflect = modwt(truk,'reflection');
```

Obtain the cross correlation sequences.

```
xcseq_default = modwtxcorr(wsoi_default,wtruk_default);
xcseq_reflect = modwtxcorr(wsoi_reflect,wtruk_reflect,'reflection');
```

Compare the number of elements in the cell array output for both boundary conditions.

```
cellfun(@numel,xcseq_reflect)
cellfun(@numel,xcseq_default)
```

```
ans =

       25981
       25953
       25897
       25785
       25561
       25113
       24217
       22425
       18841
       11673


ans =
```

```
25981
25953
25897
25785
25561
25113
24217
22425
18841
11673
```

- “Wavelet Analysis of Financial Data”

Input Arguments

w1 — MODWT transform of signal 1

matrix

MODWT transform of signal 1, specified as a matrix of doubles. The input `w1` must be the same size as `w2` and must have been obtained with the same wavelet. By default, `modwtxcorr` assumes that you obtained the MODWT using the symlet wavelet with four vanishing moments, `'sym4'`).

w2 — MODWT transform of signal 2

matrix

MODWT transform of signal 2, specified as a matrix of doubles. The input `w2` must be the same size as `w1` and must have been obtained with the same wavelet. By default, `modwtxcorr` assumes that you obtained the MODWT using the symlet wavelet with four vanishing moments (`'sym4'`).

wav — Wavelet

`'sym4'` (default) | string | positive even integer

Wavelet, specified as a string, indicating a valid wavelet, or as a positive even integer indicating the length of the wavelet and scaling filters. If `wav` is unspecified or specified as an empty, `[]`, `wav` defaults to `'sym4'`.

conflevel — Confidence level

0.95 (default) | positive scalar less than 1

Confidence level, specified as a positive scalar less than 1. `conflevel` determines the coverage probability of the confidence intervals in `xcseqci`. If unspecified, or specified as empty, `[]`, `conflevel` defaults to 0.95.

Output Arguments

`xcseq` — Cross-correlation sequences by scale

cell array of vectors

Cross-correlation sequences by scale, returned as a cell array of vectors. The vectors are of size $2NJ$-by-1, where NJ is the number of nonboundary coefficients by level (scale). This level is the minimum of `size(w1,1)` and `floor(log2(N/(L-1)+1))` where N is the length of the data and L is the filter length. If there are enough nonboundary coefficients at the final level, `modwtxcorr` returns the scaling cross-correlation sequence estimate in the final cell of `xcseq`.

`xcseqci` — Confidence intervals by scale

cell array of matrices

Confidence intervals by scale, returned as a cell array of matrices. The size of each matrix is $(2NJ\text{-}1)$-by-2, where NJ is the number of nonboundary coefficients by level (scale).

- For the .95 default value, the first column of the i^{th} element of `xcseqci` contains the lower 95% confidence bound for the cross-correlation coefficient at each lag.
- For the .95 default value, the second column contains the upper 95% confidence bound.

Confidence bounds are computed using Fisher's Z-transformation. The standard error of Fisher's Z statistic is the square root of $N–3$. In this case, N is the equivalent number of coefficients in the critically sampled discrete wavelet transform (DWT), `floor(size(w1,2)/2^LEV)`, where `LEV` is the level of the wavelet transform. `modwtcorr` returns NaNs for the confidence bounds when N^{-3} is less than or equal to zero.

`lags` — Lags for the cross-correlation sequences

cell array of vectors

Lags for the cross-correlation sequences, returned as a cell array of vectors. `lags` is a cell array of column vectors the same length as `xcseq`. The elements in each cell of `xcseq`

correspond to the cross-correlation sequence estimates at lags from -(*NJ*-1) to (*NJ*-1), where *NJ* is the number of nonboundary coefficients at level *J*. The 0^{th} lag element is located at the index floor((2*NJ-1)/2)+1.

References

[1] >Percival, D. B., and Walden, A. T. *Wavelet Methods for Time Series Analysis*. Cambridge, U.K: Cambridge University Press, 2000.

[2] Whitcher, B., P. Guttorp, and D. B. Percival. "Wavelet analysis of covariance with application to atmospheric time series." *Journal of Geophysical Research*, Vol. 105, 2000, pp. 14941–14962.

See Also

`imodwt` | `modwt` | `modwtcorr` | `modwtmra` | `modwtvar`

Introduced in R2015b

morlet

Morlet wavelet

Syntax

```
[PSI,X] = morlet(LB,UB,N)
```

Description

`[PSI,X] = morlet(LB,UB,N)` returns values of the Morlet wavelet on an `N` point regular grid in the interval `[LB,UB]`.

Output arguments are the wavelet function `PSI` computed on the grid `X`, and the grid `X`.

This wavelet has [-4 4] as effective support. Although [-4 4] is the correct theoretical effective support, a wider effective support, [-8 8], is used in the computation to provide more accurate results.

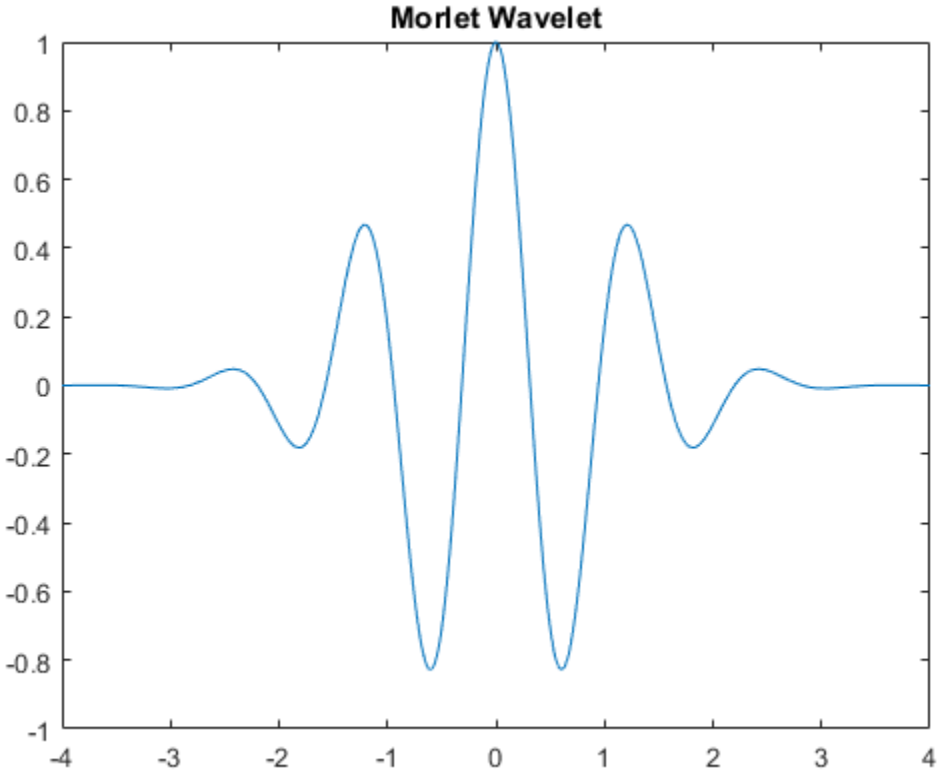
$$\psi(x) = e^{-x^2/2}\cos(5x)$$

Examples

Morlet Wavelet

This example shows how to create a Morlet wavelet. The wavelet has an effective support of [-4,4]. Use 1,000 sample points.

```
lb = -4;
ub = 4;
n = 1000;
[psi,xval] = morlet(lb,ub,n);
plot(xval,psi)
title('Morlet Wavelet');
```



See Also

`waveinfo`

Introduced before R2006a

mswcmp

Multisignal 1-D compression using wavelets

Syntax

```
[XC,DECCMP,THRESH] = mswcmp('cmp',DEC,METH)
[XC,DECCMP,THRESH] = mswcmp('cmp',DEC,METH,PARAM)
[XC,THRESH] = mswcmp('cmpsig',...)
[DECCMP,THRESH] = mswcmp('cmpdec',...)
THRESH = mswcmp('thr',...)
[...] = mswcmp(OPTION,DIRDEC,X,WNAME,LEV,METH)
[...] = mswcmp(OPTION,DIRDEC,X,WNAME,LEV,METH,PARAM)
[...] = mswcmp(...,S_OR_H)
[...] = mswcmp(...,S_OR_H,KEEPAPP)
[...] = mswcmp(...,S_OR_H,KEEPAPP,IDXSIG)
```

Description

`mswcmp` computes thresholds and, depending on the selected option, performs compression of 1-D signals using wavelets.

`[XC,DECCMP,THRESH] = mswcmp('cmp',DEC,METH)` or `[XC,DECCMP,THRESH] = mswcmp('cmp',DEC,METH,PARAM)` returns a compressed (indicated by `'cmp'` input) version `XC` of the original multisignal matrix `X`, whose wavelet decomposition structure is `DEC`. The output `XC` is obtained by thresholding the wavelet coefficients: `DECCMP`, which is the wavelet decomposition associated with `XC` (see `mdwtdec`), and `THRESH` is the matrix of threshold values. The input `METH` is the name of the compression method and `PARAM` is the associated parameter, if required.

Valid compression methods METH are shown in the following tables. For methods that use an associated parameter, the range of allowable `PARAM` values is also shown.

`'rem_n0'`	Remove near 0
`'bal_sn'`	Balance sparsity-norm

'sqrtbal_sn'	Balance sparsity-norm (sqrt)
'scarce'	Scarce, PARAM (any number)
'scarcehi'	Scarce high, 2.5 ≤ PARAM ≤ 10
'scarceme'	Scarce medium, 1.5 ≤ PARAM ≤ 2.5
'scarcelo'	Scarce low, 1 ≤ PARAM ≤ 2

PARAM is a sparsity parameter, and it should be such that: 1 ≤ PARAM ≤ 10. For scarce method no control is done.

'L2_perf'	Energy ratio
'NO_perf'	Zero coefficients ratio

PARAM is a real number which represents the required performance:

0 ≤ PARAM ≤ 100.

'glb_thr'	Global threshold

PARAM is a real positive number.

'man_thr'	Manual method

PARAM is an NbSIG-by-NbLEV matrix or NbSIG-by-(NbLEV+1) matrix such that:

- - PARAM(i,j) is the threshold for the detail coefficients of level j for the ith signal (1 ≤ j ≤ NbLEV).
- - PARAM(i,NbLEV+1) is the threshold for the approximation coefficients for the ith signal (if KEEPAPP is 0).

Where NbSIG is the number of signals and NbLEV the number of levels of decomposition.

[XC,THRESH] = mswcmp('cmpsig',...) or
[DECCMP,THRESH] = mswcmp('cmpdec',...) or
THRESH = mswcmp('thr',...) Instead of the 'cmp' input OPTION, you can use 'cmpsig', 'cmpdec' or 'thr' to select other output arguments. 'thr' returns the computed thresholds, but compression is not performed.

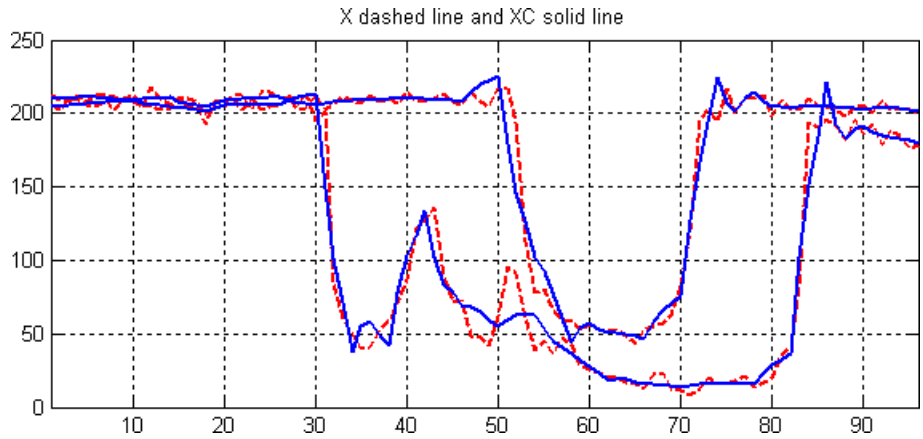
`[...] = mswcmp(OPTION,DIRDEC,X,WNAME,LEV,METH)`
`[...] = mswcmp(OPTION,DIRDEC,X,WNAME,LEV,METH,PARAM)` The decomposition structure input argument `DEC` can be replaced by four arguments: `DIRDEC`, `X`, `WNAME`, and `LEV`. Before performing a compression or computing thresholds, the multisignal matrix `X` is decomposed at level `LEV` using the wavelet `WNAME`, in the direction `DIRDEC`.

`[...] = mswcmp(...,S_OR_H)`
`[...] = mswcmp(...,S_OR_H,KEEPAPP)`
`[...] = mswcmp(...,S_OR_H,KEEPAPP,IDXSIG)` Three more optional inputs may be used:

- `S_OR_H ('s' or 'h')` stands for soft or hard thresholding (see `mswthresh` for more details). Default is `'h'`.
- `KEEPAPP (true or false)` indicates whether to keep approximation coefficients (`true`) or not (`false`). Default is `false`.
- `IDXSIG` is a vector which contains the indices of the initial signals, or the string `'all'`. Default is `'all'`.

Examples

```
% Load original 1D-multisignal.
load thinker

% Perform a decomposition at level 2 using wavelet db2.
dec = mdwtdec('r',X,2,'db2');

% Compress the signals to obtain a percentage of zeros
% near 95% for the wavelet coefficients.
[XC,decCMP,THRESH] = mswcmp('cmp',dec,'NO_perf',95);
[Ecmp,PECcmp,PECFScmp] = wdecenergy(decCMP);

% Plot the original signals 1 and 31, and
% the corresponding compressed signals.
figure;
plot(X([1 31],:)','r--','linewidth',2);   hold on
plot(XC([1 31],:)','b','linewidth',2);
grid; set(gca,'Xlim',[1,96])
title('X dashed line and XC solid line')
```



References

Birgé L.; P. Massart (1997), "From Model Selection to Adaptive Estimation," in D. Pollard (ed), *Festchrift for L. Le Cam*, Springer, pp. 55–88.

DeVore, R.A.; B. Jawerth, B.J. Lucier (1992), "Image Compression Through Wavelet Transform Coding," *IEEE Trans. on Inf. Theory*, vol. 38, No 2, pp. 719–746.

Donoho, D.L. (1993), "Progress in Wavelet Analysis and WVD: a Ten Minute Tour," in Progress in Wavelet Analysis and Applications, Y. Meyer, S. Roques, pp. 109–128. Frontières Ed.

Donoho, D.L.; I.M. Johnstone(1994), "Ideal Spatial Adaptation by Wavelet Shrinkage," *Biometrika*, vol. 81, pp. 425–455.

Donoho, D.L.; I.M. Johnstone, G. Kerkyacharian, D. Picard (1995), "Wavelet Shrinkage: Asymptopia," *Jour. Roy. Stat. Soc.*, *series B*, vol. 57 no. 2, pp. 301–369.

Donoho, D.L.; I.M. Johnstone, "Ideal De-noising in an Orthonormal Basis Chosen from a Library of Bases," C.R.A.S. Paris, t. 319, Ser. I, pp. 1317–1322.

Donoho, D.L. (1995), "De-noising by Soft-thresholding," *IEEE Trans. on Inf. Theory*, 41, 3, pp. 613–627.

See Also

`mdwtdec` | `mdwtrec` | `mswthresh` | `wthresh`

Introduced in R2007a

mswcmpscr

Multisignal 1-D wavelet compression scores

Syntax

```
[THR,L2SCR,NOSCR,IDXSORT] = mswcmpscr(DEC)
```

Description

`[THR,L2SCR,NOSCR,IDXSORT] = mswcmpscr(DEC)` computes four matrices: thresholds `THR`, compression scores `L2SCR` and `NOSCR`, and indices `IDXSORT`. The decomposition `DEC` corresponds to a matrix of wavelet coefficients `CFS` obtained by concatenation of detail and (optionally) approximation coefficients, where

`CFS = [cd{DEC.level}, ... , cd{1}]` or `CFS = [ca, cd{DEC.level}, ... , cd{1}]`

The concatenation is made rowwise if `DEC.dirDec` is equal to `'r'` or columnwise if `DEC.dirDec` is equal to `'c'` .

If `NbSIG` is the number of original signals and `NbCFS` the number of coefficients for each signal (all or only the detail coefficients), then `CFS` is an `NbSIG`-by-`NbCFS` matrix. Therefore,

- `THR`, `L2SCR`, `NOSCR` are `NbSIG`-by-(`NbCFS+1`) matrices
- `IDXSORT` is an `NbSIG`-by-`NbCFS` matrix
- `THR(:,2:end)` is equal to `CFS` sorted by row in ascending order with respect to the absolute value.
- For each row, `IDXSORT` contains the order of coefficients and `THR(:,1)=0`.

For the ith signal:

- `L2SCR(i,j)` is the percentage of preserved energy (L2-norm), corresponding to a threshold equal to `CFS(i,j-1)` ($2 \le j \le$ `NbCFS`), and `L2SCR(:,1)=100`.
- `NOSCR(i,j)` is the percentage of zeros corresponding to a threshold equal to `CFS(i,j-1)` ($2 \le j \le$ `NbCFS`), and `NOSCR(:,1)=0`.

Three more optional inputs may be used:

```
[...] = mswcmpscr(...,S_OR_H,KEEPAPP,IDXSIG)
```

- `S_OR_H ('s' or 'h')` stands for soft or hard thresholding (see `mswthresh` for more details).
- `KEEPAPP (true or false)` indicates whether to keep approximation coefficients (`true`) or not (`false`).
- `IDXSIG` is a vector that contains the indices of the initial signals, or the string `'all'`.

The defaults are, respectively, `'h'`, false and `'all'`.

Examples

```
% Load original 1D-multisignal.
load thinker

% Perform a decomposition at level 2 using wavelet db2.
dec = mdwtdec('r',X,2,'db2');

% Compute compression performances for soft an hard thresholding.
[THR_S,L2SCR_S,NOSCR_S] = mswcmpscr(dec,'s');
[THR_H,L2SCR_H,NOSCR_H] = mswcmpscr(dec,'h');
```

References

Daubechies, I. (1992), *Ten lectures on wavelets*, CBMS-NSF conference series in applied mathematics. SIAM Ed.

Mallat, S. (1989), "A theory for multiresolution signal decomposition: the wavelet representation," *IEEE Pattern Anal. and Machine Intell.*, vol. 11, no. 7, pp. 674–693.

Meyer, Y. (1990), *Ondelettes et opérateurs*, Tome 1, Hermann Ed. (English translation: *Wavelets and operators*, Cambridge Univ. Press. 1993.)

See Also

`mdwtdec` | `mdwtrec` | `ddencmp` | `wdencmp`

Introduced in R2007a

mswcmptp

Multisignal 1-D compression thresholds and performances

Syntax

```
[THR_VAL,L2_Perf,NO_Perf] = mswcmptp(DEC,METH)
[THR_VAL,L2_Perf,NO_Perf] = mswcmptp(DEC,METH,PARAM)
```

Description

`[THR_VAL,L2_Perf,NO_Perf] = mswcmptp(DEC,METH)` or `[THR_VAL,L2_Perf,NO_Perf] = mswcmptp(DEC,METH,PARAM)` computes the vectors `THR_VAL`, `L2_Perf` and `NO_Perf` obtained after a compression using the `METH` method and, if required, the `PARAM` parameter (see `mswcmp` for more information on `METH` and `PARAM`).

For the ith signal:

- `THR_VAL(i)` is the threshold applied to the wavelet coefficients. For a level dependent method, `THR_VAL(i,j)` is the threshold applied to the detail coefficients at level `j`
- `L2_Perf(i)` is the percentage of energy (L2_norm) preserved after compression.
- `NO_Perf(i)` is the percentage of zeros obtained after compression.

You can use three more optional inputs:

```
[...] = mswcmptp(...,S_OR_H,KEEPAPP,IDXSIG)
```

- `S_OR_H ('s' or 'h')` stands for soft or hard thresholding (see `mswthresh` for more details).
- `KEEPAPP (true or false)` indicates whether to keep approximation coefficients (`true`) or not (`false`)
- `IDXSIG` is a vector which contains the indices of the initial signals, or the string `'all'`.

The defaults are, respectively, `'h'`, false and `'all'`.

Examples

```
% Load original 1D-multisignal.
load thinker

% Perform a decomposition at level 2 using wavelet db2.
dec = mdwtdec('r',X,2,'db2');

% Compute compression thresholds and exact performances
% obtained after a compression using the method 'NO_perf' and
% requiring a percentage of zeros near 95% for the wavelet
% coefficients.
[THR_VAL,L2_Perf,NO_Perf] = mswcmptp(dec,'NO_perf',95);
```

References

Daubechies, I. (1992), *Ten lectures on wavelets*, CBMS-NSF conference series in applied mathematics. SIAM Ed.

Mallat, S. (1989), "A theory for multiresolution signal decomposition: the wavelet representation," *IEEE Pattern Anal. and Machine Intell.*, vol. 11, no. 7, pp. 674–693.

Meyer, Y. (1990), *Ondelettes et opérateurs*, Tome 1, Hermann Ed. (English translation: *Wavelets and operators*, Cambridge Univ. Press. 1993.)

See Also

`mdwtdec` | `mdwtrec` | `ddencmp` | `wdencmp`

Introduced in R2007a

mswden

Multisignal 1-D denoising using wavelets

Syntax

```
[XD,DECDEN,THRESH] = mswden('den',...)
[XD,THRESH] = mswden('densig',...)
[DECDEN,THRESH] = mswden('dendec',...)
THRESH = mswden('thr',...)
[...] = mswden(OPTION,DIRDEC,X,WNAME,LEV,METH,PARAM)
[...] = mswden(...,S_OR_H)
[...] = mswden(...,S_OR_H,KEEPAPP)
[...] = mswden(...,S_OR_H,KEEPAPP,IDXSIG)
```

Description

`mswden` computes thresholds and, depending on the selected option, performs denoising of 1-D signals using wavelets.

`[XD,DECDEN,THRESH] = mswden('den',...)` returns a denoised version `XD` of the original multisignal matrix `X`, whose wavelet decomposition structure is `DEC`. The output `XD` is obtained by thresholding the wavelet coefficients, `DECDEN` is the wavelet decomposition associated to `XD` (see `mdwtdec`), and `THRESH` is the matrix of threshold values. The input `METH` is the name of the denoising method and `PARAM` is the associated parameter, if required.

Valid denoising methods `METH` and associated parameters `PARAM` are:

`'rigrsure'`	Principle of Stein's Unbiased Risk
`'heursure'`	Heuristic variant of the first option
`'sqtwolog'`	Universal threshold `sqrt(2*log(.))`
`'minimaxi'`	Minimax thresholding (see `thselect`)

For these methods `PARAM` defines the multiplicative threshold rescaling:

'one'	No rescaling
'sln'	Rescaling using a single estimation of level noise based on first level coefficients
'mln'	Rescaling using a level dependent estimation of level noise

Penalization methods

'penal'	Penal
'penalhi'	Penal high, 2.5 ℜ≤ PARAM ℜ≤ 10
'penalme'	Penal medium, 1.5 ℜ≤ PARAM ℜ≤ 2.5
'penallo'	Penal low, 1 ℜ≤ PARAM ℜ≤ 2

PARAM is a sparsity parameter, and it should be such that: $1 \leq PARAM \leq 10$. For penal method, no control is done.

Manual method

'man_thr'	Manual method

PARAM is an NbSIG-by-NbLEV matrix or NbSIG-by-(NbLEV+1) matrix such that:

- PARAM(i,j) is the threshold for the detail coefficients of level j for the ith signal ($1 \leq j \leq NbLEV$).
- PARAM(i,NbLEV+1) is the threshold for the approximation coefficients for the ith signal (if KEEPAPP is 0).

where NbSIG is the number of signals and NbLEV the number of levels of decomposition.

Instead of the 'den' input OPTION, you can use 'densig', 'dendec' or 'thr' OPTION to select output arguments:

```
[XD,THRESH] = mswden('densig',...) or [DECDEN,THRESH] =
mswden('dendec',...)
```

THRESH = mswden('thr',...) returns the computed thresholds, but denoising is not performed.

The decomposition structure input argument DEC can be replaced by four arguments: DIRDEC, X, WNAME and LEV.

[...] = mswden(OPTION,DIRDEC,X,WNAME,LEV,METH,PARAM) before performing a denoising or computing thresholds, the multisignal matrix X is decomposed at level LEV using the wavelet WNAME, in the direction DIRDEC.

You can use three more optional inputs:

```
[...] = mswden(...,S_OR_H) or
[...] = mswden(...,S_OR_H,KEEPAPP) or
[...] = mswden(...,S_OR_H,KEEPAPP,IDXSIG)
```

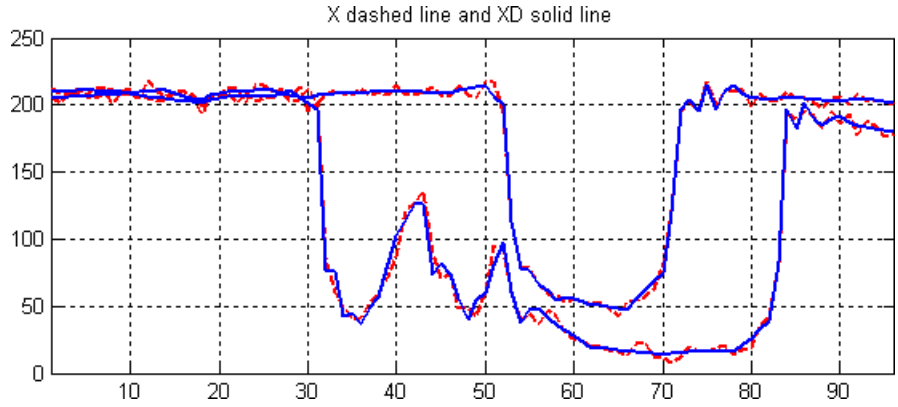
- S_OR_H ('s' or 'h') stands for soft or hard thresholding (see mswthresh for more details).
- KEEPAPP (true or false) indicates whether to keep approximation coefficients (true) or not (false).
- IDXSIG is a vector that contains the indices of the initial signals, or the string 'all'.

The defaults are, respectively, 'h', false and 'all'.

Examples

```
% Load original 1D-multisignal.
load thinker

% Perform a decomposition at level 2 using the wavelet db2.
dec = mdwtdec('r',X,2,'db2');

% Denoise signals using the universal method
% of thresholding (sqtwolog) and the 'sln'
% threshold rescaling (with a single estimation
% of level noise, based on first level coefficients).
[XD,decDEN,THRESH] = mswden('den',dec,'sqtwolog','sln');

% Plot the original signals 1 and 31, and the
% corresponding denoised signals.
figure;
plot(X([1 31],:)','r--','linewidth',2);   hold on
plot(XD([1 31],:)','b','linewidth',2);
```

```
grid; set(gca,'Xlim',[1,96])
title('X dashed line and XD solid line')
```



References

Birgé, L.; P. Massart (1997), "From model selection to adaptive estimation," in D. Pollard (ed), *Festchrift for L. Le Cam*, Springer, pp. 55–88.

DeVore, R.A.; B. Jawerth, B.J. Lucier (1992), "Image compression through wavelet transform coding," *IEEE Trans. on Inf. Theory*, vol. 38, No 2, pp. 719–746.

Donoho, D.L. (1993), "Progress in wavelet analysis and WVD: a ten minute tour," in Progress in wavelet analysis and applications, Y. Meyer, S. Roques, pp. 109–128. Frontières Ed.

Donoho, D.L.; I.M. Johnstone(1994), "Ideal spatial adaptation by wavelet shrinkage," *Biometrika*, vol. 81, pp. 425–455.

Donoho, D.L.; I.M. Johnstone, G. Kerkyacharian, D. Picard (1995), "Wavelet shrinkage: asymptopia," *Jour. Roy. Stat. Soc.,series B*, vol. 57 no. 2, pp. 301–369.

Donoho, D.L.; I.M. Johnstone, "Ideal de-noising in an orthonormal basis chosen from a library of bases," C.R.A.S. Paris, t. 319, Ser. I, pp. 1317–1322.

Donoho, D.L. (1995), "De-noising by soft-thresholding," *IEEE Trans. on Inf. Theory*, 41, 3, pp. 613–627.

See Also

mdwtdec | mdwtrec | mswthresh | wthresh

Introduced in R2007a

mswthresh

Perform multisignal 1-D thresholding

Syntax

```
Y = mswthresh(X,SORH,T)
Y = mswthresh(X,SORH,T,'c')
Y = mswthresh(X,'s',T)
Y = mswthresh(X,'h',T)
```

Description

`Y = mswthresh(X,SORH,T)` returns soft (if `SORH='s'`) or hard (if `SORH='h'`) `T`-thresholding of the input matrix `X`. `T` can be a single value, a matrix of the same size as `X` or a vector. In this last case, thresholding is performed rowwise and `LT = length(T)` must be such that `size(X,1)` ≤ `LT`.

`Y = mswthresh(X,SORH,T,'c')` performs a columnwise thresholding and `size(X,2)` ≤ `LT`.

`Y = mswthresh(X,'s',T)` returns `Y = SIGN(X).(|X|-T)+`, soft thresholding is shrinkage.

`Y = mswthresh(X,'h',T)` returns `Y = X.1_(|X|>T)`, hard thresholding is cruder.

See Also

`mswden` | `mswcmp` | `wthresh` | `wden` | `wdencmp` | `wpdencmp`

Introduced in R2007a

nodeasc

Node ascendants

Syntax

```
A = nodeasc(T,N)
```

Description

`nodeasc` is a tree-management utility.

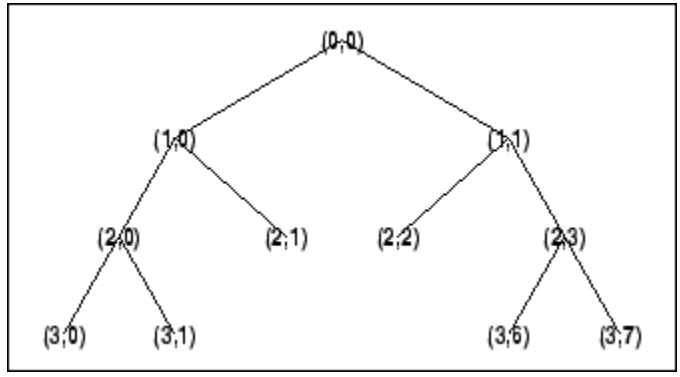
`A = nodeasc(T,N)` returns the indices of all the ascendants of the node *N* in the tree *T* where `N` can be the index node or the depth and position of the node. `A` is a column vector with `A`(1) = index of node *N*.

`A = nodeasc(T,N,'deppos')` is a matrix, which contains the depths and positions of all ascendants. `A(i,1)` is the depth of the `i`-th ascendant and `A(i,2)` is the position of the `i`-th ascendant.

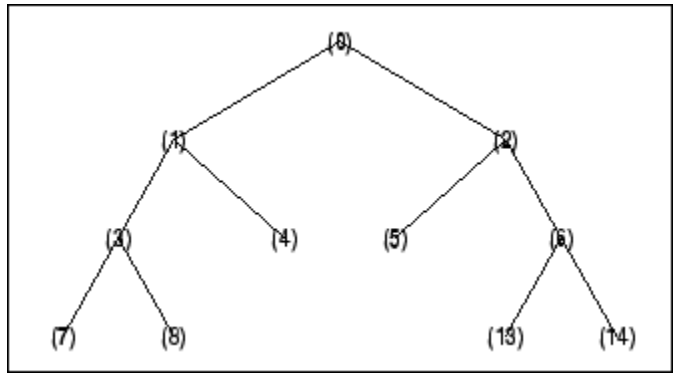
The nodes are numbered from left to right and from top to bottom. The root index is 0.

Examples

```
% Create binary tree of depth 3.
t = ntree(2,3);
t = nodejoin(t,5);
t = nodejoin(t,4);
plot(t)
```




```
% Change Node Label from Depth_Position to Index
% (see the plot function).
```




```
nodeasc(t,[2 2])
ans =
    5
    2
    0

nodeasc(t,[2 2],'deppos')
ans =
    2     2
    1     1
    0     0
```

See Also

`nodedesc` | `nodepar` | `wtreemgr`

Introduced before R2006a

nodedesc

Node descendants

Syntax

```
D = nodedesc(T,N)
D = nodedesc(T,N,'deppos')
```

Description

`nodedesc` is a tree-management utility.

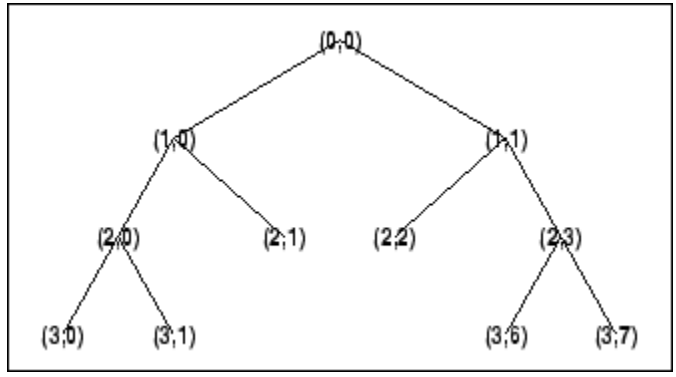
`D = nodedesc(T,N)` returns the indices of all the descendants of the node *N* in the tree *T* where *N* can be the index node or the depth and position of node. `D` is a column vector with `D(1)` = index of node *N*.

`D = nodedesc(T,N,'deppos')` is a matrix that contains the depths and positions of all descendants. `D(i,1)` is the depth of the `i`-th descendant and `D(i,2)` is the position of the `i`-th descendant.

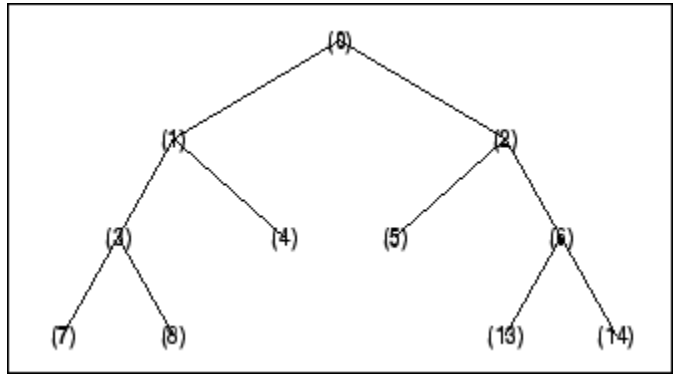
The nodes are numbered from left to right and from top to bottom. The root index is 0.

Examples

```
% Create binary tree of depth 3.
t = ntree(2,3);
t = nodejoin(t,5);
t = nodejoin(t,4);
plot(t)
```




```
% Change Node Label from Depth_Position to Index
% (see the plot function).
```




```
% Node descendants.
nodedesc(t,2)
ans =
     2
     5
     6
    13
    14

nodedesc(t,2,'deppos')
ans =
     1     1
     2     2
```

```
    2     3
    3     6
    3     7

nodedesc(t,[1 1],'deppos')
ans =
    1     1
    2     2
    2     3
    3     6
    3     7

nodedesc(t,[1 1])
ans =
    2
    5
    6
   13
   14
```

See Also

`nodeasc` | `nodepar` | `wtreemgr`

Introduced before R2006a

nodejoin

Recompose node

Syntax

```
T = nodejoin(T,N)
T = nodejoin(T)
T = nodejoin(T,0)
```

Description

`nodejoin` is a tree-management utility.

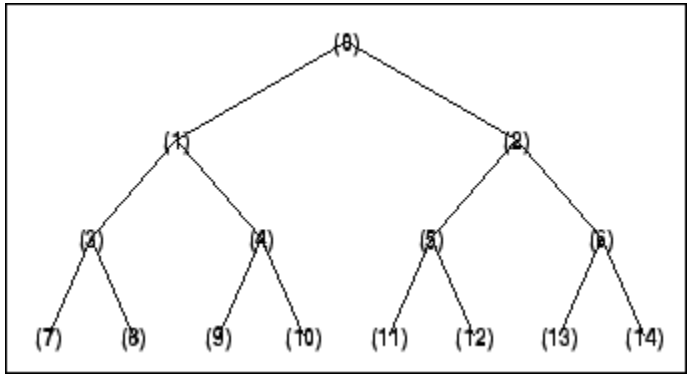
`T = nodejoin(T,N)` returns the modified tree *T* corresponding to a recomposition of the node *N*.

The nodes are numbered from left to right and from top to bottom. The root index is 0.

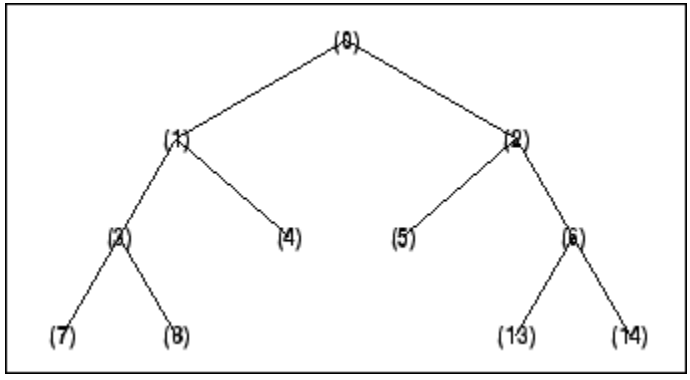
`T = nodejoin(T)` is equivalent to `T = nodejoin(T,0)`.

Examples

```
% Create binary tree of depth 3.
t = ntree(2,3);

% Plot tree t.
plot(t)

% Change Node Label from Depth_Position to Index
% (see the plot function).
```




```
% Merge nodes of indices 4 and 5.
t = nodejoin(t,5);
t = nodejoin(t,4);
% Plot new tree t.
plot(t)

% Change Node Label from Depth_Position to Index
% (see the plot function).
```



See Also

nodesplt

Introduced before R2006a

nodepar

Node parent

Syntax

```
F = nodepar(T,N)
F = nodepar(T,N,'deppos')
```

Description

`nodepar` is a tree-management utility.

`F = nodepar(T,N)` returns the indices of the Äúparent(s)Äù of the nodes *N* in the tree *T* where *N* can be a column vector containing the indices of nodes or a matrix that contains the depths and positions of nodes. In the last case, `N(i,1)` is the depth of the `i`-th node and `N(i,2)` is the position of the `i`-th node.

`F = nodepar(T,N,'deppos')` is a matrix that contains the depths and positions of returned nodes. `F(i,1)` is the depth of the `i`-th node and `F(i,2)` is the position of the `i`-th node.

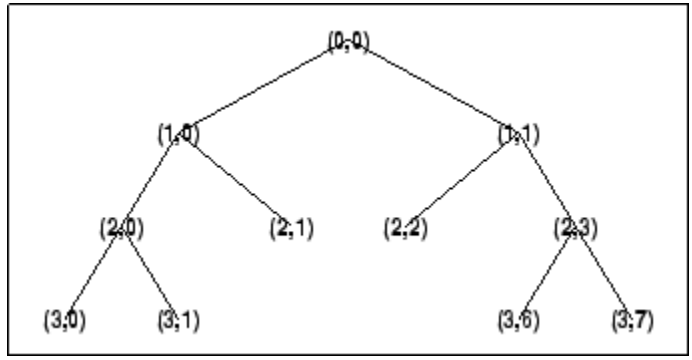
`nodepar(T,0)` or `nodepar(T,[0,0])` returns `-1`.

`nodepar(T,0,'deppos')` or `nodepar(T,[0,0],'deppos')` returns `[-1,0]`.

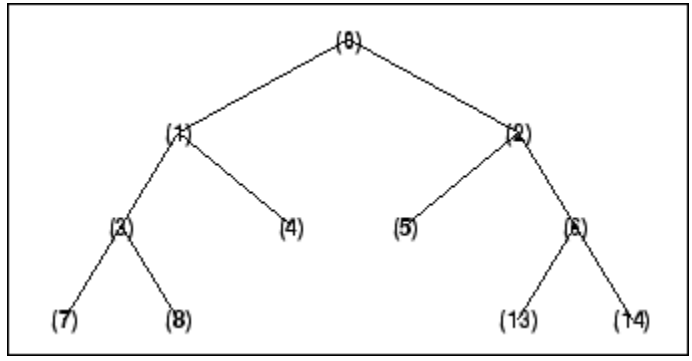
The nodes are numbered from left to right and from top to bottom. The root index is 0.

Examples

```
% Create binary tree of depth 3.
t = ntree(2,3);
t = nodejoin(t,5);
t = nodejoin(t,4);
plot(t)
```




```
% Change Node Label from Depth_Position to Index
% (see the plot function).
```




```
% Nodes parent.
nodepar(t,[2 2],'deppos')

ans =
    1     1

nodepar(t,[1;7;14])

ans =
    0
    3
    6
```

See Also

nodeasc | nodedesc | wtreemgr

Introduced before R2006a

nodesplt

Split (decompose) node

Syntax

```
T = nodesplt(T,N)
```

Description

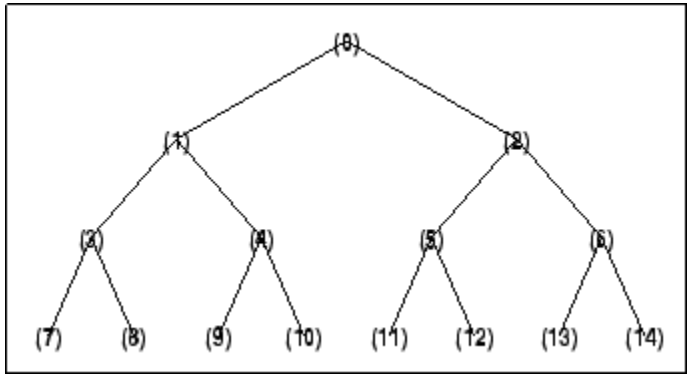
`nodesplt` is a tree-management utility.

`T = nodesplt(T,N)` returns the modified tree *T* corresponding to the decomposition of the node *N*.

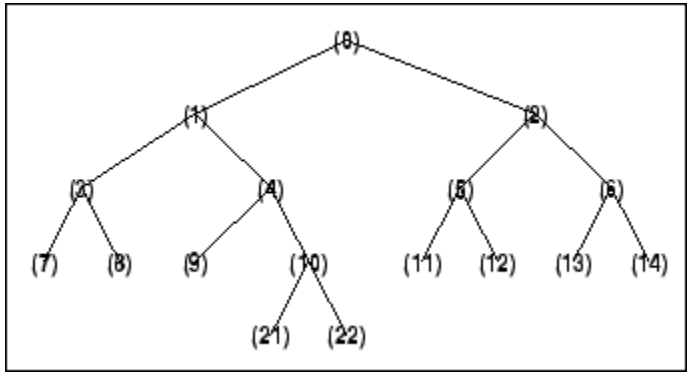
The nodes are numbered from left to right and from top to bottom. The root index is 0.

Examples

```
% Create binary tree (tree of order 2) of depth 3.
t = ntree(2,3);

% Plot tree t.
plot(t)

% Change Node Label from Depth_Position to Index
% (see the plot function).
```




```
% Split node of index 10.
t = nodesplt(t,10);

% Plot new tree t.
plot(t)
% Change Node Label from Depth_Position to Index
% (see the plot function).
```



See Also

nodejoin

Introduced before R2006a

noleaves

Determine nonterminal nodes

Syntax

```
N = noleaves(T)
N = noleaves(T,'dp')
```

Description

`N = noleaves(T)` returns the indices of nonterminal nodes of the tree *T* (i.e., nodes that are not leaves). `N` is a column vector.

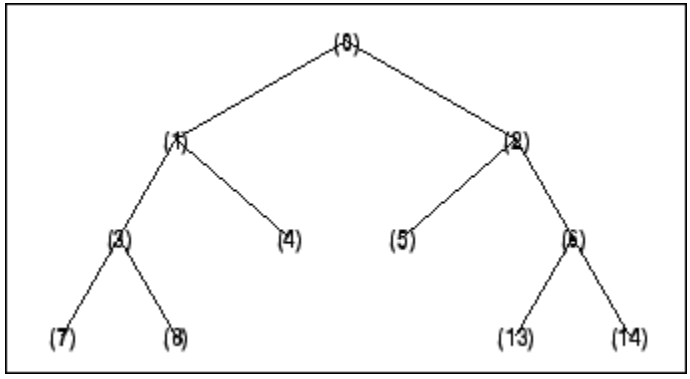
The nodes are ordered from left to right as in tree *T*.

`N = noleaves(T,'dp')` returns a matrix `N`, which contains the depths and positions of nonterminal nodes.

`N(i,1)` is the depth of the `i`-th nonterminal node and
`N(i,2)` is the position of the `i`-th nonterminal node.

Examples

```
% Create initial tree.
ord = 2;
t = ntree(ord,3);        % binary tree of depth 3.
t=nodejoin(t,5);
t=nodejoin(t,4);
plot(t)

% Change Node Label from Depth_Position to Index
% (see the plot function).
```




```
% List nonterminal nodes (index).
ntnodes_ind = noleaves(t)

ntnodes_ind =
     0
     1
     2
     3
     6

% List nonterminal nodes (Depth_Position).
ntnodes_depo = noleaves(t,'dp')

ntnodes_depo =
     0     0
     1     0
     1     1
     2     0
     2     3
```

See Also

`leaves`

Introduced before R2006a

ntnode

Number of terminal nodes

Syntax

```
NB = ntnode(T)
```

Description

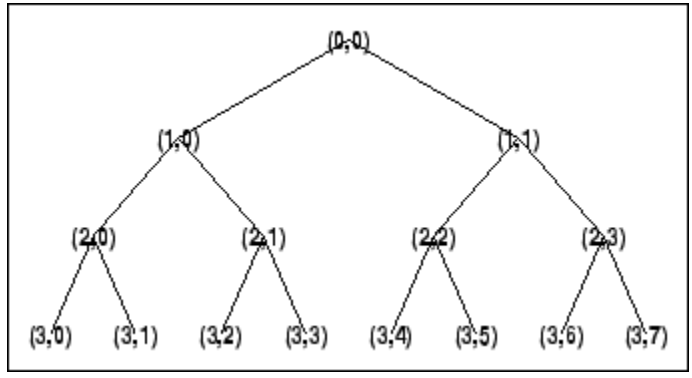
`ntnode` is a tree-management utility.

`NB = ntnode(T)` returns the number of terminal nodes in the tree *T*.

The nodes are numbered from left to right and from top to bottom. The root index is 0.

Examples

```
% Create binary tree (tree of order 2) of depth 3.
t = ntree(2,3);

% Plot tree t.
plot(t)
```



```
% Number of terminal nodes.
ntnode(t)

ans =
    8
```

See Also

wtreemgr

Introduced before R2006a

ntree

NTREE constructor

Syntax

```
T = ntree(ORD,D)
T = ntree
T = ntree(2,0)
T = ntree(ORD)
T = ntree(ORD,0)
T = ntree(ORD,D,S)
T = ntree(ORD,D,S,U)
```

Description

`T = ntree(ORD,D)` returns an NTREE object, which is a complete tree of order `ORD` and depth `D`.

`T = ntree` is equivalent to `T = ntree(2,0)`.

`T = ntree(ORD)` is equivalent to `T = ntree(ORD,0)`.

With `T = ntree(ORD,D,S)` you can set a "split scheme" for nodes. The split scheme field `S` is a logical array of size `ORD` by 1.

The root of the tree can be split and it has `ORD` children. You can split the `j`-th child if `S(j) = 1`.

Each node that you can split has the same property as the root node.

With `T = ntree(ORD,D,S,U)` you can, in addition, set a userdata field.

Inputs can be given in another way:

`T = ntree('order',ORD,'depth',D,'spsch',S,'ud',U)`. For "missing" inputs the defaults are `ORD = 2` and `D = 0 , S = ones([1:ORD]) , U = {}`.

`[T,NB] = ntree( ... )` returns also the number of terminal nodes (leaves) of `T`.

For more information on object fields, type `help ntree/get`.

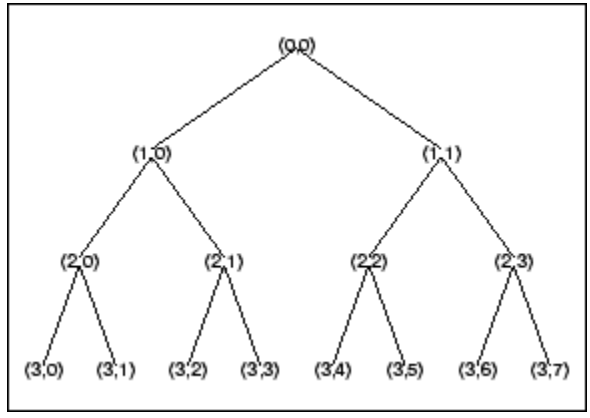
Class NTREE (Parent class: WTBO)

Fields

`wtbo`	Parent object
`order`	Tree order
`depth`	Tree depth
`spsch`	Split scheme for nodes
`tn`	Column vector with terminal node indices

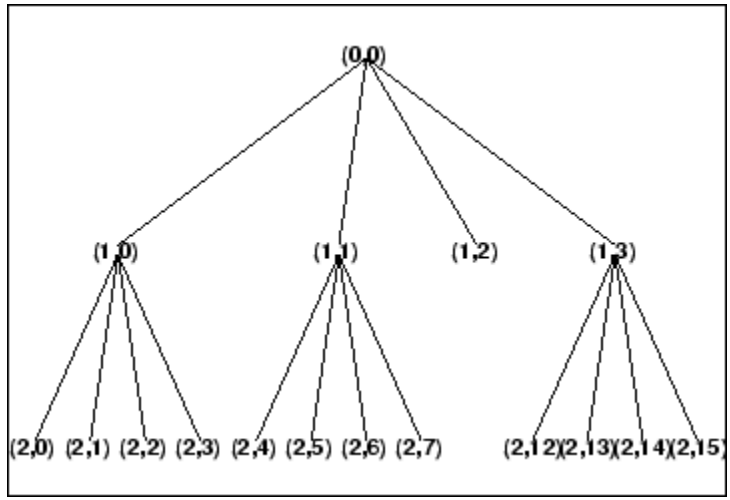
Examples

```
% Create binary tree (tree of order 2) of depth 3.
t2 = ntree(2,3);

% Plot tree t2.
plot(t2)
```

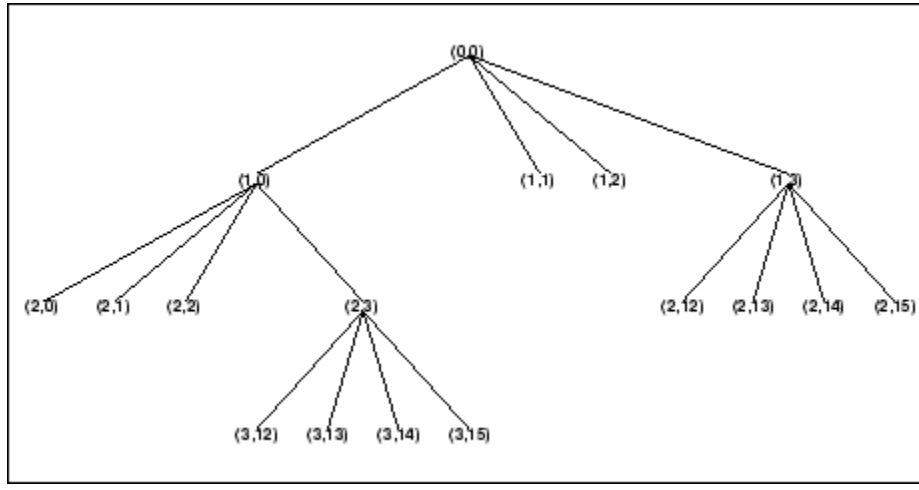



```
% Create a quadtree (tree of order 4) of depth 2.
```

```
t4 = ntree(4,2,[1 1 0 1]);

% Plot tree t4.
plot(t4)
```




```
% Split and merge some nodes using the gui
% generated by plot (see the plot function).
% The figure becomes:
```



See Also

`wtbo`

Introduced before R2006a

orthfilt

Orthogonal wavelet filter set

Syntax

```
[Lo_D,Hi_D,Lo_R,Hi_R] = orthfilt(W)
```

Description

`[Lo_D,Hi_D,Lo_R,Hi_R] = orthfilt(W)` computes the four filters associated with the scaling filter `W` corresponding to a wavelet:

`Lo_D`	Decomposition low-pass filter
`Hi_D`	Decomposition high-pass filter
`Lo_R`	Reconstruction low-pass filter
`Hi_R`	Reconstruction high-pass filter

For an orthogonal wavelet, in the multiresolution framework, we start with the scaling function ϕ and the wavelet function ψ. One of the fundamental relations is the twin-scale relation:

$$\frac{1}{2}\phi\left(\frac{x}{2}\right) = \sum_{n \in Z} w_n \phi(x-n)$$

All the filters used in `dwt` and `idwt` are intimately related to the sequence $(w_n)_{n \in Z}$. Clearly if ϕ is compactly supported, the sequence (w_n) is finite and can be viewed as a FIR filter. The scaling filter `W` is

- A low-pass FIR filter
- Of length $2N$
- Of sum 1

- Of norm $\frac{1}{\sqrt{2}}$

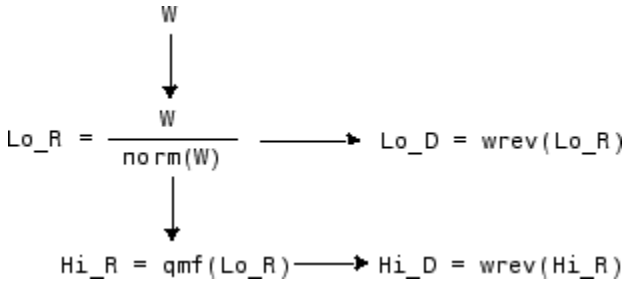
For example, for the `db3` scaling filter,

```
load db3
db3
db3 =
    0.2352 0.5706 0.3252 -0.0955 -0.0604 0.0249

sum(db3)
ans =
    1.000
    norm(db3)
ans =
    0.7071
```

From filter *W*, we define four FIR filters, of length 2N and norm 1, organized as follows:

Filters	Low-Pass	High-Pass
Decomposition	`Lo_D`	`Hi_D`
Reconstruction	`Lo_R`	`Hi_R`

The four filters are computed using the following scheme:

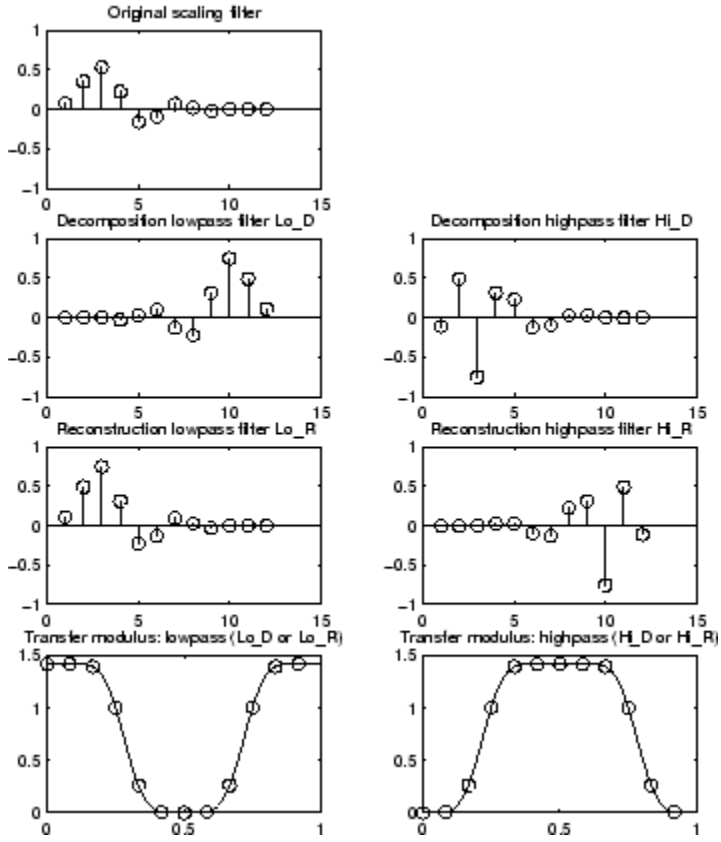


where `qmf` is such that `Hi_R` and `Lo_R` are quadrature mirror filters (i.e., `Hi_R(k) = (-1)^k Lo_R(2N + 1 - k)`, for `k = 1, 2, Ä, 2N`), and where `wrev` flips the filter coefficients. So `Hi_D` and `Lo_D` are also quadrature mirror filters. The computation of these filters is performed using `orthfilt`.

Examples

```
% Load scaling filter.
load db8; w = db8;
subplot(421); stem(w);
title('Original scaling filter');

% Compute the four filters.
[Lo_D,Hi_D,Lo_R,Hi_R] = orthfilt(w);
subplot(423); stem(Lo_D);
title('Decomposition low-pass filter');
subplot(424); stem(Hi_D);
title('Decomposition high-pass filter');
subplot(425); stem(Lo_R);
title('Reconstruction low-pass filter');
subplot(426); stem(Hi_R);
title('Reconstruction high-pass filter');

% Check for orthonormality.
df = [Lo_D;Hi_D];
rf = [Lo_R;Hi_R];
id = df*df'

id =
    1.0000         0
         0    1.0000

id = rf*rf'

id =
    1.0000         0
         0    1.0000

% Check for orthogonality by dyadic translation, for example:
df = [Lo_D 0 0;Hi_D 0 0];
dft = [0 0 Lo_D; 0 0 Hi_D];
zer = df*dft'

zer =

    1.0e-12 *
    -0.1883 0.0000
    -0.0000 -0.1883
```

```
% High- and low-frequency illustration.
fftld = fft(Lo_D); ffthd = fft(Hi_D);
freq = [1:length(Lo_D)]/length(Lo_D);
subplot(427); plot(freq,abs(fftld));
title('Transfer modulus: low-pass');
subplot(428); plot(freq,abs(ffthd));
title('Transfer modulus: high-pass')
% Editing some graphical properties,
% the following figure is generated.
```



References

Daubechies, I. (1992), *Ten lectures on wavelets*, CBMS-NSF conference series in applied mathematics, SIAM Ed. pp. 117–119, 137, 152.

See Also

`biorfilt` | `qmf` | `wfilters`

Introduced before R2006a

otnodes

Order terminal nodes of binary wavelet packet tree

Syntax

```
[Tn_Pal,Tn_Seq] = otnodes(WPT)
[Tn_Pal,Tn_Seq,I,J] = otnodes(WPT)
[DP_Pal,DP_Seq] = otnodes(WPT,'dp')
```

Description

`[Tn_Pal,Tn_Seq] = otnodes(WPT)` returns the terminal nodes of the binary wavelet packet tree, `WPT`, in Paley (natural) ordering, `Tn_Pal`, and sequency (frequency) ordering, `Tn_Seq`. `Tn_Pal` and `Tn_Seq` are *N*-by-1 column vectors where *N* is the number of terminal nodes.

`[Tn_Pal,Tn_Seq,I,J] = otnodes(WPT)` returns the permutations of the terminal node indices such that `Tn_Seq = Tn_Pal(I)` and `Tn_Pal = Tn_Seq(J)`.

`[DP_Pal,DP_Seq] = otnodes(WPT,'dp')` returns the Paley and frequency-ordered terminal nodes in node depth-position format. `DP_Pal` and `DP_Seq` are *N*-by-2 matrices. The first column contains the depth index, and the second column contains the position index.

Input Arguments

`WPT`

Binary wavelet packet tree. You can use `treeord` to determine the order of your wavelet packet tree.

`dp`

String variable indicating that the Paley-ordered or sequency-ordered nodes are returned in depth-position format.

Output Arguments

`Tn_Pal`

Terminal nodes in Paley (natural) ordering

`Tn_Seq`

Terminal nodes in sequency ordering

`DP_Pal`

Paley-ordered terminal nodes in depth-position format. This output argument only applies when you use the `'dp'` input argument.

`DP_Seq`

Sequency-ordered terminal nodes in depth-position format. This output argument only applies when you use the `'dp'` input argument.

Examples

Order terminal nodes with Paley and frequency ordering:

```
x = randn(8,1);
wpt = wpdec(x,2,'haar');
[Tn_Pal,Tn_Seq] = otnodes(wpt);
% Tn_Pal is [3 4 5 6]
% Tn_Seq is [3 4 6 5]
```

Return permutations for Paley and frequency ordering:

```
load noisdopp;
wpt = wpdec(noisdopp,6,'sym4');
[Tn_Pal,Tn_Seq,I,J] = otnodes(wpt);
isequal(Tn_Seq(J),Tn_Pal)
isequal(Tn_Seq,Tn_Pal(I))
```

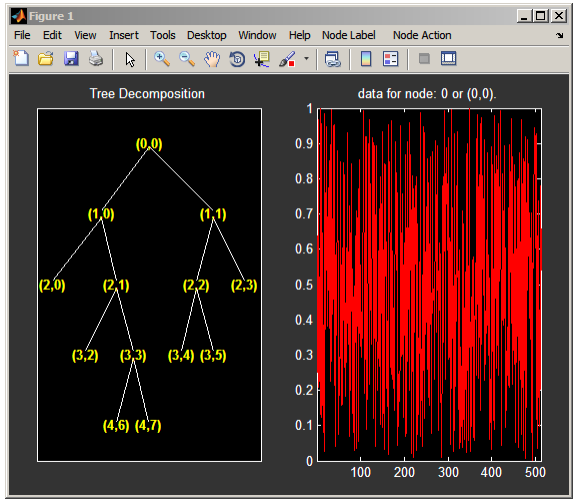
Order terminal nodes by depth and position:

```
x = randn(8,1);
```

```
wpt = wpdec(x,2,'haar');
[DP_Pal,DP_Seq] = otnodes(wpt,'dp');
```

Order terminal nodes from a modified wavelet packet tree:

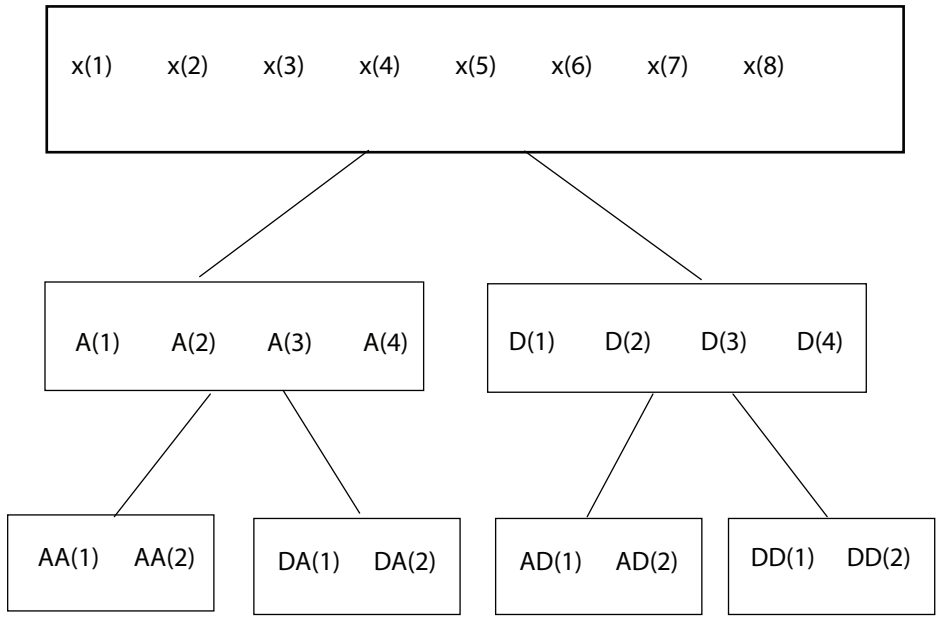
```
t = wptree(2,2,rand(1,512),'haar');
t = wpsplt(t,4);
t = wpsplt(t,5);
t = wpsplt(t,10);
plot(t);
[tn_Pal,tn_Seq,I,J] = otnodes(t);
```



More About

Paley (Natural) and Sequency (Frequency) Ordering

The discrete wavelet packet transform iterates on both approximation and detail coefficients at each level. In this transform, *A* denotes the lowpass (approximation) filter followed by downsampling. *D* denotes the highpass (detail) filter followed by downsampling. The following figure represents a wavelet packet transform in Paley ordering acting on a time series of length 8. The transform has a depth of two.



Because of aliasing introduced by downsampling, the frequency content extracted by the operator *AD* is higher than the frequency content extracted by the *DD* operator. Therefore, the terminal nodes in frequency (sequency) order are: *AA,DA,DD,AD*. The terminal nodes in Paley order have the following indices: 3,4,5,6. The frequency order has the indices: 3,4,6,5.

References

Wickerhauser, M.V. *Lectures on Wavelet Packet Algorithms*, Technical Report, Washington University, Department of Mathematics, 1992.

See Also

`leaves` | `treeord`

Introduced in R2010b

pat2cwav

Build wavelet from pattern

Syntax

```
[PSI,XVAL,NC] = pat2cwav(YPAT,METHOD,POLDEGREE,REGULARITY)
```

Description

`[PSI,XVAL,NC] = pat2cwav(YPAT,METHOD,POLDEGREE,REGULARITY)` computes an admissible wavelet for CWT (given by `XVAL` and `PSI`) adapted to the pattern defined by the vector `YPAT`, and of norm equal to 1.

The underlying x-values pattern is set to

```
xpat = linspace(0,1,length(YPAT))
```

The constant `NC` is such that `NC*PSI` approximates `YPAT` on the interval `[0,1]` by least squares fitting using

- a polynomial of degree `POLDEGREE` when `METHOD` is equal to `'polynomial'`
- a projection on the space of functions orthogonal to constants when `METHOD` is equal to `'orthconst'`

The `REGULARITY` parameter defines the boundary constraints at the points 0 and 1. Allowable values are `'continuous'`, `'differentiable'`, and `'none'`.

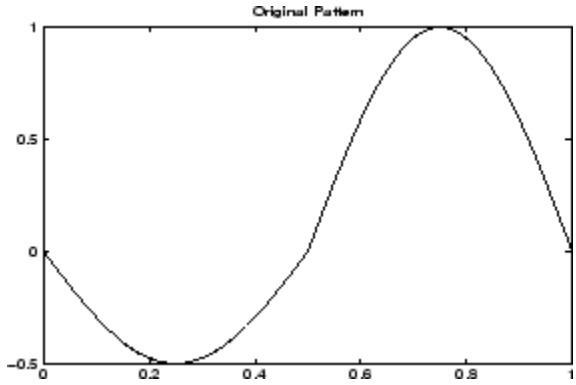
When `METHOD` is equal to `'polynomial'`

- if `REGULARITY` is equal to `'continuous'`, `POLDEGREE` must be greater than or equal to 3.
- if `REGULARITY` is equal to `'differentiable'`, `POLDEGREE` must be greater than or equal to 5.

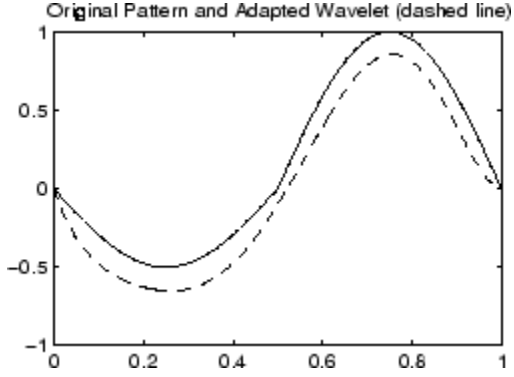
Examples

The principle for designing a new wavelet for CWT is to approximate a given pattern using least squares optimization under constraints leading to an admissible wavelet well suited for the pattern detection using the continuous wavelet transform (see Misiti et al.).

```
load ptpssin1;
plot(X,Y), title('Original Pattern')
```




```
[psi,xval,nc] = pat2cwav(Y, 'polynomial',6, 'continuous') ;
plot(X,Y,'-',xval,nc*psi,'--'),
title('Original Pattern and Adapted Wavelet (dashed line)')
```



You can check that `psi` satisfies the definition of a wavelet by noting that it integrates to zero and that its L_2 norm is equal to 1.

```
dx = xval(2)-xval(1);
Mu = sum(psi*dx)
L2norm = sum(abs(psi).^2*dx)
```

References

Misiti, M., Y. Misiti, G. Oppenheim, J.-M. Poggi (2003), “Les ondelettes et leurs applications,” Hermes.

Introduced before R2006a

plot

Plot tree GUI

Syntax

```
plot(T)
plot(T,FIG)
```

Description

`plot` is a graphical tree-management utility.

`plot(T)` plots the tree *T*.

The figure that contains the tree is a GUI tool. It lets you change the **Node Label** to **Depth_Position** or **Index**, and **Node Action** to **Split-Merge** or **Visualize**.

The default values are **Depth_Position** and **Visualize**.

You can click the nodes to execute the current **Node Action**.

`plot(T,FIG)` plots the tree *T* in the figure whose handle is `FIG`. This figure was already used to plot a tree, for example using the command

```
FIG = plot(T)
```

After some split or merge actions, you can get the new tree using its parent figure handle. The following syntax lets you perform this functionality:

```
NEWT = plot(T,'read',FIG)
```

In fact, the first argument is dummy. The most general syntax is

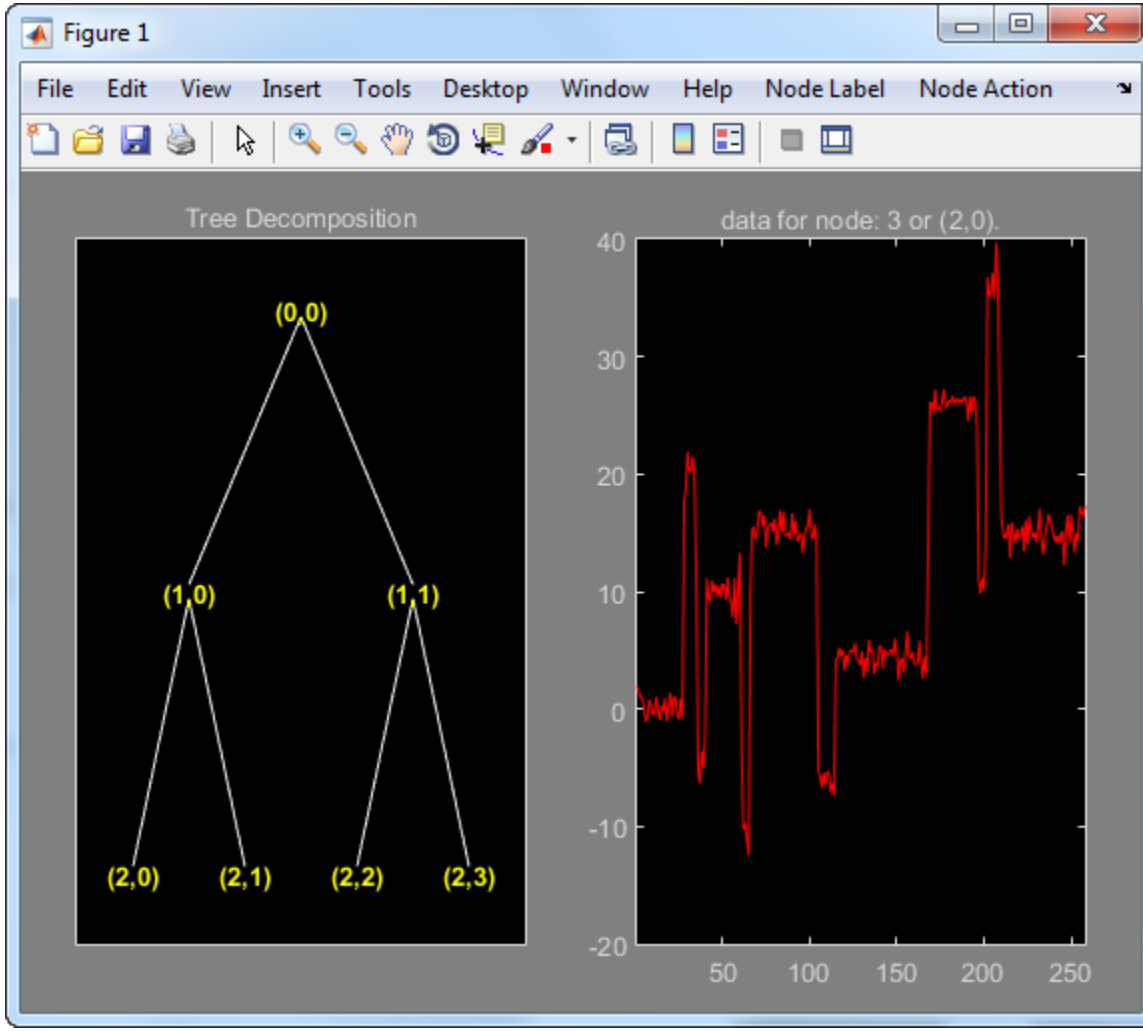
```
NEWT = plot(DUMMY,'read',FIG)
```

where `DUMMY` is any object parented by an NTREE object. More generally, `DUMMY` can be any object constructor name returning an NTREE parented object. For example:

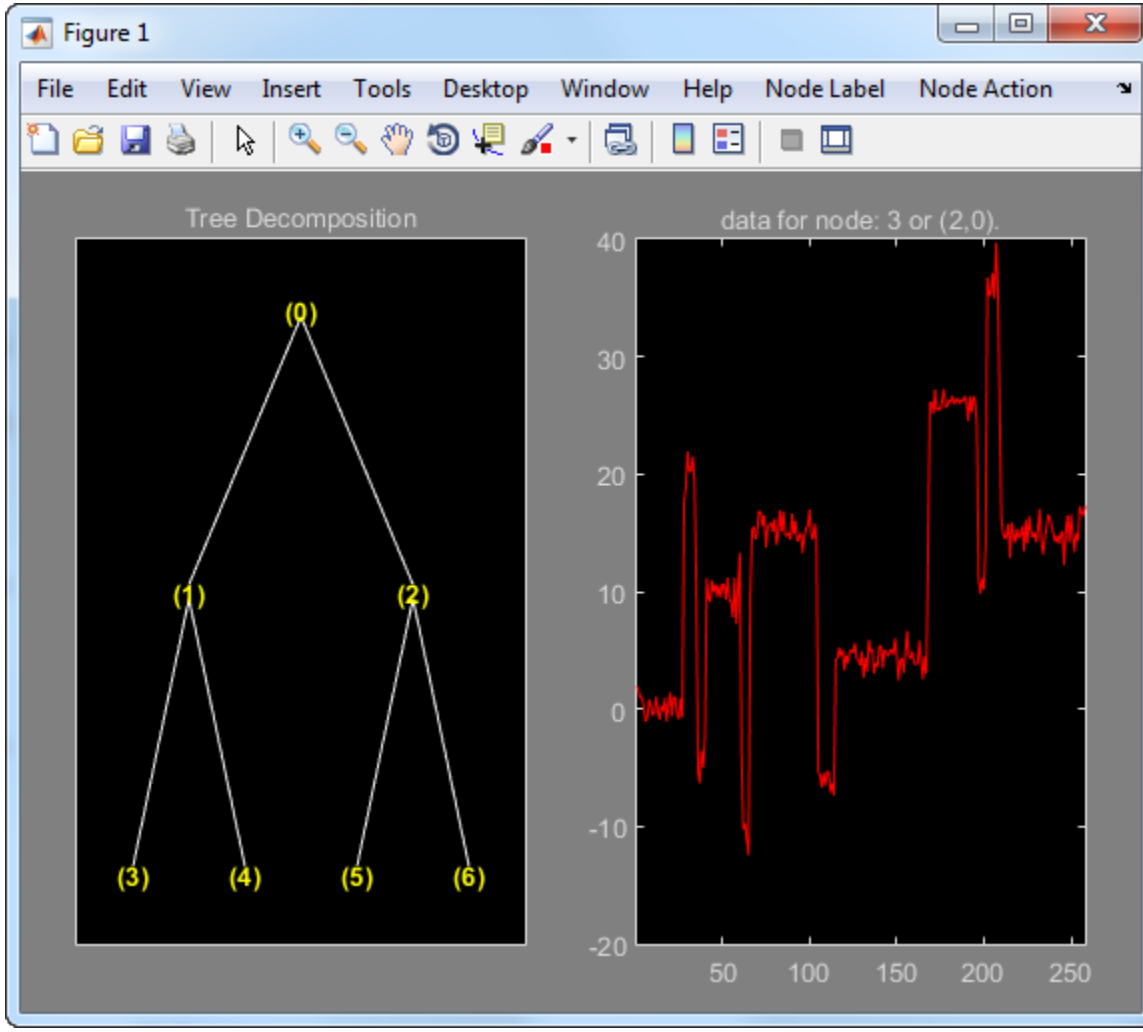
```
NEWT = plot(ntree,'read',FIG)
NEWT = plot(dtree,'read',FIG)
NEWT = plot(wptree,'read',FIG)
```

Examples

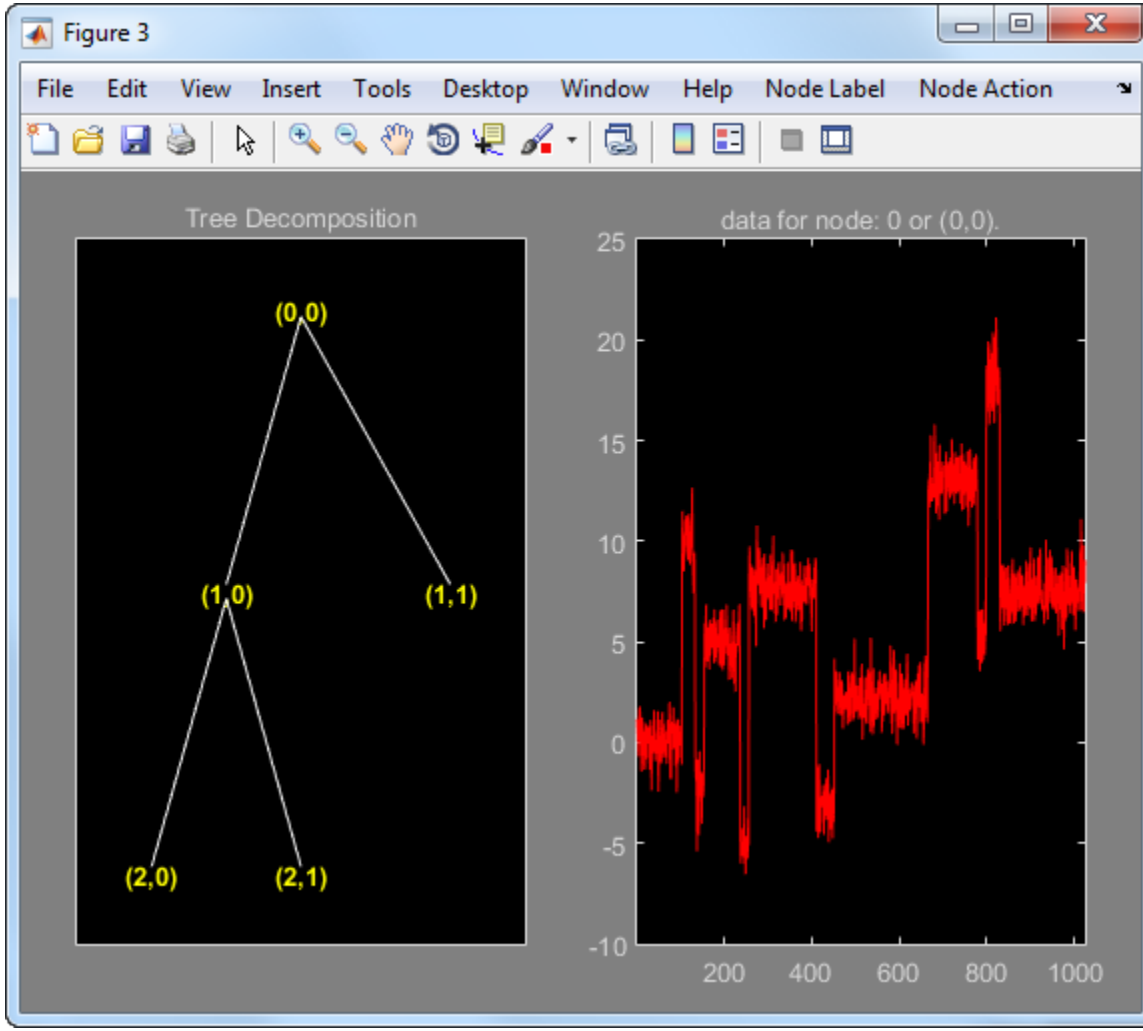
```
% Create a wavelet packets tree (1-D)
load noisbloc
x = noisbloc;
t = wpdec(x,2,'db2');

% Plot tree t.
plot(t)
```




```
% Change Node Label from Depth_Position to Index.

% Click the node (3). You get the following figure.
```



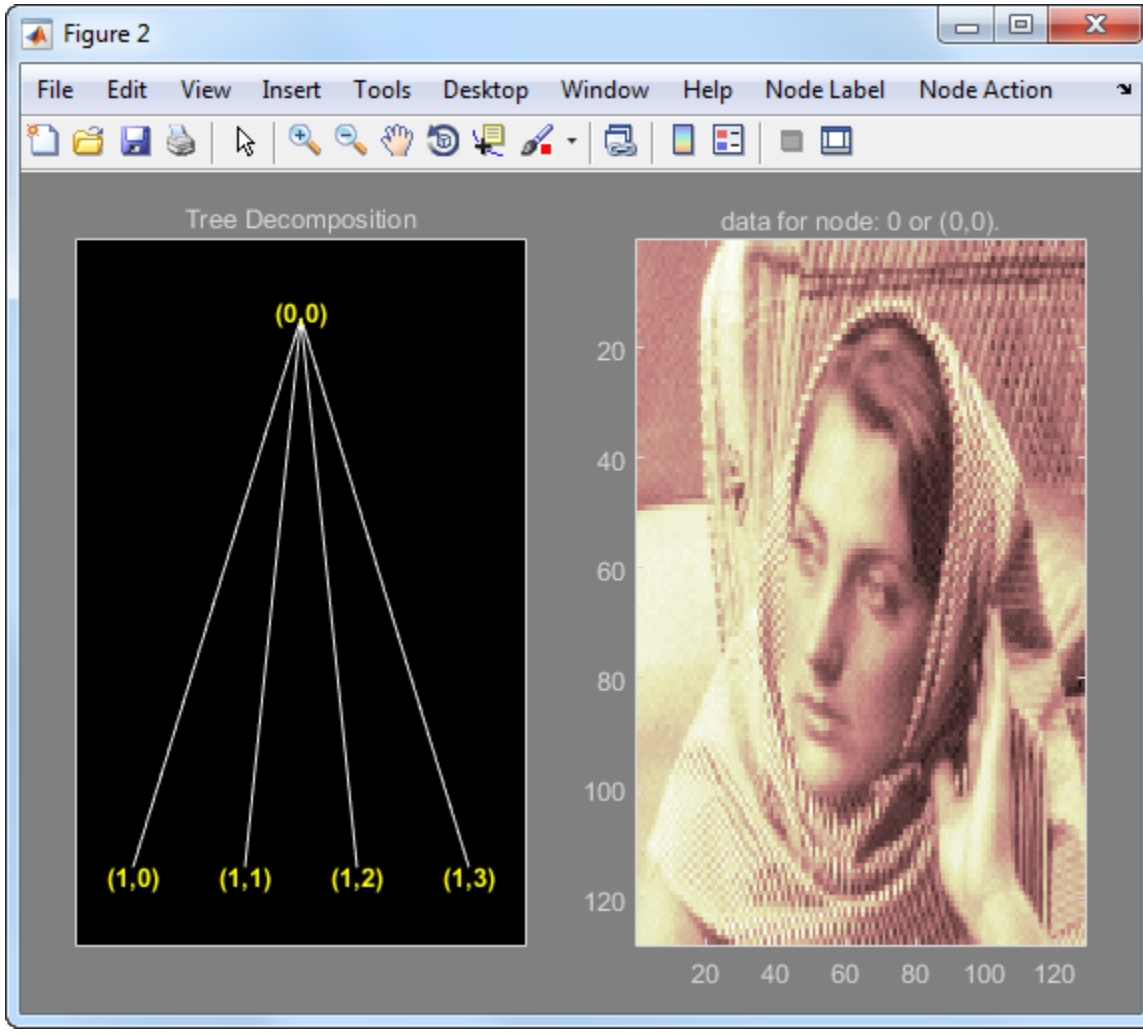
Now set the **Node Label** back to **Depth_Position**. Change **Node Action** to **Split-Merge**. Click on the `(1,1)` node.



The above figure now shows the discrete wavelet transform down to level 2.

```
% Create a wavelet packets tree (2-D)
load woman2
t = wpdec2(X,1,'sym4');

% Plot tree t.
plot(t)
```

```
% Change Node Label from Depth_Position to Index.
% Click the node (1). You get the following figure.
```



Introduced before R2006a

plotdt

Plot dual-tree or double-density wavelet transform

Syntax

```
plotdt(wt)
```

Description

`plotdt(wt)` plots the coefficients of the 1-D or 2-D wavelet filter bank decomposition, `wt`.

Examples

Plot Complex Dual-Tree Wavelet Transform of 1-D Signal

Plot the complex dual-tree wavelet transform of the noisy Doppler signal.

Load the noisy Doppler signal. Obtain the complex dual-tree wavelet transform down to level 4.

```
load noisdopp;
wt = dddtree('cplxdt',noisdopp,4,'dtf1');
```

Plot the coefficients.

```
plotdt(wt)
```

Plot Complex Oriented Dual-Tree Wavelet Transform of 2-D Image

Plot the complex oriented dual-tree wavelet transform of an image.

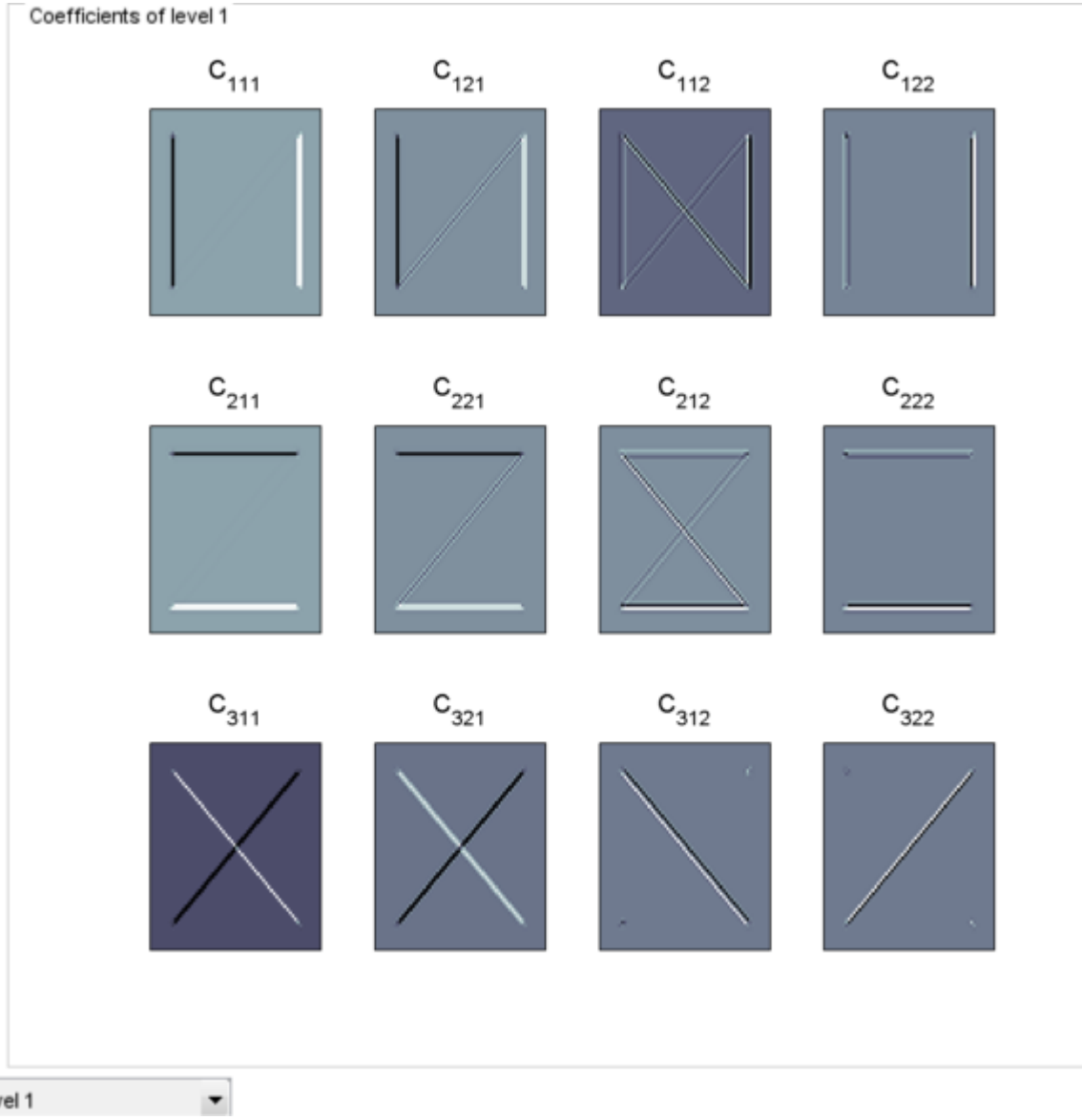
Load the “xbox” image. Obtain the complex oriented dual-tree wavelet transform down to level 3.

```
load xbox;
wt = dddtree2('cplxdt',xbox,3,'dtf1');
```

Plot the coefficients.

```
plotdt(wt)
```

Select the level-one detail coefficients from the drop-down list in the lower left corner.



- “Analytic Wavelets Using the Dual-Tree Wavelet Transform”

Input Arguments

`wt` — Wavelet transform

structure

Wavelet transform, returned as a structure from `dddtree` or `dddtree2` with these fields:

`type` — Type of wavelet decomposition (filter bank)

`'dwt' | 'ddt' | 'realdt' | 'cplxdt' | 'realdddt' | 'cplxdddt'`

Type of wavelet decomposition (filter bank), specified as one of `'dwt'`, `'ddt'`, `'realdt'`, `'cplxdt'`,, `'realdddt'`, or `'cplxdddt'`. `'realdt'` and `'realdddt'` are only valid for the 2-D wavelet transform. The type, `'dwt'`, is a critically sampled (nonredundant) discrete wavelet transform for 1-D data or 2-D images. The other decomposition types are oversampled wavelet transforms. For details about transform types see `dddtree` for 1-D wavelet transforms and `dddtree2` for 2-D wavelet transforms.

`level` — Level of the wavelet decomposition

positive integer

Level of the wavelet decomposition, specified as a positive integer.

`filters` — Decomposition (analysis) and reconstruction (synthesis) filters

structure

Decomposition (analysis) and reconstruction (synthesis) filters, specified as a structure with these fields:

`Fdf` — First-stage analysis filters

matrix | cell array

First level decomposition filters specified as an *N*-by-2 or *N*-by-3 matrix for single-tree wavelet transforms, or a 1-by-2 cell array of two *N*-by-2 or *N*-by-3 matrices for dual-tree wavelet transforms. The matrices are *N*-by-3 for the double-density wavelet transforms. For an *N*-by-2 matrix, the first column of the matrix is the scaling (lowpass) filter and the second column is the wavelet (highpass) filter. For an *N*-by-3 matrix, the first column of the matrix is the scaling (lowpass) filter and the second and third columns are the wavelet (highpass) filters. For the dual-tree transforms, each element of the cell array contains the first-stage analysis filters for the corresponding tree.

`Df` — Analysis filters for levels > 1

matrix | cell array

Analysis filters for levels > 1, specified as an *N*-by-2 or *N*-by-3 matrix for single-tree wavelet transforms, or a 1-by-2 cell array of two *N*-by-2 or *N*-by-3 matrices for dual-tree wavelet transforms. The matrices are *N*-by-3 for the double-density wavelet transforms. For an *N*-by-2 matrix, the first column of the matrix is the scaling (lowpass) filter and the second column is the wavelet (highpass) filter. For an *N*-by-3 matrix, the first column of the matrix is the scaling (lowpass) filter and the second and third columns are the wavelet (highpass) filters. For the dual-tree transforms, each element of the cell array contains the analysis filters for the corresponding tree.

`Frf` — First-level reconstruction filters

matrix | cell array

First-level reconstruction filters, specified as an *N*-by-2 or *N*-by-3 matrix for single-tree wavelet transforms, or a 1-by-2 cell array of two *N*-by-2 or *N*-by-3 matrices for dual-tree wavelet transforms. The matrices are *N*-by-3 for the double-density wavelet transforms. For an *N*-by-2 matrix, the first column of the matrix is the scaling (lowpass) filter and the second column is the wavelet (highpass) filter. For an *N*-by-3 matrix, the first column of the matrix is the scaling (lowpass) filter and the second and third columns are the wavelet (highpass) filters. For the dual-tree transforms, each element of the cell array contains the first-stage synthesis filters for the corresponding tree.

`Rf` — Reconstruction filters for levels > 1

matrix | cell array

Reconstruction filters for levels > 1, specified as an *N*-by-2 or *N*-by-3 matrix for single-tree wavelet transforms, or a 1-by-2 cell array of two *N*-by-2 or *N*-by-3 matrices for dual-tree wavelet transforms. The matrices are *N*-by-3 for the double-density wavelet transforms. For an *N*-by-2 matrix, the first column of the matrix is the scaling (lowpass) filter and the second column is the wavelet (highpass) filter. For an *N*-by-3 matrix, the first column of the matrix is the scaling (lowpass) filter and the second and third columns are the wavelet (highpass) filters. For the dual-tree transforms, each element of the cell array contains the first-stage synthesis filters for the corresponding tree.

`cfs` — Wavelet transform coefficients

cell array of matrices

Wavelet transform coefficients, specified as a 1-by-(`level`+1) cell array of matrices. The size and structure of the matrix elements of the cell array depend on the type of wavelet transform and whether the decomposition is 1-D or 2-D. For a 1-D wavelet transform, the coefficients are organized by transform type as follows:

- `'dwt'` — `cfs{j}`

  - j = 1,2,...`level` is the level.
  - `cfs{level+1}` are the lowpass, or scaling, coefficients.
- `'ddt'` — `cfs{j}(:,:,k)`
  - j = 1,2,... `level` is the level.
  - k = 1,2 is the wavelet filter.
  - `cfs{level+1}(:,:)` are the lowpass, or scaling, coefficients.
- `'cplxdt'` — `cfs{j}(:,:,m)`
  - j = 1,2,... `level` is the level.
  - m = 1,2 are the real and imaginary parts.
  - `cfs{level+1}(:,:)` are the lowpass, or scaling, coefficients.
- `'realdddt'` — `cfs{j}(:,:,d,k)`
  - j = 1,2,... `level` is the level.
  - d = 1,2,3 is the orientation.
  - k = 1,2 is the wavelet transform tree.
  - `cfs{level+1}(:,:)` are the lowpass, or scaling, coefficients.
- `'cplxdddt'` — `cfs{j}(:,:,d,k,m)`
  - j = 1,2,... `level` is the level.
  - k = 1,2 is the wavelet transform tree.
  - m = 1,2 are the real and imaginary parts.
  - `cfs{level+1}(:,:)` are the lowpass, or scaling, coefficients.

For a 2-D wavelet transform, the coefficients are organized by transform type as follows:

- `'dwt'` — `cfs{j}(:,:,d)`
  - j = 1,2,... `level` is the level.
  - d = 1,2,3 is the orientation.
  - `cfs{level+1}(:,:)` are the lowpass, or scaling, coefficients.
- `'ddt'` — `cfs{j}(:,:,d)`
  - j = 1,2,... `level` is the level.

- d = 1,2,3,4,5,6,7,8 is the orientation.
- `cfs{level+1}(:,:)` are the lowpass, or scaling, coefficients.

- `'realdt'` — `cfs{j}(:,:,d,k)`
  - j = 1,2,... `level` is the level.
  - d = 1,2,3 is the orientation.
  - k = 1,2 is the wavelet transform tree.
  - `cfs{level+1}(:,:)` are the lowpass, or scaling, coefficients.
- `'cplxdt'` — `cfs{j}(:,:,d,k,m)`
  - j = 1,2,... `level` is the level.
  - d = 1,2,3 is the orientation.
  - k = 1,2 is the wavelet transform tree.
  - m = 1,2 are the real and imaginary parts.
  - `cfs{level+1}(:,:)` are the lowpass, or scaling, coefficients.
- `'realdddt'` — `cfs{j}(:,:,d,k)`
  - j = 1,2,... `level` is the level.
  - d = 1,2,3 is the orientation.
  - k = 1,2 is the wavelet transform tree.
  - `cfs{level+1}(:,:)` are the lowpass, or scaling, coefficients.
- `'cplxdddt'` — `cfs{j}(:,:,d,k,m)`
  - j = 1,2,... `level` is the level.
  - d = 1,2,3 is the orientation.
  - k = 1,2 is the wavelet transform tree.
  - m = 1,2 are the real and imaginary parts.
  - `cfs{level+1}(:,:)` are the lowpass, or scaling, coefficients.

More About

- "Critically Sampled and Oversampled Wavelet Filter Banks"

See Also

`dddtree` | `dddtree2` | `dddtreecfs`

Introduced in R2013b

qmf

Scaling and Wavelet Filter

Syntax

```
Y = qmf(X,P)
Y = qmf(X)
Y = qmf(X,0)
```

Description

`Y = qmf(X,P)` changes the signs of the even index entries of the reversed vector filter coefficients `X` if `P` is even. If `P` is odd the same holds for odd index entries. `Y = qmf(X)` is equivalent to `Y = qmf(X,0)`.

Let `x` be a finite energy signal. Two filters F_0 and F_1 are quadrature mirror filters (QMF) if, for any x,

$$\|y_0\|^2 + \|y_1\|^2 = \|x\|^2$$

where y_0 is a decimated version of the signal x filtered with F_0 so y_0 defined by $x_0 = F_0(x)$ and $y_0(n) = x_0(2n)$, and similarly, y_1 is defined by $x_1 = F_1(x)$ and $y_1(n) = x_1(2n)$. This property ensures a perfect reconstruction of the associated two-channel filter banks scheme (see Strang-Nguyen p. 103).

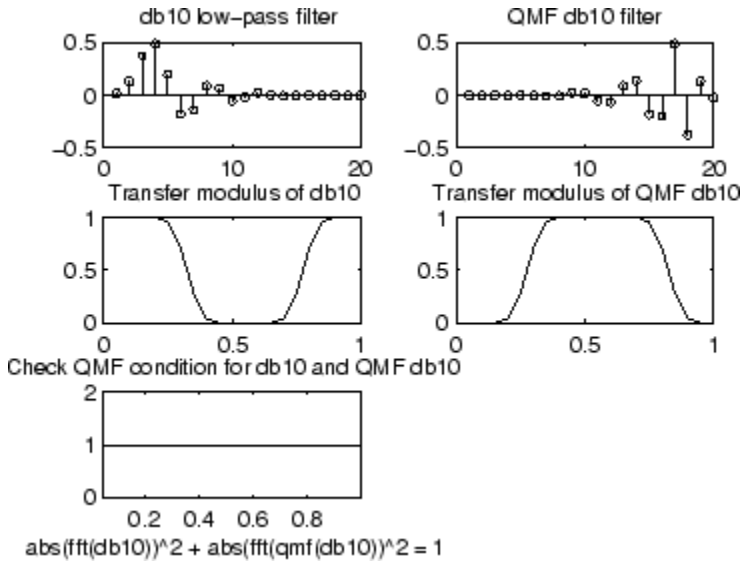
For example, if F_0 is a Daubechies scaling filter and $F_1 = \texttt{qmf}(F_0)$, then the transfer functions $F_0(z)$ and $F_1(z)$ of the filters F_0 and F_1 satisfy the condition (see the example for *db*10):

$$|F_0(z)|^2 + |F_1(z)|^2 = 1$$

Examples

```
% Load scaling filter associated with an orthogonal wavelet.
load db10;
```

```
subplot(321); stem(db10); title('db10 low-pass filter');

% Compute the quadrature mirror filter.
qmfdb10 = qmf(db10);
subplot(322); stem(qmfdb10); title('QMF db10 filter');

% Check for frequency condition (necessary for orthogonality):
% abs(fft(filter))^2 + abs(fft(qmf(filter))^2 = 1 at each
% frequency.
m = fft(db10);
mt = fft(qmfdb10);
freq = [1:length(db10)]/length(db10);
subplot(323); plot(freq,abs(m));
title('Transfer modulus of db10')
subplot(324); plot(freq,abs(mt));
title('Transfer modulus of QMF db10')
subplot(325); plot(freq,abs(m).^2 + abs(mt).^2);
title('Check QMF condition for db10 and QMF db10')
xlabel(' abs(fft(db10))^2 + abs(fft(qmf(db10))^2 = 1')

% Editing some graphical properties,
% the following figure is generated.
```




```
% Check for orthonormality.
```

```
df = [db10;qmfdb10]*sqrt(2);
id = df*df'

id =
    1.0000 0.0000
    0.0000 1.0000
```

References

Strang, G.; T. Nguyen (1996), *Wavelets and Filter Banks*, Wellesley-Cambridge Press.

Introduced before R2006a

rbiowavf

Reverse biorthogonal spline wavelet filters

Syntax

```
[RF,DF] = rbiowavf(W)
```

Description

`[RF,DF] = rbiowavf(W)` returns the two scaling filters associated with the biorthogonal wavelet specified by the string *W*.

`W = 'rbioNd.Nr'` where possible values for `Nd` and `Nr` are

`Nd = 1`	`Nr = 1 , 3 or 5`
`Nd = 2`	`Nr = 2 , 4 , 6 or 8`
`Nd = 3`	`Nr = 1 , 3 , 5 , 7 or 9`
`Nd = 4`	`Nr = 4`
`Nd = 5`	`Nr = 5`
`Nd = 6`	`Nr = 8`

The output arguments are filters.

- `RF` is the reconstruction filter.
- `DF` is the decomposition filter.

Reverse Biorthgonal Scaling Filter

Obtain the reverse biorthogonal reconstruction and decomposition scaling filters for the `'rbio3.1'` wavelet. The `'rbio3.1'` wavelet has 3 vanishing moments for the decomposition (analysis) wavelet and 1 vanishing moment for the reconstruction (synthesis) wavelet.

```
[RF,DF] = rbiowavf('rbio3.1');
```

The reconstruction scaling filter, RF, and the decomposition filter, DF, are equal to the filters returned by `wfilters` scaled by $\sqrt{2}$.

```
[LoD,HiD,LoR,HiR] = wfilters('rbio3.1');
max(abs(sqrt(2)*DF-LoD))
max(abs(sqrt(2)*RF-LoR))
```

```
ans =

     0


ans =

     0
```

See Also

`biorfilt` | `waveinfo`

Introduced before R2006a

read

Read values of WPTREE

Syntax

```
VARARGOUT = read(T,VARARGIN)
```

Description

`VARARGOUT = read(T,VARARGIN)` is the most general syntax to read one or more property values from the fields of a WPTREE object .

The different ways to call the `read` function are

```
PropValue = read(T,'PropName') or
PropValue = read(T,'PropName','PropParam')
```

or any combination of the previous syntaxes:

```
[PropValue1,PropValue2, ] =
read(T,'PropName1','PropParam1','PropName2','PropParam2', )
```

where '*PropParam*' is optional.

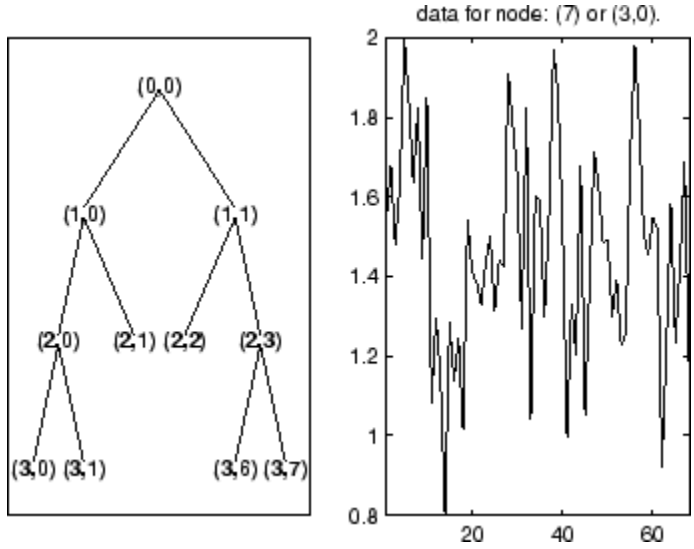
The valid choices for '*PropName*' and '*PropParam*' are listed in this table.

PropName	***PropParam***
`'ent'`, `'ento'` or `'sizes'` (see `wptree`)	Without '*PropParam*' or with '*PropParam*' = Vector of node indices, `PropValue` contains the entropy (or optimal entropy, or size) of the tree nodes in ascending node index order.
`'cfs'`	With '*PropParam*' = One terminal node index. `cfs = read(T,'cfs',NODE)` is equivalent to cfs = read(T,'data',NODE) and returns the coefficients of the terminal node `NODE`.

PropName	*PropParam*
'entName', 'entPar', 'wavName' (see wptree) or 'allcfs'	Without '*PropParam*'. cfs = read(T,'allcfs') is equivalent to cfs = read(T,'data'). PropValue contains the desired information in ascending node index order of the tree nodes.
'wfilters' (see wfilters)	Without '*PropParam*' or with '*PropParam*' = 'd','r','l','h'.
'data'	Without '*PropParam*' or with '*PropParam*' = One terminal node index or '*PropParam*' = Column vector of terminal node indices.In this last case, PropValue is a cell array. Without '*PropParam*', PropValue contains the coefficients of the tree nodes in ascending node index order.

Examples

```
% Create a wavelet packet tree.
x = rand(1,512);
t = wpdec(x,3,'db3');
t = wpjoin(t,[4;5]);
plot(t);

% Click the node (3,0), (see the plot function).
l% Read values.

sAll = read(t,'sizes');
sNod = read(t,'sizes',[0,4,5]);
eAll = read(t,'ent');
eNod = read(t,'ent',[0,4,5]);
dAll = read(t,'data');
dNod = read(t,'data',[4;5]);
[lo_D,hi_D,lo_R,hi_R] = read(t,'wfilters');
[lo_D,lo_R,hi_D,hi_R] = read(t,'wfilters','l','wfilters','h');
[ent,ento,cfs4,cfs5]  = read(t,'ent','ento','cfs',4,'cfs',5);
```



See Also

`disp` | `get` | `set` | `wptree` | `write`

Introduced before R2006a

readtree

Read wavelet packet decomposition tree from figure

Syntax

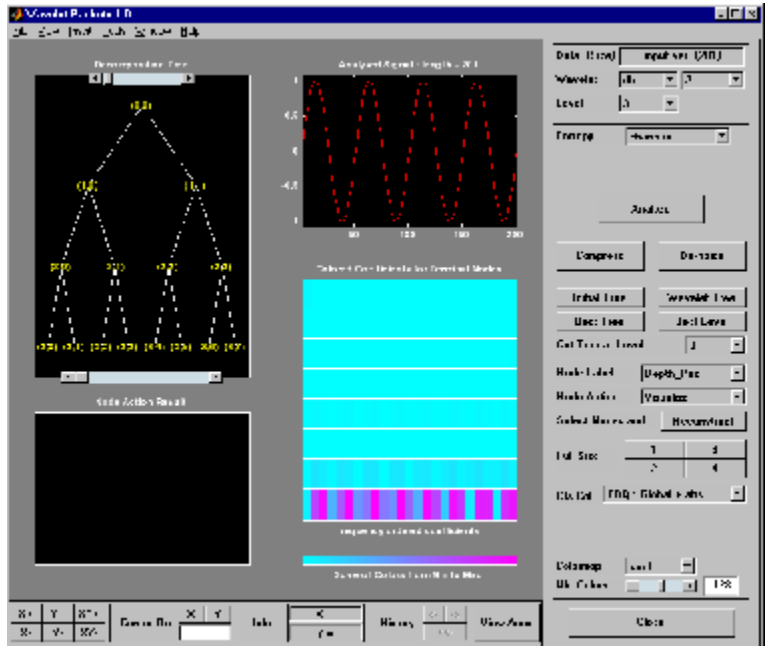
```
T = readtree(F)
```

Description

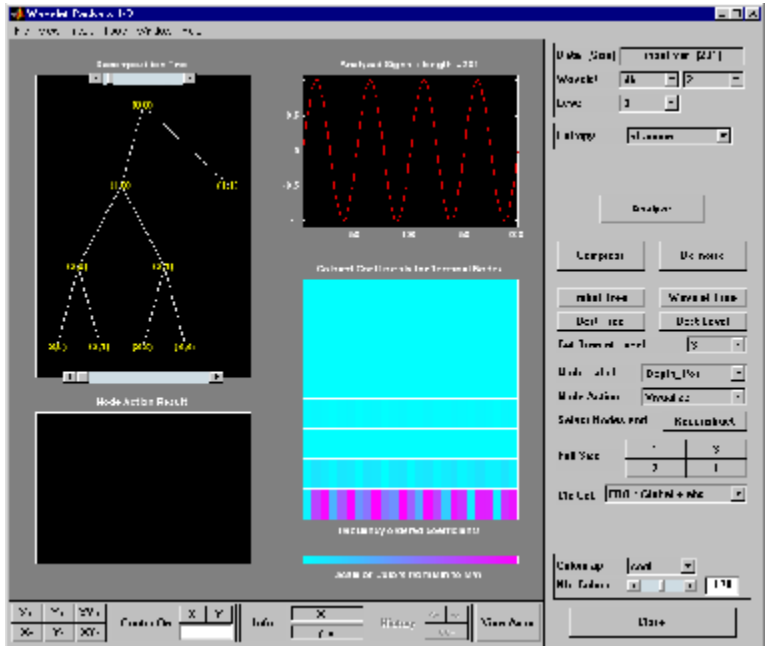
`T = readtree(F)` reads the wavelet packet decomposition tree from the figure whose handle is *F*.

Examples

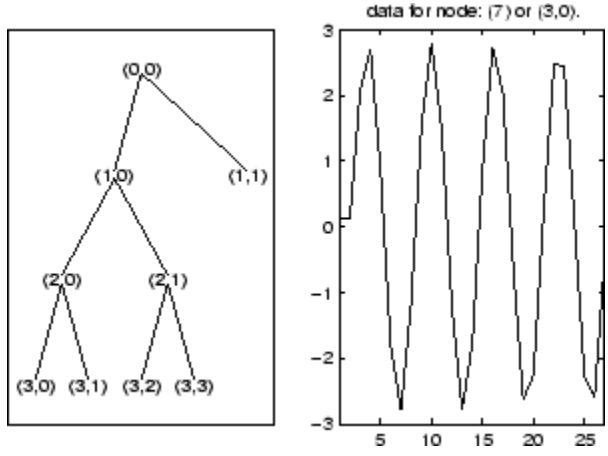
```
% Create a wavelet packet tree.
x   = sin(8*pi*[0:0.005:1]);
t   = wpdec(x,3,'db2');

% Display the generated tree in a Wavelet Packet 1-D GUI window.
fig = drawtree(t);
```



```
%-------------------------------------
% Use the GUI to split or merge Nodes.
%-------------------------------------
```



```
t = readtree(fig);
plot(t)

% Click the node (3,0), (see the plot function).
```



See Also

`drawtree`

Introduced before R2006a

scal2frq

Scale to frequency

Syntax

```
F = scal2frq(A,'wname',DELTA)
scal2frq(A,'wname')
scal2frq(A,'wname',1)
```

Description

`F = scal2frq(A,'wname',DELTA)` returns the pseudo-frequencies corresponding to the scales given by `A` and the wavelet function `'wname'` (see `wavefun` for more information) and the sampling period `DELTA`.

`scal2frq(A,'wname')` is equivalent to `scal2frq(A,'wname',1)`.

There is only an approximate answer for the relationship between scale and frequency.

In wavelet analysis, the way to relate scale to frequency is to determine the center frequency of the wavelet, F_c, and use the following relationship.

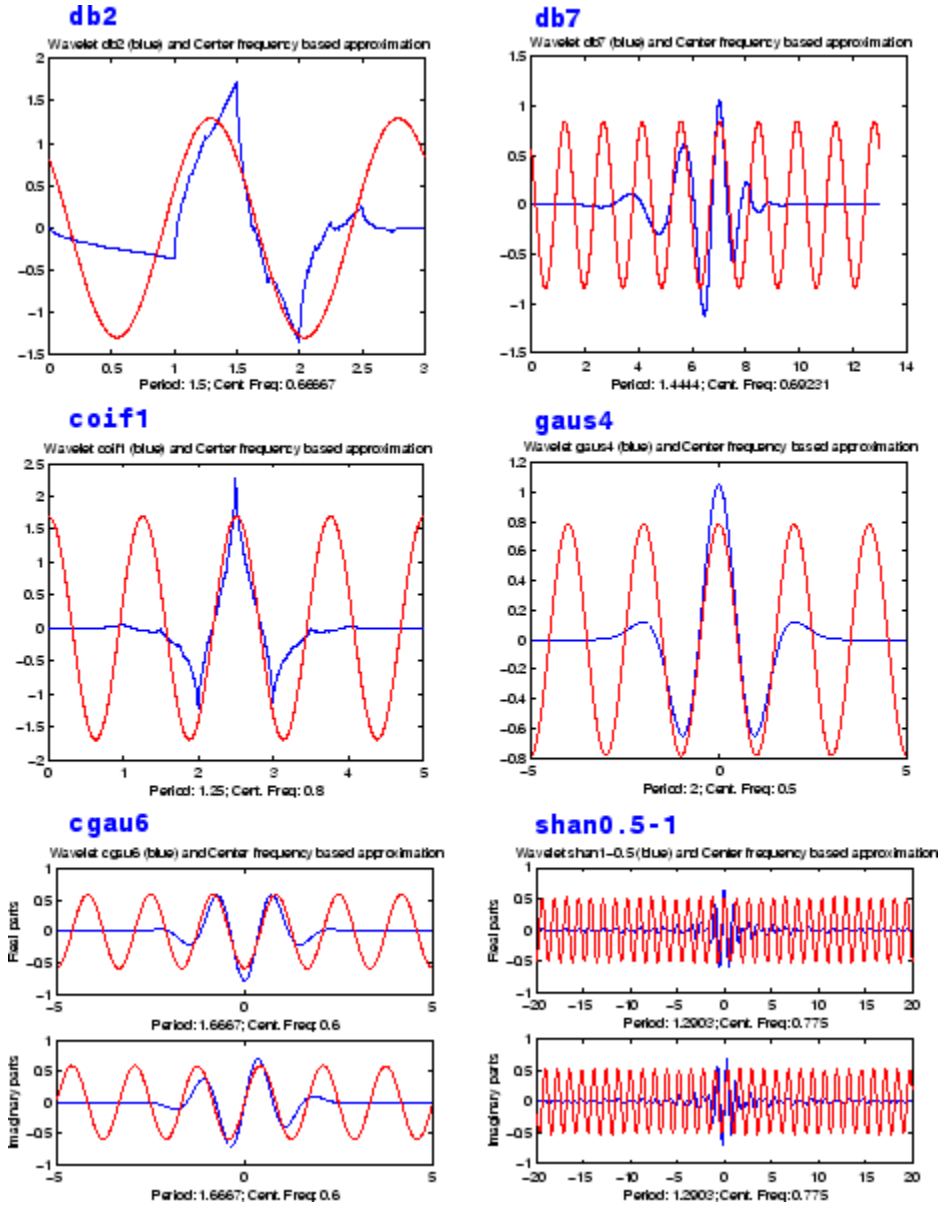
$$F_a = \frac{F_c}{a \cdot \Delta}$$

where

- a is a scale.
- Δ is the sampling period.
- F_c is the center frequency of a wavelet in Hz.
- F_a is the pseudo-frequency corresponding to the scale a, in Hz.

The idea is to associate with a given wavelet a purely periodic signal of frequency F_c. The frequency maximizing the Fourier transform of the wavelet modulus is F_c. `centfrq` computes the center frequency for a specified wavelet.

Some examples of the correspondence between the center frequency and the wavelet are shown in the following figure.



Center Frequencies for Real and Complex Wavelets

As you can see, the center frequency-based approximation captures the main wavelet oscillations. Therefore, the center frequency is a convenient and simple characterization of the dominant frequency of the wavelet.

Dilating the wavelet by a, changes the center frequency to F_c / a. If the underlying sampling period is Δ, the scale a corresponds to the frequency

$$F_a = \frac{F_c}{a \cdot \Delta}$$

`scal2frq` computes this correspondence.

Examples

Scales To Frequencies

Construct a vector of scales with 10 voices per octave over five octaves. Assume the data are sampled at 10 kHz.

```
voicesperoctave = 10;
numoctaves = 5;
a0 = 2^(1/voicesperoctave);
Fs = 1e4;
scales = ...
    a0.^(voicesperoctave:1/voicesperoctave:numoctaves*voicesperoctave);
```

Convert the scales to approximate frequencies in hertz for the Morlet wavelet.

```
Frq = scal2frq(scales,'morl',1/Fs);
```

Determine the corresponding periods. Construct a table with the scales, the corresponding frequencies, and periods. Display the smallest 20 scales along with their corresponding frequencies and periods.

```
Frq = Frq(:);
scales = scales(:);
T = [scales.*(1/Fs) Frq 1./Frq];
T = array2table(T,'VariableNames',{'Scale','Frequency','Period'});
T(1:20,:)
```

```
ans =

      Scale       Frequency      Period
    __________    _________    __________

       0.0002      4062.5       0.00024615
    0.00020139     4034.4       0.00024787
    0.00020279     4006.6       0.00024959
     0.0002042     3978.9       0.00025133
    0.00020562     3951.4       0.00025307
    0.00020705     3924.1       0.00025483
    0.00020849       3897       0.00025661
    0.00020994     3870.1       0.00025839
     0.0002114     3843.4       0.00026019
    0.00021287     3816.8         0.000262
    0.00021435     3790.4       0.00026382
    0.00021585     3764.3       0.00026566
    0.00021735     3738.3        0.0002675
    0.00021886     3712.4       0.00026936
    0.00022038     3686.8       0.00027124
    0.00022191     3661.3       0.00027312
    0.00022346       3636       0.00027502
    0.00022501     3610.9       0.00027694
    0.00022658       3586       0.00027886
    0.00022815     3561.2        0.0002808
```

Plot CWT with Frequencies instead of Scales

The example shows how to plot the CWT using approximate frequencies in Hz instead of scales. This creates a time-frequency plot instead of a time-scale plot.

Create a signal consisting of two sine waves with disjoint support in additive noise. Assume the signal is sampled at 1 kHz.

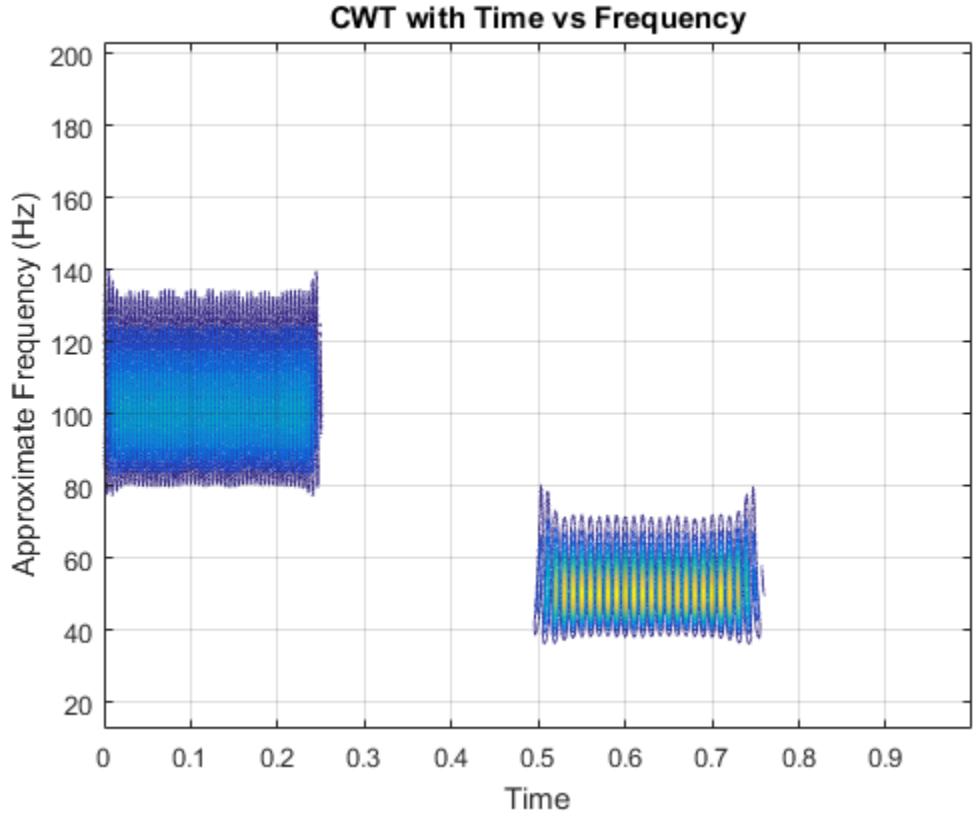
```
Fs = 1000;
t = 0:1/Fs:1-1/Fs;
x = 1.5*cos(2*pi*100*t).*(t<0.25)+1.5*cos(2*pi*50*t).*(t>0.5 & t<=0.75);
x = x+0.05*randn(size(t));
```

Obtain the CWT of the input signal with the Morlet wavelet. Set the number of voices per octave to 32. Create a scale vector to cover 6 octaves. The initial scale is $4\Delta t$ where Δt is the sampling interval.

```
numvoices = 32;
a0 = 2^(1/numvoices);
numoctaves = 6;
scales = a0.^(2*numvoices:1/numvoices:numoctaves*numvoices);
cfs = cwt(x,scales,'morl');
```

Convert the scales to approximate frequencies and plot the result.

```
pfreq = scal2frq(scales,'morl',1/Fs);
contour(t,pfreq,abs(cfs).^2);
axis tight;
grid on;
xlabel('Time');
ylabel('Approximate Frequency (Hz)');
title('CWT with Time vs Frequency');
```



References

Abry, P. (1997), *Ondelettes et turbulence. Multirésolutions, algorithmes de décomposition, invariance d'échelles*, Diderot Editeur, Paris.

See Also

`centfrq`

Introduced before R2006a

set

WPTREE field contents

Syntax

```
T = set(T,'FieldName1',FieldValue1,'FieldName2',FieldValue2, ...)
```

Description

`T = set(T,'FieldName1',FieldValue1,'FieldName2',FieldValue2, ...)` sets the content of the specified fields for the WPTREE object `T`.

For the fields that are objects or structures, you can set the subfield contents, giving the name of these subfields as `'FieldName'` values.

The valid choices for `'FieldName'` are

`'dtree'`	DTREE parent object
`'wavInfo'`	Structure (wavelet information)

The fields of the wavelet information structure, `'wavInfo'`, are also valid for `'FieldName'`:

`'wavName'`	Wavelet name
`'Lo_D'`	Low Decomposition filter
`'Hi_D'`	High Decomposition filter
`'Lo_R'`	Low Reconstruction filter
`'Hi_R'`	High Reconstruction filter

`'entInfo'`	Structure (entropy information)

The fields of the entropy information structure, `'entInfo'`, are also valid for `'FieldName'`:

'entName'	Entropy name
'entPar'	Entropy parameter

Or fields of DTREE parent object:

'ntree'	NTREE parent object
'allNI'	All nodes information
'terNI'	Terminal nodes information

Or fields of NTREE parent object:

'wtbo'	WTBO parent object
'order'	Order of the tree
'depth'	Depth of the tree
'spsch'	Split scheme for nodes
'tn'	Array of terminal nodes of the tree

Or fields of WTBO parent object:

'wtboInfo'	Object information
'ud'	Userdata field

Caution The `set` function should only be used to set the field *'ud'*.

See Also

`disp` | `get` | `read` | `write`

Introduced before R2006a

shanwavf

Complex Shannon wavelet

Syntax

```
[PSI,X] = shanwavf(LB,UB,N,FB,FC)
```

Description

`[PSI,X] = shanwavf(LB,UB,N,FB,FC)` returns values of the complex Shannon wavelet. The complex Shannon wavelet is defined by a bandwidth parameter `FB`, a wavelet center frequency `FC`, and the expression

```
PSI(X) = (FB^0.5)*(sinc(FB*X).*exp(2*i*pi*FC*X))
```

on an `N` point regular grid in the interval `[LB,UB]`.

`FB` and `FC` must be such that `FC > 0` and `FB > 0`.

Output arguments are the wavelet function `PSI` computed on the grid `X`.

Examples

Complex Shannon Wavelet

Obtain and plot a complex Shannon wavelet. Set the bandwidth and center frequency parameters.

```
fb = 1; fc = 1.5;
```

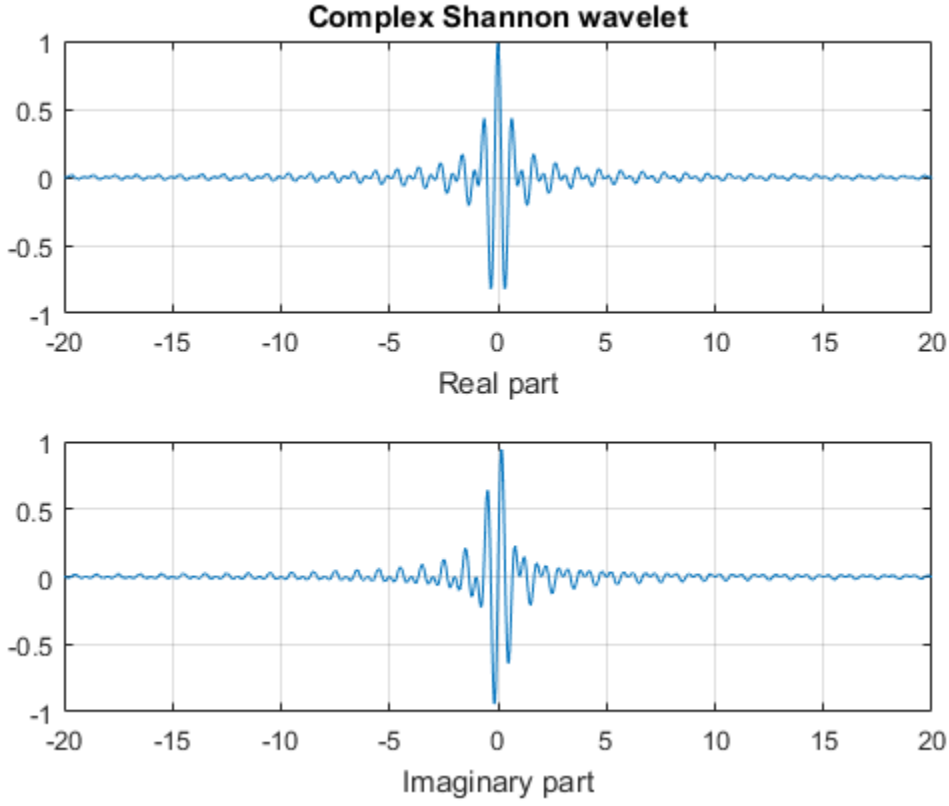
Set the effective support and number of sample points.

```
lb = -20;
ub = 20;
n = 1000;
```

Obtain the complex-valued Shannon wavelet and plot the real and imaginary parts.

```
[psi,x] = shanwavf(lb,ub,n,fb,fc);
```

```
subplot(211)
plot(x,real(psi))
title('Complex Shannon wavelet')
xlabel('Real part');
grid on;
subplot(212)
plot(x,imag(psi))
xlabel('Imaginary part')
grid on;
```



References

Teolis, A. (1998), *Computational signal processing with wavelets*, Birkäuser, p. 62.

See Also

`waveinfo`

Introduced before R2006a

swt

Discrete stationary wavelet transform 1-D

Syntax

```
SWC = swt(X,N,'wname')
SWC = swt(X,N,Lo_D,Hi_D)
```

Description

`swt` performs a multilevel 1-D stationary wavelet decomposition using either a specific orthogonal wavelet (`'wname'`, see `wfilters` for more information) or specific orthogonal wavelet decomposition filters.

`SWC = swt(X,N,'wname')` computes the stationary wavelet decomposition of the signal `X` at level `N`, using `'wname'`.

`N` must be a strictly positive integer (see `wmaxlev` for more information) and `length(X)` must be a multiple of 2^N .

`SWC = swt(X,N,Lo_D,Hi_D)` computes the stationary wavelet decomposition as above, given these filters as input:

- `Lo_D` is the decomposition low-pass filter.
- `Hi_D` is the decomposition high-pass filter.

`Lo_D` and `Hi_D` must be the same length.

The output matrix `SWC` contains the vectors of coefficients stored row-wise:

For `1 ≤ i ≤ N`, the output matrix `SWC(i,:)` contains the detail coefficients of level `i` and `SWC(N+1,:)` contains the approximation coefficients of level `N`.

`[SWA,SWD] = swt( )` computes approximations, `SWA`, and details, `SWD`, stationary wavelet coefficients.

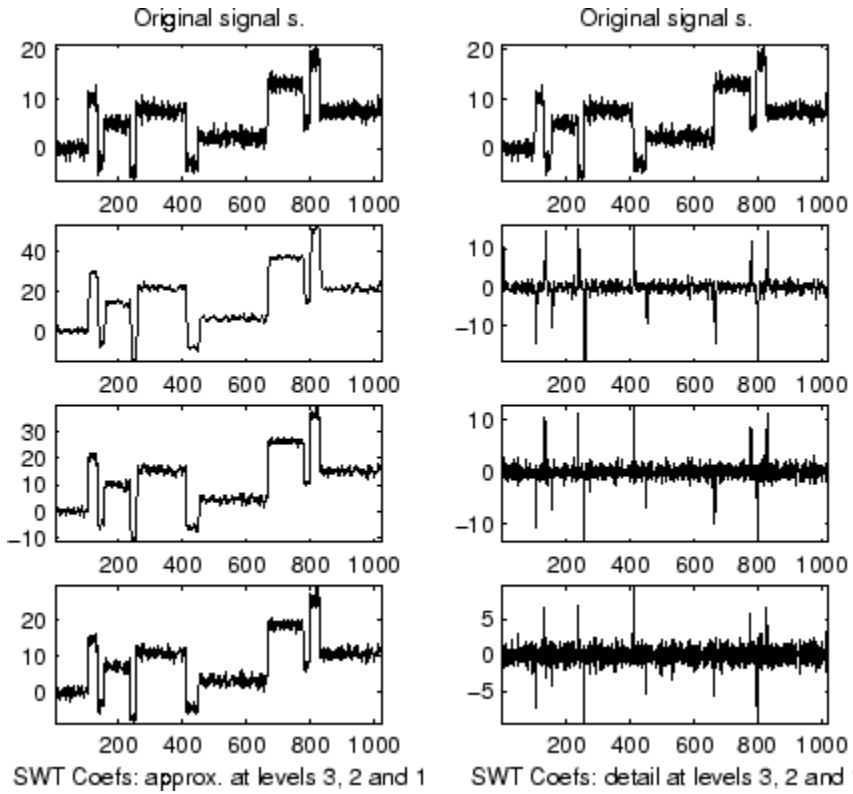
The vectors of coefficients are stored row-wise:

For `1 ≤ i ≤ N`, the output matrix `SWA(i,:)` contains the approximation coefficients of level `i` and the output matrix `SWD(i,:)` contains the detail coefficients of level `i`.

Note `swt` is defined using `dwt` with periodic extension.

Examples

```
% Load original 1D signal.
load noisbloc; s = noisbloc;

% Perform SWT decomposition at level 3 of s using db1.
[swa,swd] = swt(s,3,'db1');

% Plots of SWT coefficients of approximations and details
% at levels 3 to 1.

% Using some plotting commands,
% the following figure is generated.
```

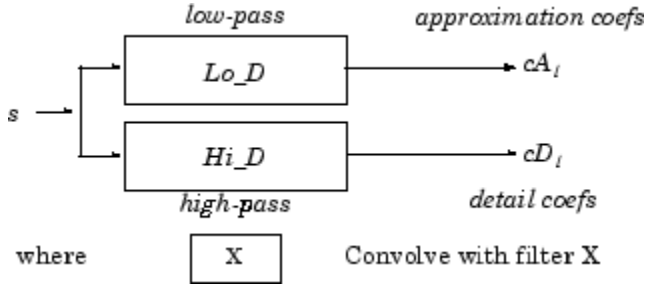


More About

Algorithms

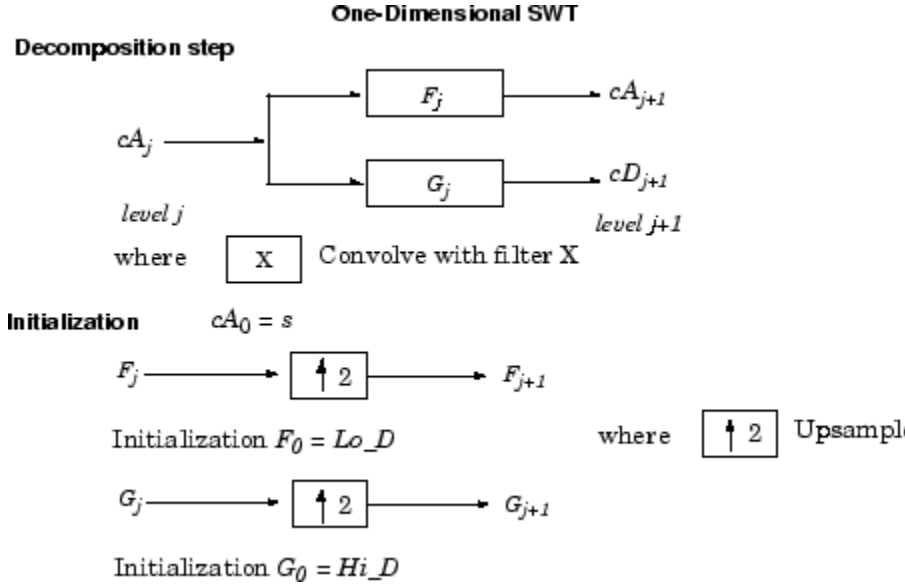
Given a signal s of length N, the first step of the SWT produces, starting from s, two sets of coefficients: approximation coefficients cA_1 and detail coefficients cD_1. These vectors are obtained by convolving s with the low-pass filter `Lo_D` for approximation, and with the high-pass filter `Hi_D` for detail.

More precisely, the first step is



Note cA_1 and cD_1 are of length `N` instead of `N/2` as in the DWT case.

The next step splits the approximation coefficients cA_1 in two parts using the same scheme, but with modified filters obtained by upsampling the filters used for the previous step and replacing s by cA_1. Then, the SWT produces cA_2 and cD_2. More generally,



References

Nason, G.P.; B.W. Silverman (1995), "The stationary wavelet transform and some statistical applications," *Lecture Notes in Statistics*, 103, pp. 281–299.

Coifman, R.R.; Donoho, D.L. (1995), "Translation invariant de-noising," *Lecture Notes in Statistics*, 103, pp. 125–150.

Pesquet, J.C.; H. Krim, H. Carfatan (1996), "Time-invariant orthonormal wavelet representations," *IEEE Trans. Sign. Proc.*, vol. 44, 8, pp. 1964–1970.

See Also

`dwt` | `wavedec`

Introduced before R2006a

swt2

Discrete stationary wavelet transform 2-D

Syntax

```
SWC = swt2(X,N,'wname')
[A,H,V,D] = swt2(X,N,'wname')
SWC = swt2(X,N,Lo_D,Hi_D)
[A,H,V,D] = swt2(X,N,Lo_D,Hi_D)
```

Description

`swt2` performs a multilevel 2-D stationary wavelet decomposition using either a specific orthogonal wavelet (`'wname'` — see `wfilters` for more information) or specific orthogonal wavelet decomposition filters.

`SWC = swt2(X,N,'wname')` or `[A,H,V,D] = swt2(X,N,'wname')` compute the stationary wavelet decomposition of the matrix `X` at level `N`, using `'wname'`.

`N` must be a strictly positive integer (see `wmaxlev` for more information), and 2^N must divide `size(X,1)` and `size(X,2)`.

Outputs `[A,H,V,D]` are 3-D arrays, which contain the coefficients:

- For $1 \leq$ `i` $\leq$ `N`, the output matrix `A(:,:,i)` contains the coefficients of approximation of level `i`.
- The output matrices `H(:,:,i)`, `V(:,:,i)` and `D(:,:,i)` contain the coefficients of details of level `i` (horizontal, vertical, and diagonal):

  ```
  SWC = [H(:,:,1:N) ; V(:,:,1:N) ; D(:,:,1:N) ; A(:,:,N)]
  ```

`SWC = swt2(X,N,Lo_D,Hi_D)` or `[A,H,V,D] = swt2(X,N,Lo_D,Hi_D)`, computes the stationary wavelet decomposition as in the previous syntax, given these filters as input:

- `Lo_D` is the decomposition low-pass filter.

- `Hi_D` is the decomposition high-pass filter.

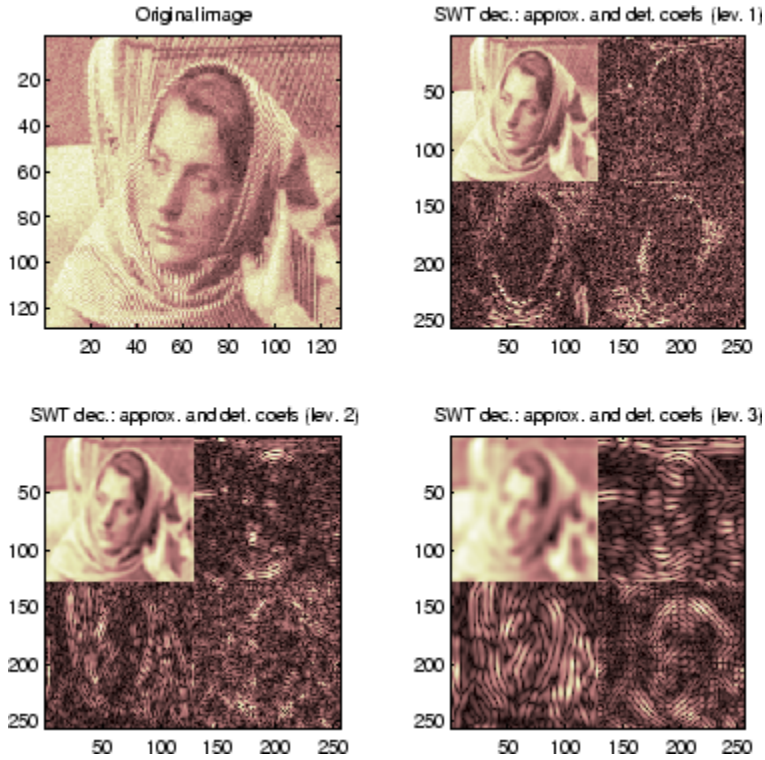
`Lo_D` and `Hi_D` must be the same length.

Note `swt2` is defined using `dwt` with periodic extension.

Examples

```
% Load original image.
load nbarb1;

% Image coding.
nbcol = size(map,1);
cod_X = wcodemat(X,nbcol);

% Visualize the original image.
subplot(221)
image(cod_X)
title('Original image');
colormap(map)

% Perform SWT decomposition
% of X at level 3 using sym4.
[ca,chd,cvd,cdd] = swt2(X,3,'sym4');

% Visualize the decomposition.

for k = 1:3
    % Images coding for level k.
    cod_ca  = wcodemat(ca(:,:,k),nbcol);
    cod_chd = wcodemat(chd(:,:,k),nbcol);
    cod_cvd = wcodemat(cvd(:,:,k),nbcol);
    cod_cdd = wcodemat(cdd(:,:,k),nbcol);
    decl = [cod_ca,cod_chd;cod_cvd,cod_cdd];

    % Visualize the coefficients of the decomposition
    % at level k.
    subplot(2,2,k+1)
    image(decl)

    title(['SWT dec.: approx. ', ...
   'and det. coefs (lev. ',num2str(k),')']);
```

```
    colormap(map)
end
% Editing some graphical properties,
% the following figure is generated.
```



More About

Tips

When X represents an indexed image, X is an m-by-n matrix and the output arrays SWC or cA,cH,cV, and cD are m-by-n-by-p arrays.

When X represents a truecolor image, it becomes an m-by-n-by-3 array. This array is an m-by-n-by-3 array, where each m-by-n matrix represents a red, green, or blue color plane

concatenated along the third dimension. The output arrays SWC or cA,cH,cV, and cD are `m`-by-`n`-by-`p`-by-3 arrays.

For more information on image formats, see the `image` and `imfinfo` reference pages.

Algorithms

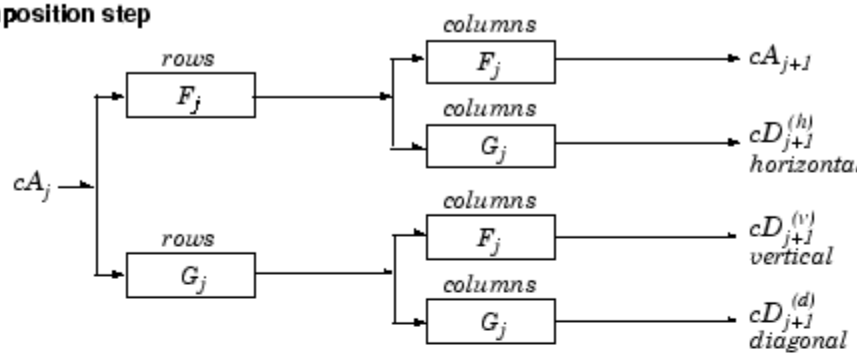
For images, an algorithm similar to the one-dimensional case is possible for two-dimensional wavelets and scaling functions obtained from one-dimensional ones by tensor product.

This kind of two-dimensional SWT leads to a decomposition of approximation coefficients at level*j* in four components: the approximation at level *j*+1, and the details in three orientations (horizontal, vertical, and diagonal).

The following chart describes the basic decomposition step for images:

Two-Dimensional SWT

Decomposition step

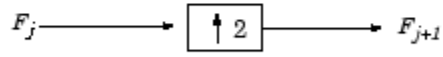


where

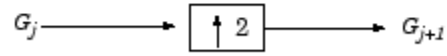
rows
X Convolve with filter X the rows of the entry

columns
X Convolve with filter X the columns of the entry

Initialization $cA_0 = s$ for the decomposition initialization



Initialization $F_0 = Lo\_D$ where 2 Upsample



Initialization $G_0 = Hi\_D$

Note $size(cA_j) = size(cD_j^{(h)}) = size(cD_j^{(v)}) = size(cD_j^{(d)}) = s$

where s = *size of the analyzed image*

References

Nason, G.P.; B.W. Silverman (1995), "The stationary wavelet transform and some statistical applications," *Lecture Notes in Statistics*, 103, pp. 281–299.

Coifman, R.R.; Donoho, D.L. (1995), "Translation invariant de-noising," *Lecture Notes in Statistics*, 103, pp. 125–150.

Pesquet, J.C.; H. Krim, H. Carfatan (1996), "Time-invariant orthonormal wavelet representations," *IEEE Trans. Sign. Proc.*, vol. 44, 8, pp. 1964–1970.

See Also

`dwt2` | `iswt2` | `wavedec2`

Introduced before R2006a

symaux

Symlet wavelet filter computation

Syntax

```
W = SYMAUX(N,SUMW)
W = SYMAUX(N)
W = SYMAUX(N,1)
W = SYMAUX(N,0)
W = SYMAUX(N,1)
```

Description

Symlets are the Äúleast asymmetricÄù Daubechies wavelets.

`W = SYMAUX(N,SUMW)` is the order `N` Symlet scaling filter such that `SUM(W) = SUMW`. Possible values for `N` are 1, 2, 3, ...

Note Instability may occur when `N` is too large.

`W = SYMAUX(N)` is equivalent to `W = SYMAUX(N,1)`.

`W = SYMAUX(N,0)` is equivalent to `W = SYMAUX(N,1)`.

Examples

```
% Generate wdb4 the order 4 Daubechies scaling filter.
wdb4 = dbaux(4)

wdb4 =

  Columns 1 through 7

    0.1629  0.5055  0.4461  -0.0198  -0.1323  0.0218  0.0233
```

```
  Column 8

   -0.0075


% wdb4 is a solution of the equation: P = conv(wrev(w),w)*2,
% where P is the "Lagrange  trous" filter for N=4.
% wdb4 is a minimum phase solution of the previous equation,
% based on the roots of P (see dbaux).
P = conv(wrev(wdb4),wdb4)*2;

% Generate wsym4 the order 4 symlet scaling filter.
% The Symlets are the "least asymmetric" Daubechies'
% wavelets obtained from another choice between the roots of P.
wsym4 = symaux(4)

wsym4 =

  Columns 1 through 7

    0.0228  -0.0089  -0.0702  0.2106  0.5683  0.3519  -0.0210

  Column 8

   -0.0536


% Compute conv(wrev(wsym4),wsym4) * 2 and check that wsym4
% is another solution of the equation P = conv(wrev(w),w)*2.
Psym = conv(wrev(wsym4),wsym4)*2;
err = norm(P-Psym)

err =

  7.4988e-016
```

See Also

symwavf | wfilters

Introduced before R2006a

symwavf

Symlet wavelet filter

Syntax

```
F = symwavf(W)
```

Description

`F = symwavf(W)` returns the scaling filter associated with the symlet wavelet specified by the string *W* where `W = 'symN'`. Possible values for `N` are 2, 3, ..., 45.

Examples

```
% Compute the scaling filter corresponding to wavelet sym4.
w = symwavf('sym4')

w =
 Columns 1 through 7
    0.0228 -0.0089 -0.0702 0.2106 0.5683 0.3519 -0.0210
 Column 8
    -0.0536
```

See Also

`symaux` | `waveinfo`

Introduced before R2006a

thselect

Threshold selection for de-noising

Syntax

```
THR = thselect(X,TPTR)
```

Description

`thselect` is a one-dimensional de-noising oriented function.

`THR = thselect(X,TPTR)` returns threshold `X`-adapted value using selection rule defined by string `TPTR`.

Available selection rules are

- `TPTR = 'rigrsure'`, adaptive threshold selection using principle of Stein's Unbiased Risk Estimate.
- `TPTR = 'heursure'`, heuristic variant of the first option.
- `TPTR = 'sqtwolog'`, threshold is `sqrt(2*log(length(X)))`.
- `TPTR = 'minimaxi'`, minimax thresholding.

Threshold selection rules are based on the underlying model $y = f(t) + e$ where e is a white noise $N(0,1)$. Dealing with unscaled or nonwhite noise can be handled using rescaling output threshold `THR` (see `SCAL` parameter in `wden` for more information).

Available options are

- `tptr = 'rigrsure'` uses for the soft threshold estimator, a threshold selection rule based on SteinÄôs Unbiased Estimate of Risk (quadratic loss function). One gets an estimate of the risk for a particular threshold value (t). Minimizing the risks in (t) gives a selection of the threshold value.
- `tptr = 'sqtwolog'` uses a fixed-form threshold yielding minimax performance multiplied by a small factor proportional to `log(length(X))`.
- `tptr = 'heursure'` is a mixture of the two previous options. As a result, if the signal to noise ratio is very small, the SURE estimate is very noisy. If such a situation is detected, the fixed form threshold is used.

- `tptr = 'minimaxi'` uses a fixed threshold chosen to yield minimax performance for mean square error against an ideal procedure. The minimax principle is used in statistics in order to design estimators. Since the de-noised signal can be assimilated to the estimator of the unknown regression function, the minimax estimator is the one that realizes the minimum of the maximum mean square error obtained for the worst function in a given set.

Examples

```
% The current extension mode is zero-padding (see dwtmode).
% Generate Gaussian white noise.
x = randn(1,1000);

% Find threshold for each selection rule.
% Adaptive threshold using SURE.
thr = thselect(x,'rigrsure')
% Fixed form threshold.
thr = thselect(x,'sqtwolog')
% Heuristic variant of the first option.
thr = thselect(x,'heursure')
% Minimax threshold.
thr = thselect(x,'minimaxi')
```

References

Donoho, D.L. (1993), "Progress in wavelet analysis and WVD: a ten minute tour," in *Progress in wavelet analysis and applications*, Y. Meyer, S. Roques, pp. 109–128. Frontières Ed.

Donoho, D.L., I.M. Johnstone (1994), "Ideal spatial adaptation by wavelet shrinkage," *Biometrika*, vol 81, pp. 425–455.

Donoho, D.L. (1995), "De-noising by soft-thresholding," *IEEE Trans. on Inf. Theory*, 41, 3, pp. 613–627.

See Also

`wden`

Introduced before R2006a

tnodes

Determine terminal nodes

Syntax

```
N = tnodes(T)
N = tnodes(T,'deppos')
[N,K] = tnodes(T)
[N,K] = tnodes(T,'deppos'), M = N(K)
```

Description

`tnodes` is a tree-management utility.

`N = tnodes(T)` returns the indices of terminal nodes of the tree *T*. `N` is a column vector.

The nodes are numbered from left to right and from top to bottom. The root index is 0.

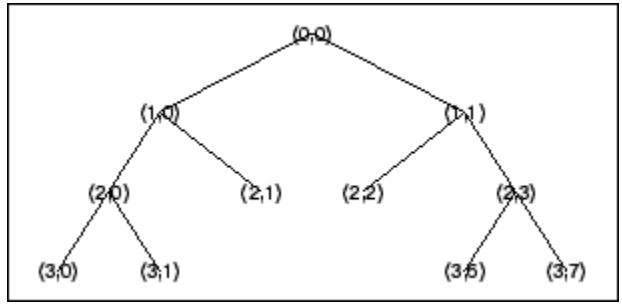
`N = tnodes(T,'deppos')` returns a matrix `N`, which contains the depths and positions of terminal nodes.

`N(i,1)` is the depth of the `i`-th terminal node. `N(i,2)` is the position of the `i`-th terminal node.

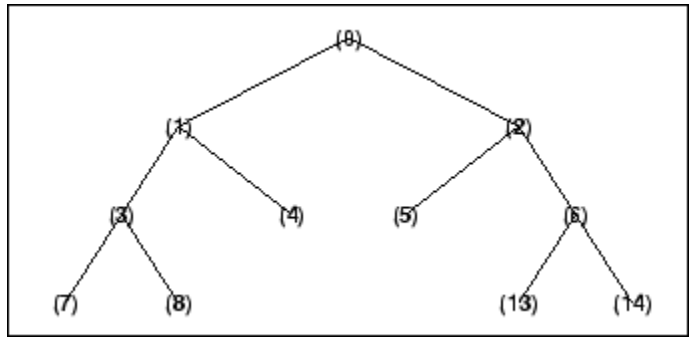
For `[N,K] = tnodes(T)` or `[N,K] = tnodes(T,'deppos'), M = N(K)` are the indices reordered as in tree *T*, from left to right.

Examples

```
% Create initial tree.
ord = 2;
t = ntree(ord,3);      % Binary tree of depth 3.
t = nodejoin(t,5);
t = nodejoin(t,4);
plot(t)
```




```
% Change Node Label from Depth_Position to Index
% (see the plot function).
```




```
% List terminal nodes (index).
tnodes(t)

ans =
     4
     5
     7
     8
     13
     14
% List terminal nodes (Depth_Position).
tnodes(t,'deppos')
ans =
     2    1
     2    2
     3    0
     3    1
```

```
    3    6
    3    7
```

See Also

`leaves` | `noleaves` | `wtreemgr`

Introduced before R2006a

treedpth

Tree depth

Syntax

```
D = treedpth(T)
```

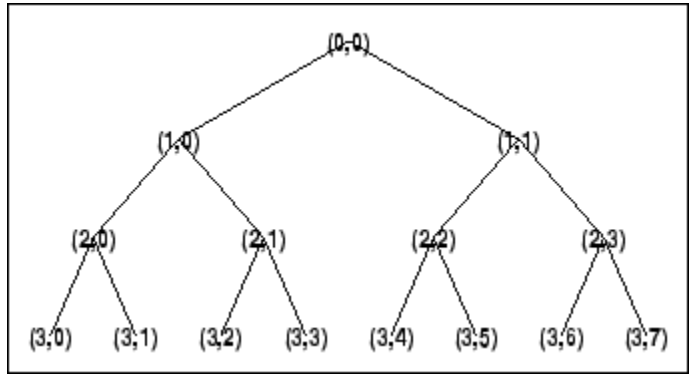
Description

`treedpth` is a tree-management utility.

`D = treedpth(T)` returns the depth `D` of the tree *T*.

Examples

```
% Create binary tree (tree of order 2) of depth 3.
t = ntree(2,3);

% Plot tree t.
plot(t)
```




```
% Tree depth.
treedpth(t)
```

```
ans =
    3
```

See Also

`wtreemgr`

Introduced before R2006a

treeord

Tree order

Syntax

```
ORD = treeord(T)
```

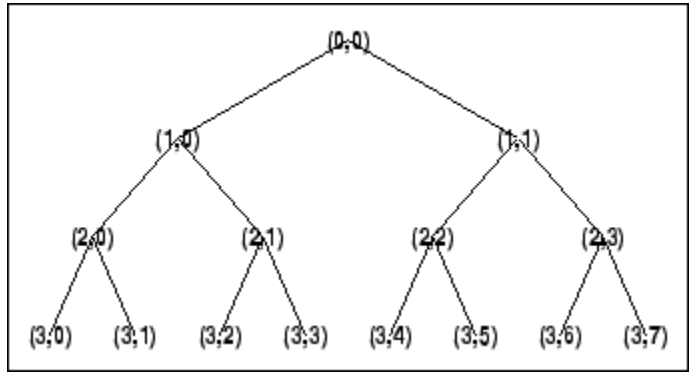
Description

`treeord` is a tree-management utility.

`ORD = treeord(T)` returns the order `ORD` of the tree *T*.

Examples

```
% Create binary tree (tree of order 2) of depth 3.
t = ntree(2,3);

% Plot tree t.
plot(t)
```




```
% Tree order.
treeord(t)
```

```
ans =
    2
```

See Also

`wtreemgr`

Introduced before R2006a

upcoef

Direct reconstruction from 1-D wavelet coefficients

Syntax

```
Y = upcoef(O,X,'wname',N)
Y = upcoef(O,X,'wname',N,L)
Y = upcoef(O,X,Lo_R,Hi_R,N)
Y = upcoef(O,X,Lo_R,Hi_R,N,L)
Y = upcoef(O,X,'wname'')
Y = upcoef(O,X,'wname'',1)
Y = upcoef(O,X,Lo_R,Hi_R)
Y = upcoef(O,X,Lo_R,Hi_R,1)
```

Description

`upcoef` is a one-dimensional wavelet analysis function.

`Y = upcoef(O,X,'wname',N)` computes the `N`-step reconstructed coefficients of vector `X`.

`'wname'` is a string containing the wavelet name. See `wfilters` for more information.

`N` must be a strictly positive integer.

If `O = 'a'`, approximation coefficients are reconstructed.

If `O = 'd'`, detail coefficients are reconstructed.

`Y = upcoef(O,X,'wname',N,L)` computes the `N`-step reconstructed coefficients of vector `X` and takes the length-`L` central portion of the result.

Instead of giving the wavelet name, you can give the filters.

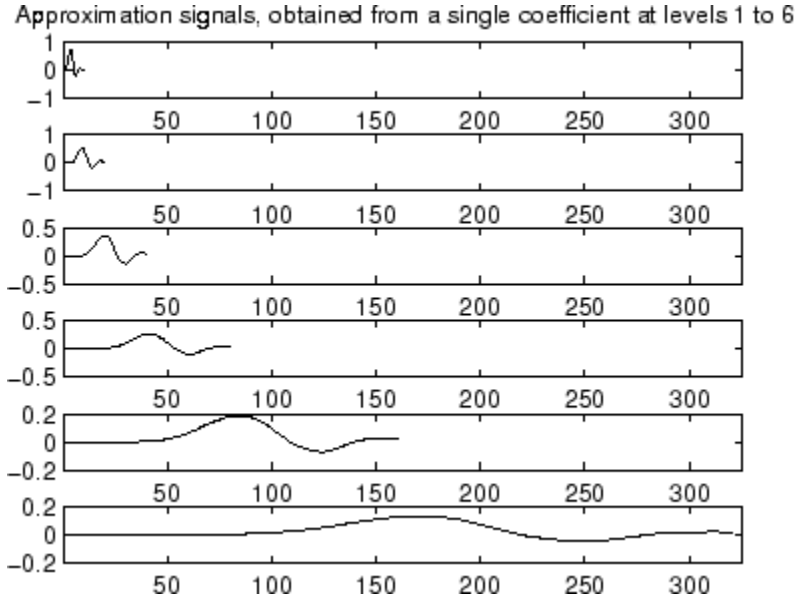
For `Y = upcoef(O,X,Lo_R,Hi_R,N)` or `Y = upcoef(O,X,Lo_R,Hi_R,N,L)`, `Lo_R` is the reconstruction low-pass filter and `Hi_R` is the reconstruction high-pass filter.

`Y = upcoef(O,X,'wname'')` is equivalent to `Y = upcoef(O,X,'wname'',1)`.

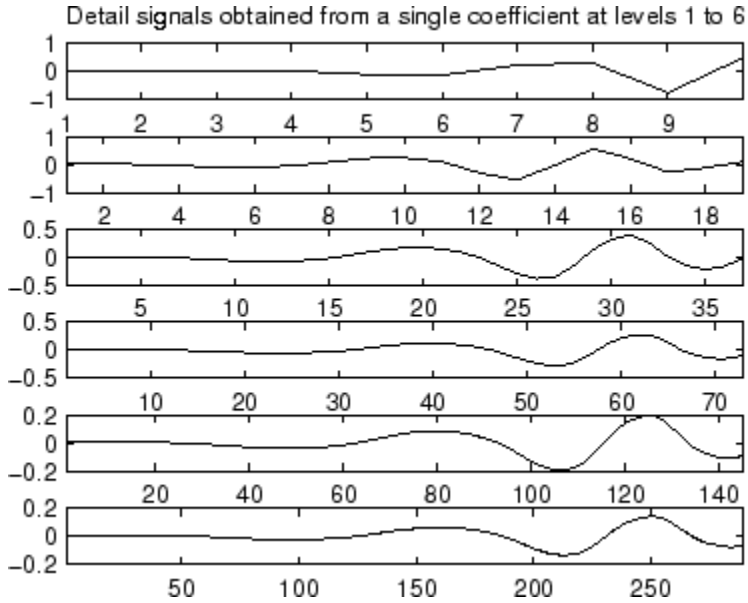
`Y = upcoef(O,X,Lo_R,Hi_R)` is equivalent to `Y = upcoef(O,X,Lo_R,Hi_R,1)`.

Examples

```
% The current extension mode is zero-padding (see dwtmode).

% Approximation signals, obtained from a single coefficient
% at levels 1 to 6.
cfs = [1];  % Decomposition reduced a single coefficient.
essup = 10; % Essential support of the scaling filter db6.
figure(1)
for i=1:6
    % Reconstruct at the top level an approximation
    % which is equal to zero except at level i where only
    % one coefficient is equal to 1.
    rec = upcoef('a',cfs,'db6',i);

    % essup is the essential support of the
    % reconstructed signal.
    % rec(j) is very small when j is ≥ essup.
    ax = subplot(6,1,i),h = plot(rec(1:essup));
    set(ax,'xlim',[1 325]);
    essup = essup*2;

end
subplot(611)
title(['Approximation signals, obtained from a single ' ...
     'coefficient at levels 1 to 6'])

% Editing some graphical properties,
% the following figure is generated.
```




```
% The same can be done for details.
% Details signals, obtained from a single coefficient
% at levels 1 to 6.

cfs = [1];
mi = 12; ma = 30;   % Essential support of
                    % the wavelet filter db6.
rec = upcoef('d',cfs,'db6',1);
figure(2)
subplot(611), plot(rec(3:12))
for i=2:6
    % Reconstruct at top level a single detail
    % coefficient at level i.
    rec = upcoef('d',cfs,'db6',i);
    subplot(6,1,i), plot(rec(mi*2^(i-2):ma*2^(i-2)))
end
subplot(611)
title(['Detail signals obtained from a single ' ...
    'coefficient at levels 1 to 6'])
% Editing some graphical properties,
% the following figure is generated.
```



More About

Algorithms

`upcoef` is equivalent to an `N` time repeated use of the inverse wavelet transform.

See Also

`idwt`

Introduced before R2006a

upcoef2

Direct reconstruction from 2-D wavelet coefficients

Syntax

```
Y = upcoef2(O,X,'wname',N,S)
Y = upcoef2(O,X,Lo_R,Hi_R,N,S)
Y = upcoef2(O,X,'wname',N)
Y = upcoef2(O,X,Lo_R,Hi_R,N)
Y = upcoef2(O,X,'wname')
Y = upcoef2(O,X,'wname',1)
Y = upcoef2(O,X,Lo_R,Hi_R)
Y = upcoef2(O,X,Lo_R,Hi_R,1)
```

Description

`upcoef2` is a two-dimensional wavelet analysis function.

`Y = upcoef2(O,X,'wname',N,S)` computes the `N`-step reconstructed coefficients of matrix `X` and takes the central part of size `S`. `'wname'` is a string containing the name of the wavelet. See `wfilters` for more information.

If `O = 'a'`, approximation coefficients are reconstructed; otherwise if `O = 'h'` (`'v'` or `'d'`, respectively), horizontal (vertical or diagonal, respectively) detail coefficients are reconstructed. `N` must be a strictly positive integer.

Instead of giving the wavelet name, you can give the filters.

For `Y = upcoef2(O,X,Lo_R,Hi_R,N,S)` is the reconstruction low-pass filter and `Hi_R` is the reconstruction high-pass filter.

`Y = upcoef2(O,X,'wname',N)` or `Y = upcoef2(O,X,Lo_R,Hi_R,N)` returns the computed result without any truncation.

`Y = upcoef2(O,X,'wname')` is equivalent to `Y = upcoef2(O,X,'wname',1)`.

`Y = upcoef2(O,X,Lo_R,Hi_R)` is equivalent to
`Y = upcoef2(O,X,Lo_R,Hi_R,1)`.

Examples

```
% The current extension mode is zero-padding (see dwtmode).

% Load original image.
load woman;
% X contains the loaded image.

% Perform decomposition at level 2
% of X using db4.
[c,s] = wavedec2(X,2,'db4');

% Reconstruct approximation and details
% at level 1, from coefficients.
% This can be done using wrcoef2, or
% equivalently using:
%
% Step 1: Extract coefficients from the
% decomposition structure [c,s].
%
% Step 2: Reconstruct using upcoef2.

siz = s(size(s,1),:);

ca1 = appcoef2(c,s,'db4',1);
a1 = upcoef2('a',ca1,'db4',1,siz);

chd1 = detcoef2('h',c,s,1);
hd1 = upcoef2('h',chd1,'db4',1,siz);

cvd1 = detcoef2('v',c,s,1);
vd1 = upcoef2('v',cvd1,'db4',1,siz);

cdd1 = detcoef2('d',c,s,1);
dd1 = upcoef2('d',cdd1,'db4',1,siz);
```

More About

Algorithms

See `upcoef`.

See Also

`idwt2`

Introduced before R2006a

upwlev

Single-level reconstruction of 1-D wavelet decomposition

Syntax

```
[NC,NL,cA] = upwlev(C,L,'wname')
```

Description

`upwlev` is a one-dimensional wavelet analysis function.

`[NC,NL,cA] = upwlev(C,L,'wname')` performs the single-level reconstruction of the wavelet decomposition structure `[C,L]` giving the new one `[NC,NL]`, and extracts the last approximation coefficients vector `cA`.

`[C,L]` is a decomposition at level `n = length(L)-2`, so `[NC,NL]` is the same decomposition at level `n`-1 and `cA` is the approximation coefficients vector at level `n`.

`'wname'` is a string containing the wavelet name, `C` is the original wavelet decomposition vector, and `L` the corresponding bookkeeping vector (for detailed storage information, see `wavedec`).

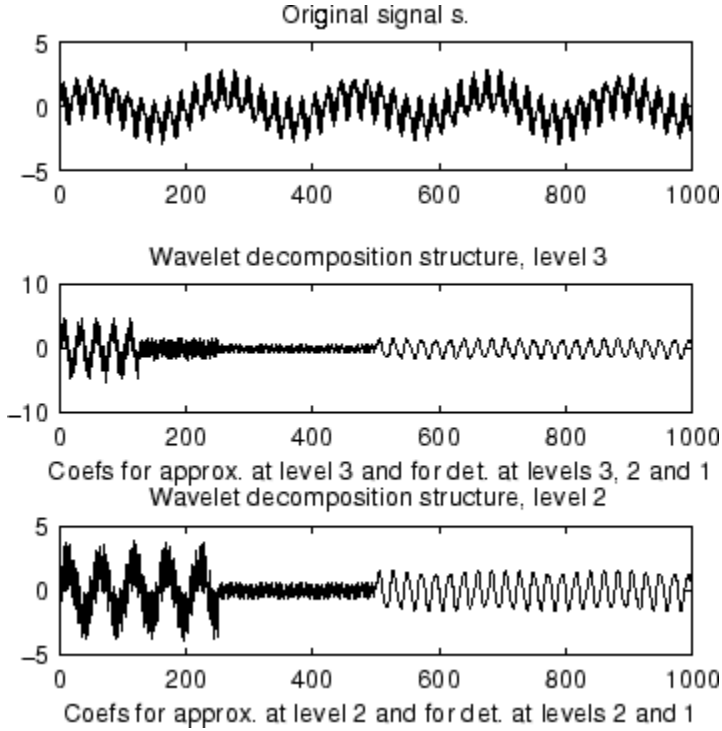
Instead of giving the wavelet name, you can give the filters.

For `[NC,NL,cA] = upwlev(C,L,Lo_R,Hi_R)`, `Lo_R` is the reconstruction low-pass filter and `Hi_R` is the reconstruction high-pass filter.

Examples

```
% The current extension mode is zero-padding (see dwtmode).

% Load original one-dimensional signal.
load sumsin; s = sumsin;

% Perform decomposition at level 3 of s using db1.
[c,l] = wavedec(s,3,'db1');
```

```
subplot(311); plot(s);
title('Original signal s.');
subplot(312); plot(c);
title('Wavelet decomposition structure, level 3')
xlabel(['Coefs for approx. at level 3 ' ...
        'and for det. at levels 3, 2 and 1'])

% One step reconstruction of the wavelet decomposition
% structure at level 3 [c,l], so the new structure [nc,nl]
% is the wavelet decomposition structure at level 2.
[nc,nl] = upwlev(c,l,'db1');
subplot(313); plot(nc);
title('Wavelet decomposition structure, level 2')
xlabel(['Coefs for approx. at level 2 ' ...
        'and for det. at levels 2 and 1'])

% Editing some graphical properties,
% the following figure is generated.
```



More About

- `upcoef`
- `wavedec`

See Also

`idwt`

Introduced before R2006a

upwlev2

Single-level reconstruction of 2-D wavelet decomposition

Syntax

```
[NC,NS,cA] = upwlev2(C,S,'wname')
[NC,NS,cA] = upwlev2(C,S,Lo_R,Hi_R)
```

Description

`upwlev2` is a two-dimensional wavelet analysis function.

`[NC,NS,cA] = upwlev2(C,S,'wname')` performs the single-level reconstruction of wavelet decomposition structure `[C,S]` giving the new one `[NC,NS]`, and extracts the last approximation coefficients matrix `cA`.

`[C,S]` is a decomposition at level `n = size(S,1)-2`, so `[NC,NS]` is the same decomposition at level `n`-1 and `cA` is the approximation matrix at level `n`.

`'wname'` is a string containing the wavelet name, `C` is the original wavelet decomposition vector, and `S` the corresponding bookkeeping matrix (for detailed storage information, see `wavedec2`).

Instead of giving the wavelet name, you can give the filters.

For `[NC,NS,cA] = upwlev2(C,S,Lo_R,Hi_R)`, `Lo_R` is the reconstruction low-pass filter and `Hi_R` is the reconstruction high-pass filter.

Examples

```
% The current extension mode is zero-padding (see dwtmode).

% Load original image.
load woman;
% X contains the loaded image.
```

```
% Perform decomposition at level 2
% of X using db1.
[c,s] = wavedec2(X,2,'db1');
sc = size(c)

sc =
    1   65536

val_s = s

val_s =
    64   64
    64   64
    128  128
    256  256

% One step reconstruction of wavelet
% decomposition structure [c,s].
[nc,ns] = upwlev2(c,s,'db1');
snc = size(nc)

snc =
    1   65536

val_ns = ns

val_ns =
    128   128
    128   128
    256   256
```

See Also

idwt2 | upcoef2 | wavedec2

Introduced before R2006a

wave2lp

Laurent polynomials associated with wavelet

Syntax

```
[Hs,Gs,Ha,Ga] = wave2lp(W)
```

Description

`[Hs,Gs,Ha,Ga] = wave2lp(W)` returns the four Laurent polynomials associated with the wavelet `W` (see `liftwave`).

The pairs `(Hs,Gs)` and `(Ha,Ga)` are the synthesis and the analysis pair respectively.

The `H`-polynomials (`G`-polynomials) are low-pass (high-pass) polynomials.

For an orthogonal wavelet, `Hs` = `Ha` and `Gs` = `Ga`.

Examples

```
% Get Laurent polynomials associated to the "lazy" wavelet.
[Hs,Gs,Ha,Ga] = wave2lp('lazy')

Hs(z) = 1

Gs(z) = z^(-1)

Ha(z) = 1

Ga(z) = z^(-1)

% Get Laurent polynomials associated to the db1 wavelet.
[Hs,Gs,Ha,Ga] = wave2lp('db1')

Hs(z) = + 0.7071 + 0.7071*z^(-1)

Gs(z) = - 0.7071 + 0.7071*z^(-1)
```

```
Ha(z) = + 0.7071 + 0.7071*z^(-1)

Ga(z) = - 0.7071 + 0.7071*z^(-1)

% Get Laurent polynomials associated to the bior1.3 wavelet.
[Hs,Gs,Ha,Ga] = wave2lp('bior1.3')

Hs(z) = + 0.7071 + 0.7071*z^(-1)

Gs(z) = ...
    + 0.08839*z^(+2) + 0.08839*z^(+1) - 0.7071 + 0.7071*z^(-1) -
0.08839*z^(-2)  ...
    - 0.08839*z^(-3)

Ha(z) = ...
    - 0.08839*z^(+2) + 0.08839*z^(+1) + 0.7071 + 0.7071*z^(-1) +
0.08839*z^(-2)  ...
    - 0.08839*z^(-3)

Ga(z) = - 0.7071 + 0.7071*z^(-1)
```

See Also

`laurpoly`

Introduced before R2006a

wavedec

Multilevel 1-D wavelet decomposition

Syntax

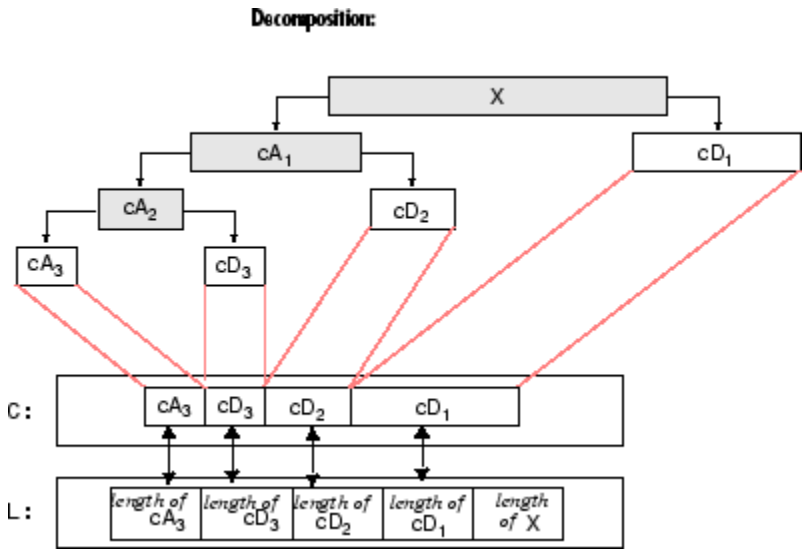
```
[C,L] = wavedec(X,N,'wname')
[C,L] = wavedec(X,N,Lo_D,Hi_D)
```

Description

`wavedec` performs a multilevel one-dimensional wavelet analysis using either a specific wavelet (`'wname'`) or a specific wavelet decomposition filters (`Lo_D` and `Hi_D`, see `wfilters`).

Note `wavedec` supports only Type 1 (orthogonal) or Type 2 (biorthogonal) wavelets.

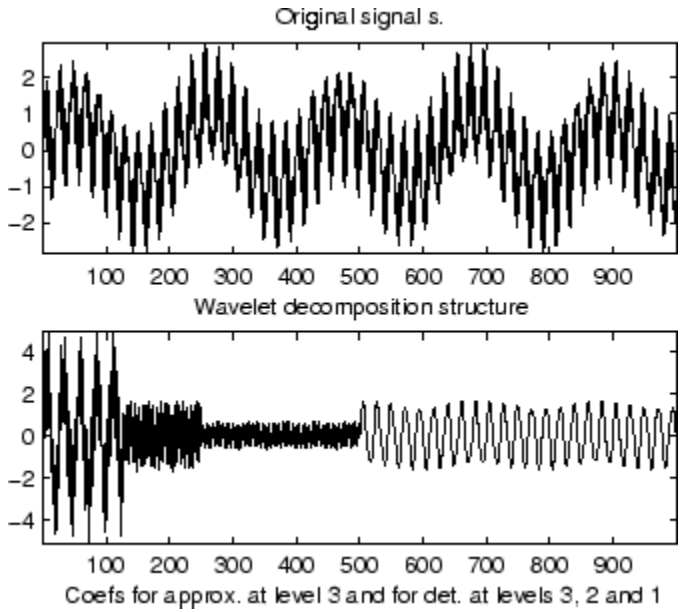
`[C,L] = wavedec(X,N,'wname')` returns the wavelet decomposition of the signal `X` at level `N`, using `'wname'`. `N` must be a strictly positive integer (see `wmaxlev` for more information). The output decomposition structure contains the wavelet decomposition vector `C` and the bookkeeping vector `L`. The structure is organized as in this level-3 decomposition example.



`[C,L] = wavedec(X,N,Lo_D,Hi_D)` returns the decomposition structure as above, given the low- and high-pass decomposition filters you specify.

Examples

```
% The current extension mode is zero-padding (see dwtmode).

% Load original one-dimensional signal.
load sumsin; s = sumsin;
% Perform decomposition at level 3 of s using db1.
[c,l] = wavedec(s,3,'db1');
% Using some plotting commands,
% the following figure is generated.
```

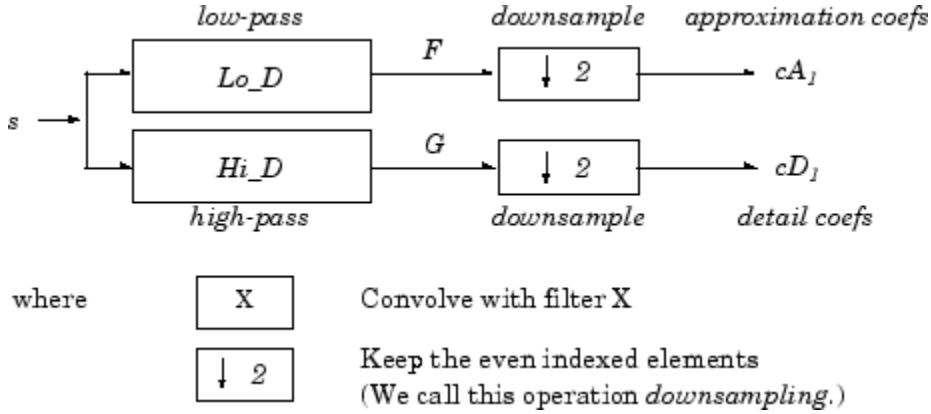


More About

Algorithms

Given a signal s of length N, the DWT consists of $\log_2 N$ stages at most. The first step produces, starting from s, two sets of coefficients: approximation coefficients CA_1, and detail coefficients CD_1. These vectors are obtained by convolving s with the low-pass filter `Lo_D` for approximation, and with the high-pass filter `Hi_D` for detail, followed by dyadic decimation (downsampling).

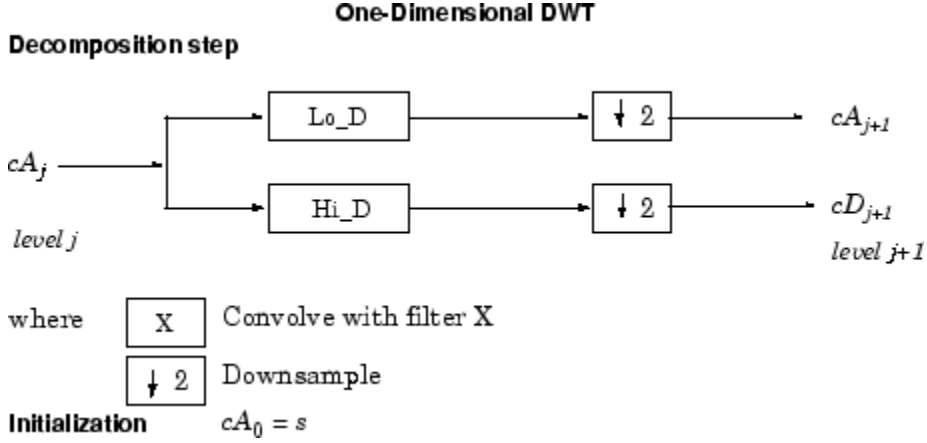
More precisely, the first step is



The length of each filter is equal to $2N$. If n = length(s), the signals F and G are of length $n + 2N - 1$ and the coefficients cA_1 and cD_1 are of length

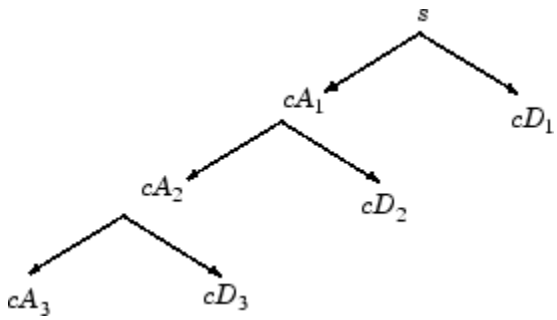
$$\text{floor}\left(\frac{n-1}{2}\right)+N$$

The next step splits the approximation coefficients cA_1 in two parts using the same scheme, replacing s by cA_1, and producing cA_2 and cD_2, and so on



The wavelet decomposition of the signal s analyzed at level j has the following structure: $[cA_j, cD_j, \ldots, cD_1]$.

This structure contains, for $J = 3$, the terminal nodes of the following tree:



References

Daubechies, I. (1992), *Ten lectures on wavelets*, CBMS-NSF conference series in applied mathematics. SIAM Ed.

Mallat, S. (1989), "A theory for multiresolution signal decomposition: the wavelet representation," *IEEE Pattern Anal. and Machine Intell.*, vol. 11, no. 7, pp 674–693.

Meyer, Y. (1990), *Ondelettes et opérateurs*, Tome 1, Hermann Ed. (English translation: *Wavelets and operators*, Cambridge Univ. Press. 1993.)

See Also

`dwt` | `waveinfo` | `waverec` | `wfilters` | `wmaxlev`

Introduced before R2006a

wavedec2

Multilevel 2-D wavelet decomposition

Syntax

```
[C,S] = wavedec2(X,N,'wname')
[C,S] = wavedec2(X,N,Lo_D,Hi_D)
```

Description

`wavedec2` is a two-dimensional wavelet analysis function.

`[C,S] = wavedec2(X,N,'wname')` returns the wavelet decomposition of the matrix `X` at level `N`, using the wavelet named in string `'wname'` (see `wfilters` for more information).

Outputs are the decomposition vector `C` and the corresponding bookkeeping matrix `S`.

`N` must be a strictly positive integer (see `wmaxlev` for more information).

Instead of giving the wavelet name, you can give the filters.

For `[C,S] = wavedec2(X,N,Lo_D,Hi_D)`, `Lo_D` is the decomposition low-pass filter and `Hi_D` is the decomposition high-pass filter.

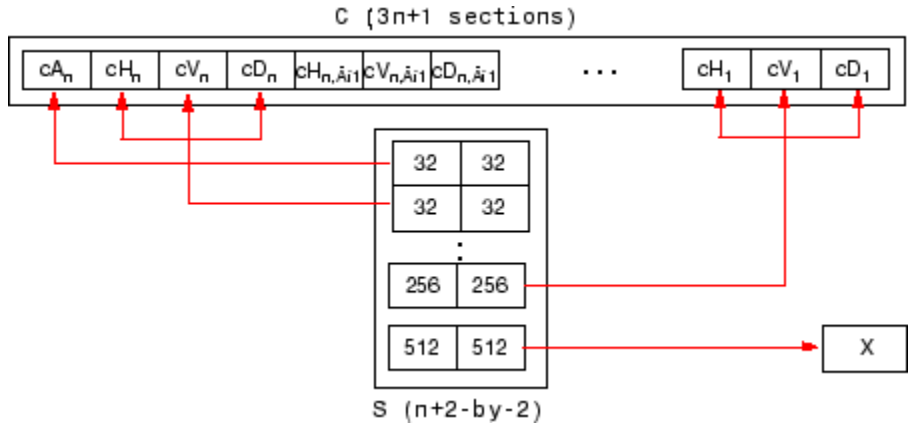
Vector `C` is organized as a vector with A(N), H(N), V(N), D(N), H(N-1), V(N-1), D(N-1), ..., H(1), V(1), D(1), where `A`, `H`, `V`, and `D` are each a row vector. Each vector is the vector column-wise storage of a matrix.

- `A` contains the approximation coefficients
- `H` contains the horizontal detail coefficients
- `V` contains the vertical detail coefficients
- `D` contains the diagonal detail coefficients
-

Matrix `S` is such that

- `S(1,:)` = size of approximation coefficients(`N`).

- `S(i,:)` = size of detail coefficients(`N-i+2`) for `i` = 2, ...`N+1` and `S(N+2,:) = size(X)`.



Examples

Decomposition Structure

This example shows the structure of the `wavedec2` output matrices.

```
% The current extension mode is zero-padding (see dwtmode).

% Load original image.
load woman;
% X contains the loaded image.

% Perform decomposition at level 2
% of X using db1.
[c,s] = wavedec2(X,2,'db1');

% Decomposition structure organization.
sizex = size(X)

sizex =
    256  256
sizec = size(c)
```

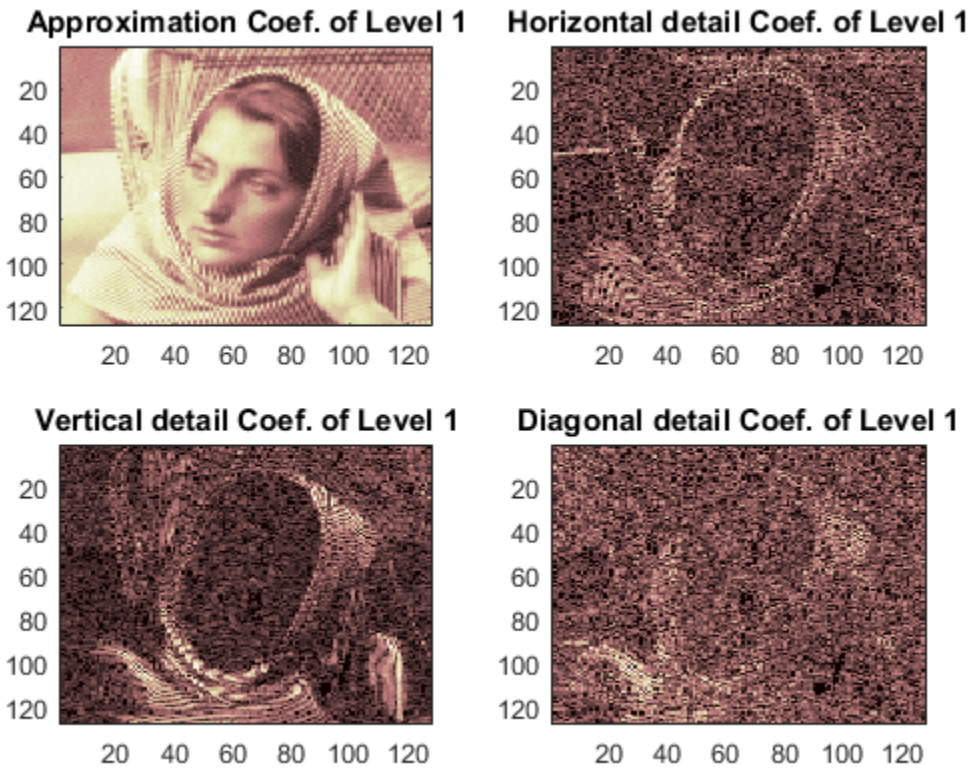
```
sizec =
    1  65536
    val_s = s

val_s =
    64  64
    64  64
    128  128
    256  256
```

Extract and Display Image Decomposition Level

Extract and display images of wavelet decomposition level details. The resulting images are similar to the visualizations in the *At level 2, with haar --> woman* indexed image example in the Wavelet 2-D interactive tool. See wavemenu to launch this tool.

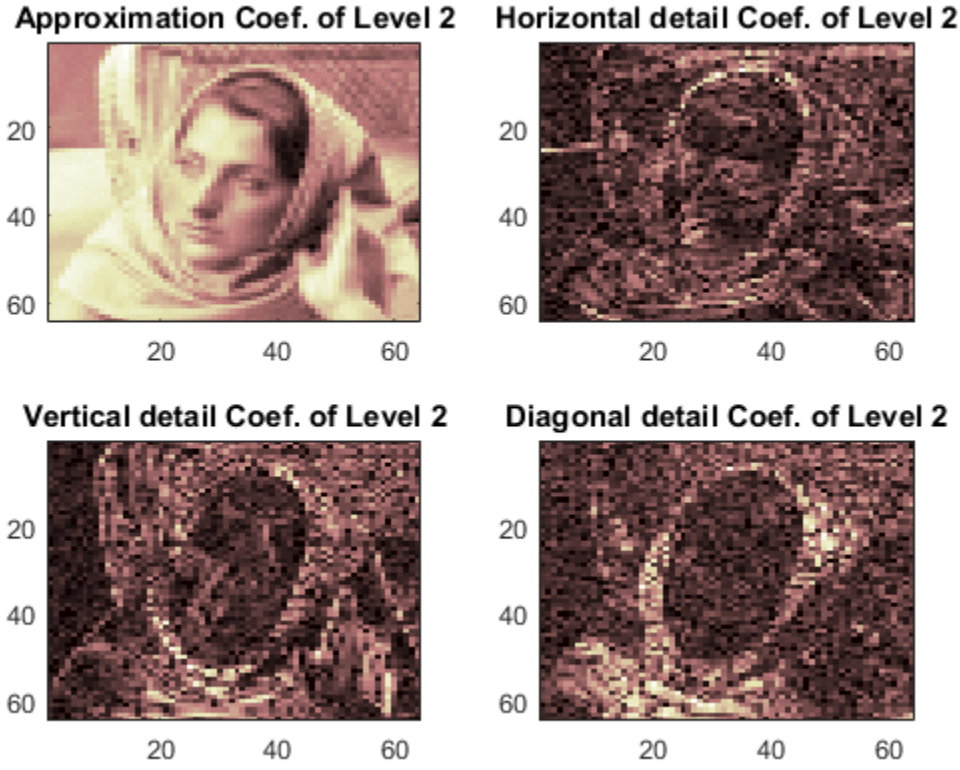
```
load woman;
[c,s]=wavedec2(X,2,'haar');

[H1,V1,D1] = detcoef2('all',c,s,1);
A1 = appcoef2(c,s,'haar',1);
V1img = wcodemat(V1,255,'mat',1);
H1img = wcodemat(H1,255,'mat',1);
D1img = wcodemat(D1,255,'mat',1);
A1img = wcodemat(A1,255,'mat',1);

[H2,V2,D2] = detcoef2('all',c,s,2);
A2 = appcoef2(c,s,'haar',2);
V2img = wcodemat(V2,255,'mat',1);
H2img = wcodemat(H2,255,'mat',1);
D2img = wcodemat(D2,255,'mat',1);
A2img = wcodemat(A2,255,'mat',1);

subplot(2,2,1);
imagesc(A1img);
colormap pink(255);
title('Approximation Coef. of Level 1');

subplot(2,2,2);
imagesc(H1img);
title('Horizontal detail Coef. of Level 1');

subplot(2,2,3);
imagesc(V1img);
title('Vertical detail Coef. of Level 1');
```

```
subplot(2,2,4);
imagesc(D1img);
title('Diagonal detail Coef. of Level 1');
```




```
figure;
subplot(2,2,1);
imagesc(A2img);
colormap pink(255);
title('Approximation Coef. of Level 2');

subplot(2,2,2)
imagesc(H2img);
title('Horizontal detail Coef. of Level 2');
```

```
subplot(2,2,3)
imagesc(V2img);
title('Vertical detail Coef. of Level 2');

subplot(2,2,4)
imagesc(D2img);
title('Diagonal detail Coef. of Level 2');
```

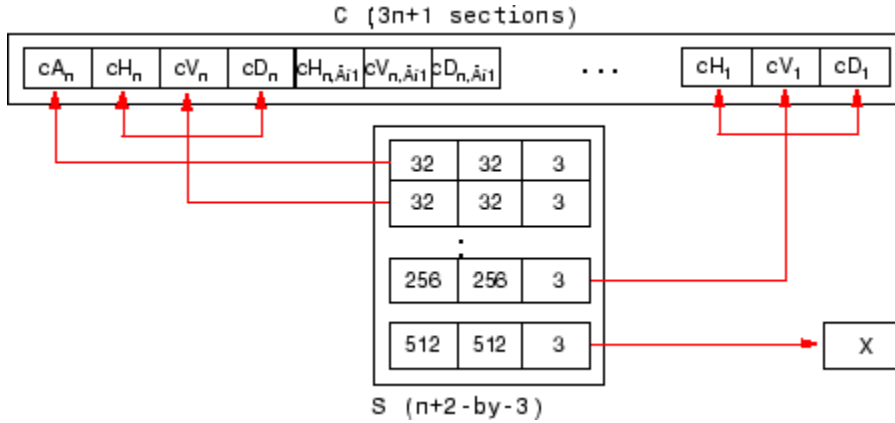


More About

Tips

When X represents an indexed image, X, as well as the output arrays cA,cH,cV, and cD are `m`-by-`n` matrices. When X represents a truecolor image, it is an `m`-by-`n`-by-3 array, where each `m`-by-`n` matrix represents a red, green, or blue color plane concatenated along the third dimension. The size of vector C and the size of matrix `S` depend on the type of analyzed image.

For a truecolor image, the decomposition vector `C` and the corresponding bookkeeping matrix `S` can be represented as follows.



For more information on image formats, see the `image` and `imfinfo` reference pages.

Algorithms

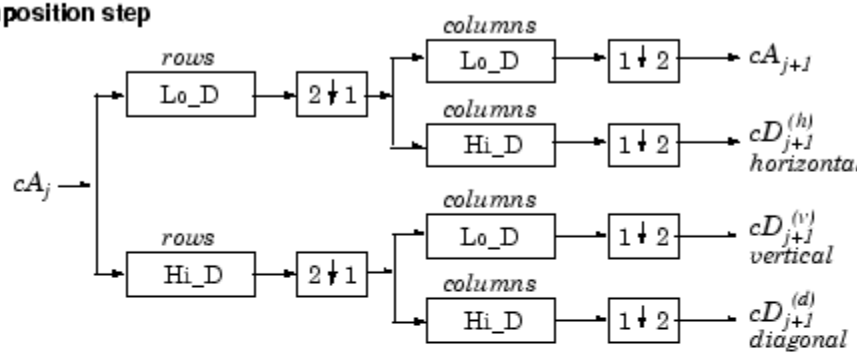
For images, an algorithm similar to the one-dimensional case is possible for two-dimensional wavelets and scaling functions obtained from one-dimensional ones by tensor product.

This kind of two-dimensional DWT leads to a decomposition of approximation coefficients at level *j* in four components: the approximation at level *j*+1, and the details in three orientations (horizontal, vertical, and diagonal).

The following chart describes the basic decomposition step for images:

Two-Dimensional DWT

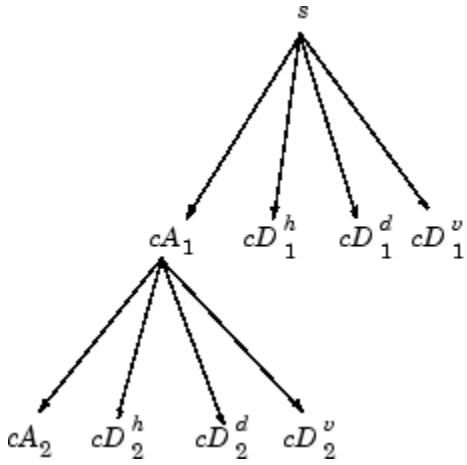
Decomposition step



where
- $2\downarrow 1$ Downsample columns: keep the even indexed columns
- $1\downarrow 2$ Downsample rows: keep the even indexed rows
- *rows* X Convolve with filter X the rows of the entry
- *columns* X Convolve with filter X the columns of the entry

Initialization $cA_0 = s$ for the decomposition initialization

So, for J=2, the two-dimensional wavelet tree has the form



References

Daubechies, I. (1992), *Ten lectures on wavelets*, CBMS-NSF conference series in applied mathematics. SIAM Ed.

Mallat, S. (1989), "A theory for multiresolution signal decomposition: the wavelet representation," *IEEE Pattern Anal. and Machine Intell.*, vol. 11, no. 7, pp. 674–693.

Meyer, Y. (1990), *Ondelettes et opérateurs*, Tome 1, Hermann Ed. (English translation: *Wavelets and operators*, Cambridge Univ. Press. 1993.)

See Also

`dwt` | `waveinfo` | `waverec2` | `wfilters` | `wmaxlev`

Introduced before R2006a

wavedec3

Multilevel 3-D wavelet decomposition

Syntax

```
WDEC = wavedec3(X,N,'wname')
WDEC = wavedec3(X,N,'wname','mode','ExtM')
WDEC = wavedec3(X,N,{LoD,HiD,LoR,HiR})
```

Description

`wavedec3` is a three-dimensional wavelet analysis function.

`WDEC = wavedec3(X,N,'wname')` returns the wavelet decomposition of the 3-D array `X` at level `N`, using the wavelet named in string `'wname'` or the particular wavelet filters you specify. It uses the default extension mode `'sym'`. See `dwtmode`. `N` must be a positive integer.

`WDEC = wavedec3(X,N,'wname','mode','ExtM')` uses the specified DWT extension mode .

`WDEC = wavedec3(X,N,{LoD,HiD,LoR,HiR})` uses the decomposition and reconstruction filters you specify in a cell array.

`WDEC` is the output decomposition structure, with the following fields:

`sizeINI`	Size of the three-dimensional array `X`
`level`	Level of the decomposition
`mode`	Name of the wavelet transform extension mode
`filters`	Structure with 4 fields, `LoD`, `HiD`, `LoR`, `HiR`, which contain the filters used for the DWT.
`dec`	`N` x 1 cell array containing the coefficients of the decomposition. `N` is equal to `7*WDEC.level+1`.

	`dec{1}` contains the lowpass component (approximation) at the level of the decomposition. The approximation is equivalent to the filtering operations `'LLL'`. `dec{k+2},...,dec{k+8}` with `k = 0,7,14,...,7*(WDEC.level-1)` contain the 3-D wavelet coefficients for the multiresolution starting with the coarsest level when `k=0`. For example, if `WDEC.level=3`, `dec{2},...,dec{8}` contain the wavelet coefficients for level 3 (`k=0`), `dec{9},...,dec{15}` contain the wavelet coefficients for level 2 (`k=7`), and `dec{16},...,dec{22}` contain the wavelet coefficients for level 1 (`k=7*(WDEC.level-1)`). At each level, the wavelet coefficients in `dec{k+2},...,dec{k+8}` are in the following order: `'HLL','LHL','HHL','LLH','HLH','LHH','HHH'`. The strings give the order in which the separable filtering operations are applied from left to right. For example, `'LHH'` means that the lowpass (scaling) filter with downsampling is applied to the rows of `X`, followed by the highpass (wavelet) filter with downsampling applied to the columns of `X`. Finally, the highpass filter with downsampling is applied to the 3rd dimension of `X`.
`sizes`	Successive sizes of the decomposition components

Examples

3-D Wavelet Transform

Find the 3-D DWT of a volume.

Construct 8-by-8-by-8 matrix. Obtain the 3-D discrete wavelet transform at level 1 using the Haar wavelet and the default whole-point symmetric extension mode.

```
% Matrix of integers 1:64
M = magic(8);
% Make data 3-D
```

```
X = repmat(M,[1 1 8]);
% Decompose X at level 1 using db1.
wd1 = wavedec3(X,1,'db1');
```

3-D Wavelet Transform Using Specified Decomposition and Reconstruction Filters

Specify the decomposition and reconstruction filters as a cell array.

Construct 8-by-8-by-8 matrix. Obtain the 3-D discrete wavelet transform down to level 2 using the Daubechies extremal phase wavelet with two vanishing moments. Input the decomposition and reconstruction filters as a cell array. Use the periodic extension mode.

```
% Matrix of integers 1:64
M = magic(8);
% Make data 3-D
X = repmat(M,[1 1 8]);
[LoD,HiD,LoR,HiR] = wfilters('db2');
wd2 = wavedec3(X,2,{LoD,HiD,LoR,HiR},'mode','per');
```

Coefficient Order in 3-D Wavelet Transform

Compare the output of `wavedec3` and `dwt3` to illustrate the ordering of the 3-D wavelet coefficients described in the `dec` field description.

```
X = reshape(1:512,8,8,8);
dwtOut = dwt3(X,'db1','mode','per');
wdec = wavedec3(X,1,'db1','mode','per');
max(abs((wdec.dec{4}(:)-dwtOut.dec{2,2,1}(:))))
max(abs((wdec.dec{5}(:)-dwtOut.dec{1,1,2}(:))))
```

See Also

`dwt3` | `dwtmode` | `waveinfo` | `waverec3` | `wfilters` | `wmaxlev`

Introduced in R2010a

wavedemo

Wavelet Toolbox software examples

Syntax

Description

`wavedemo` opens a GUI that allows you to choose between several Wavelet Toolbox examples.

Introduced before R2006a

wavefun

Wavelet and scaling functions

Syntax

```
[PHI,PSI,XVAL] = wavefun('wname',ITER)
[PHI1,PSI1,PHI2,PSI2,XVAL] = wavefun('wname',ITER)
[PHI,PSI,XVAL] = wavefun('wname',ITER)
[PSI,XVAL] = wavefun('wname',ITER)
[...] = wavefun(wname,A,B)
[...] = wavefun('wname',max(A,B))
[...] = wavefun('wname',0)
[...] = wavefun('wname',8,0)
[...] = wavefun('wname')
[...] = wavefun('wname',8)
```

Description

The function `wavefun` returns approximations of the wavelet function `'wname'` and the associated scaling function, if it exists. The positive integer `ITER` determines the number of iterations computed; thus, the refinement of the approximations.

For an orthogonal wavelet:

`[PHI,PSI,XVAL] = wavefun('wname',ITER)` returns the scaling and wavelet functions on the points grid `XVAL`.

For a biorthogonal wavelet:

`[PHI1,PSI1,PHI2,PSI2,XVAL] = wavefun('wname',ITER)` returns the scaling and wavelet functions both for decomposition `(PHI1,PSI1)` and for reconstruction `(PHI2,PSI2)`.

For a Meyer wavelet:

`[PHI,PSI,XVAL] = wavefun('wname',ITER)`

For a wavelet without scaling function (e.g., Morlet, Mexican Hat, Gaussian derivatives wavelets or complex wavelets):

```
[PSI,XVAL] = wavefun('wname',ITER)
```

`[...] = wavefun(wname,A,B)`, where `A` and `B` are positive integers, is equivalent to `[...] = wavefun('wname',max(A,B))`, and draws plots.

When `A` is set equal to the special value 0,

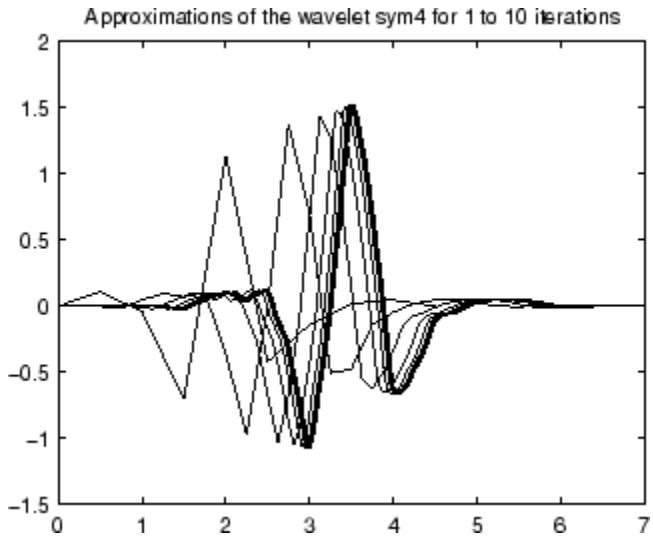
- `[...] = wavefun('wname',0)` is equivalent to
- `[...] = wavefun('wname',8,0)`.
- `[...] = wavefun('wname')` is equivalent to
- `[...] = wavefun('wname',8)`.

The output arguments are optional.

Examples

On the following graph, 10 piecewise linear approximations of the `sym4` wavelet obtained after each iteration of the cascade algorithm are shown.

```
% Set number of iterations and wavelet name.
iter = 10;
wav = 'sym4';

% Compute approximations of the wavelet function using the
% cascade algorithm.
for i = 1:iter
    [phi,psi,xval] = wavefun(wav,i);
    plot(xval,psi);
    hold on
end
title(['Approximations of the wavelet ',wav, ...
       ' for 1 to ',num2str(iter),' iterations']);
hold off
```



More About

Algorithms

For compactly supported wavelets defined by filters, in general no closed form analytic formula exists.

The algorithm used is the cascade algorithm. It uses the single-level inverse wavelet transform repeatedly.

Let us begin with the scaling function ϕ.

Since ϕ is also equal to $\phi_{0,0}$, this function is characterized by the following coefficients in the orthogonal framework:

- $<\phi, \phi_{0,n}> = 1$ only if $n = 0$ and equal to 0 otherwise
- $<\phi, \psi_{-j,k}> = 0$ for positive j, and all k.

This expansion can be viewed as a wavelet decomposition structure. Detail coefficients are all zeros and approximation coefficients are all zeros except one equal to 1.

Then we use the reconstruction algorithm to approximate the function ϕ over a dyadic grid, according to the following result:

For any dyadic rational of the form $x = n2^{-j}$ in which the function is continuous and where j is sufficiently large, we have pointwise convergence and

$$\left| \phi(x) - 2^{\frac{j}{2}} \langle \phi, \phi_{-j,\, n\,2^{j-J}} \rangle \right| \le C.2^{-j\alpha}$$

where C is a constant, and α is a positive constant depending on the wavelet regularity.

Then using a good approximation of ϕ on dyadic rationals, we can use piecewise constant or piecewise linear interpolations η on dyadic intervals, for which uniform convergence occurs with similar exponential rate:

$$\|\phi - \eta\|_{\infty} \le C.2^{-j\alpha}$$

So using a J-step reconstruction scheme, we obtain an approximation that converges exponentially towards ϕ when J goes to infinity.

Approximations are computed over a grid of dyadic rationals covering the support of the function to be approximated.

Since a scaled version of the wavelet function ψ can also be expanded on the $(\phi_{-1,n})_n$, the same scheme can be used, after a single-level reconstruction starting with the appropriate wavelet decomposition structure. Approximation coefficients are all zeros and detail coefficients are all zeros except one equal to 1.

For biorthogonal wavelets, the same ideas can be applied on each of the two multiresolution schemes in duality.

Note This algorithm may diverge if the function to be approximated is not continuous on dyadic rationals.

References

Daubechies, I., *Ten lectures on wavelets*, CBMS, SIAM, 1992, pp. 202Äì213.

Strang, G.; T. Nguyen (1996), *Wavelets and Filter Banks*, Wellesley-Cambridge Press.

See Also

`intwave` | `waveinfo` | `wfilters`

Introduced before R2006a

wavefun2

Wavelet and scaling functions 2-D

Syntax

```
[PHI,PSI,XVAL] = wavefun('wname',ITER)
[S,W1,W2,W3,XYVAL] = wavefun2('wname',ITER,'plot')
[S,W1,W2,W3,XYVAL] = wavefun2(wname,A,B)
[S,W1,W2,W3,XYVAL] = wavefun2('wname',max(A,B))
[S,W1,W2,W3,XYVAL] = wavefun2('wname',0)
[S,W1,W2,W3,XYVAL] = wavefun2('wname',4,0)
[S,W1,W2,W3,XYVAL] = wavefun2('wname')
[S,W1,W2,W3,XYVAL] = wavefun2('wname',4)
```

Description

For an orthogonal wavelet `'wname'`, `wavefun2` returns the scaling function and the three wavelet functions resulting from the tensor products of the one-dimensional scaling and wavelet functions.

If `[PHI,PSI,XVAL] = wavefun('wname',ITER)`, the scaling function `S` is the tensor product of `PHI` and `PSI`.

The wavelet functions `W1`, `W2`, and `W3` are the tensor products (`PHI`,`PSI`), (`PSI`,`PHI`), and (`PSI`,`PSI`), respectively.

The two-dimensional variable `XYVAL` is a 2^{ITER} x 2^{ITER} points grid obtained from the tensor product (`XVAL`,`XVAL`).

The positive integer `ITER` determines the number of iterations computed and thus, the refinement of the approximations.

`[S,W1,W2,W3,XYVAL] = wavefun2('wname',ITER,'plot')` computes and also plots the functions.

`[S,W1,W2,W3,XYVAL] = wavefun2(wname,A,B)`, where `A` and `B` are positive integers, is equivalent to

`[S,W1,W2,W3,XYVAL] = wavefun2('wname',max(A,B))`. The resulting functions are plotted.

When `A` is set equal to the special value 0,

- `[S,W1,W2,W3,XYVAL] = wavefun2('wname',0)` is equivalent to `[S,W1,W2,W3,XYVAL] = wavefun2('wname',4,0)`.
- `[S,W1,W2,W3,XYVAL] = wavefun2('wname')` is equivalent to `[S,W1,W2,W3,XYVAL] = wavefun2('wname',4)`.

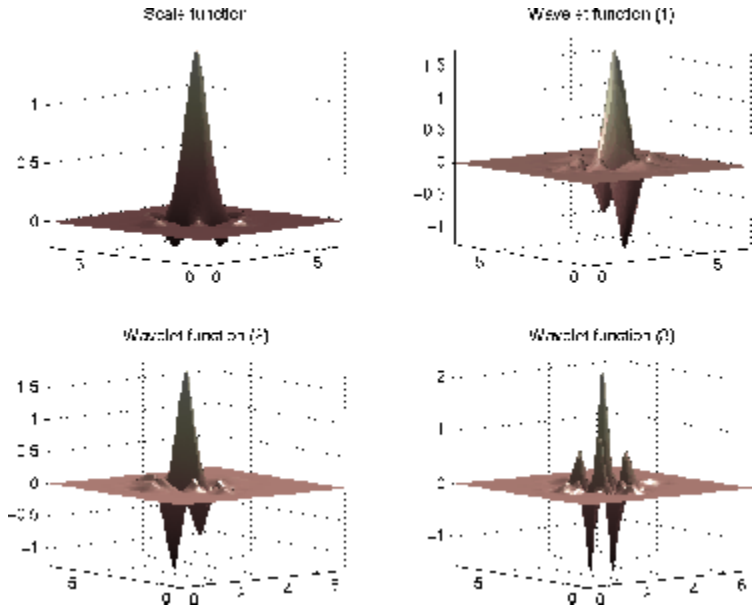
The output arguments are optional.

Note The `wavefun2` function can only be used with an orthogonal wavelet.

Examples

On the following graph, a linear approximation of the `sym4` wavelet obtained using the cascade algorithm is shown.

```
% Set number of iterations and wavelet name.
iter = 4;
wav = 'sym4';

% Compute approximations of the wavelet and scale functions using
% the cascade algorithm and plot.
[s,w1,w2,w3,xyval] = wavefun2(wav,iter,0);
```



More About

Algorithms

See `wavefun` for more information.

References

Daubechies, I., *Ten lectures on wavelets*, CBMS, SIAM, 1992, pp. 202Äì213.

Strang, G.; T. Nguyen (1996), *Wavelets and Filter Banks*, Wellesley-Cambridge Press.

See Also

`intwave` | `wavefun` | `waveinfo` | `wfilters`

Introduced before R2006a

waveinfo

Wavelets information

Syntax

```
waveinfo('wname')
```

Description

`waveinfo` provides information on all wavelets within the toolbox.

`waveinfo('wname')` provides information on the wavelet family whose short name is specified by the string `'wname'`. Available family short names are listed in the table below.

Wavelet Family Short Name	Wavelet Family Name
`'haar'`	Haar wavelet
`'db'`	Daubechies wavelets
`'sym'`	Symlets
`'coif'`	Coiflets
`'bior'`	Biorthogonal wavelets
`'fk'`	Fejer-Korovkin filters
`'rbio'`	Reverse biorthogonal wavelets
`'meyr'`	Meyer wavelet
`'dmey'`	Discrete approximation of Meyer wavelet
`'gaus'`	Gaussian wavelets
`'mexh'`	Mexican hat wavelet
`'morl'`	Morlet wavelet
`'cgau'`	Complex Gaussian wavelets

Wavelet Family Short Name	Wavelet Family Name
'shan'	Shannon wavelets
'fbsp'	Frequency B-Spline wavelets
'cmor'	Complex Morlet wavelets

The family short names can also be user-defined ones (see `wavemngr` for more information).

`waveinfo('wsys')` provides information on wavelet packets.

Examples

```
waveinfo('db')

DBINFO Information on Daubechies wavelets.
Daubechies Wavelets
General characteristics: Compactly supported
wavelets with extremal phase and highest
number of vanishing moments for a given
support width. Associated scaling filters are
minimum-phase filters.

Family          Daubechies
Short name      db
Order N         N strictly positive integer
Examples        db1 or haar, db4, db15

Orthogonal      yes
Biorthogonal    yes
Compact support yes
DWT             possible
CWT             possible

Support width   2N-1
Filters length  2N
Regularity      about 0.2 N for large N
Symmetry        far from
Number of vanishing moments for psi     N

Reference:   I. Daubechies,
Ten lectures on wavelets CBMS, SIAM, 61, 1994, 194-202.
```

See Also

wavemngr

Introduced before R2006a

waveletfamilies

Wavelet families and family members

Syntax

```
waveletfamilies('f')
waveletfamilies('n')
waveletfamilies('a')
```

Description

`waveletfamilies` or `waveletfamilies('f')` displays the names of all available wavelet families.

`waveletfamilies('n')` displays the names of all available wavelets in each family.

`waveletfamilies('a')` displays all available wavelet families with their corresponding properties.

Examples

```
waveletfamilies

==================================
Haar                haar
Daubechies          db
Symlets             sym
Coiflets            coif
BiorSplines         bior
ReverseBior         rbio
Meyer               meyr
DMeyer              dmey
Gaussian            gaus
Mexican_hat         mexh
Morlet              morl
Complex Gaussian    cgau
Shannon             shan
Frequency B-Spline  fbsp
```

```
Complex Morlet      cmor
==================================

waveletfamilies('n')

==================================
Haar                    haar
==================================
Daubechies              db
------------------------------
db1 db2 db3 db4
db5 db6 db7 db8
db9 db10    db**
==================================
Symlets                 sym
------------------------------
sym2    sym3    sym4    sym5
sym6    sym7    sym8    sym**
==================================
Coiflets                coif
------------------------------
coif1   coif2   coif3   coif4
coif5
==================================
BiorSplines             bior
------------------------------
bior1.1 bior1.3 bior1.5 bior2.2
bior2.4 bior2.6 bior2.8 bior3.1
bior3.3 bior3.5 bior3.7 bior3.9
bior4.4 bior5.5 bior6.8
==================================
ReverseBior             rbio
------------------------------
rbio1.1 rbio1.3 rbio1.5 rbio2.2
rbio2.4 rbio2.6 rbio2.8 rbio3.1
rbio3.3 rbio3.5 rbio3.7 rbio3.9
rbio4.4 rbio5.5 rbio6.8
==================================
Meyer                   meyr
==================================
DMeyer                  dmey
==================================
Gaussian                gaus
------------------------------
```

```
gaus1   gaus2   gaus3   gaus4
gaus5   gaus6   gaus7   gaus8
gaus**
==================================
Mexican_hat             mexh
==================================
Morlet                  morl
==================================
Complex Gaussian        cgau
------------------------------
cgau1   cgau2   cgau3   cgau4
cgau5   cgau**
==================================
Shannon                 shan
------------------------------
shan1-1.5   shan1-1 shan1-0.5   shan1-0.1
shan2-3 shan**
==================================
Frequency B-Spline      fbsp
------------------------------
fbsp1-1-1.5 fbsp1-1-1   fbsp1-1-0.5 fbsp2-1-1
fbsp2-1-0.5 fbsp2-1-0.1 fbsp**
==================================
Complex Morlet          cmor
------------------------------
cmor1-1.5   cmor1-1 cmor1-0.5   cmor1-1
cmor1-0.5   cmor1-0.1   cmor**
==================================

waveletfamilies('a')

Type of Wavelets
-----------------
type = 1   - orthogonals wavelets          (F.I.R.)
type = 2   - biorthogonals wavelets        (F.I.R.)
type = 3   - with scale function
type = 4   - without scale function
type = 5   - complex wavelet.
-------------------------------------------------------------

------------------------
Family Name : Haar
haar
1
```

```
no
no
dbwavf

------------------------
Family Name : Daubechies
db
1
1 2 3 4 5 6 7 8 9 10 **
integer
dbwavf

------------------------
Family Name : Symlets
sym
1
2 3 4 5 6 7 8 **
integer
symwavf

------------------------
Family Name : Coiflets
coif
1
1 2 3 4 5
integer
coifwavf

------------------------
Family Name : BiorSplines
bior
2
1.1 1.3 1.5 2.2 2.4 2.6 2.8 3.1 3.3 3.5 3.7 3.9 4.4 5.5 6.8
real
biorwavf

------------------------
Family Name : ReverseBior
rbio
2
1.1 1.3 1.5 2.2 2.4 2.6 2.8 3.1 3.3 3.5 3.7 3.9 4.4 5.5 6.8
real
rbiowavf
```

```
------------------------
Family Name : Meyer
meyr
3
no
no
meyer
-8 8
------------------------
Family Name : DMeyer
dmey
1
no
no
dmey.mat

------------------------
Family Name : Gaussian
gaus
4
1 2 3 4 5 6 7 8 **
integer
gauswavf
-5 5
------------------------
Family Name : Mexican_hat
mexh
4
no
no
mexihat
-8 8
-------------------------
Family Name : Morlet
morl
4
no
no
morlet
-8 8
------------------------
Family Name : Complex Gaussian
cgau
5
```

```
1 2 3 4 5 **
integer
cgauwavf
-5 5
------------------------
Family Name : Shannon
shan
5
1-1.5 1-1 1-0.5 1-0.1 2-3 **
string
shanwavf
-20 20
------------------------
Family Name : Frequency B-Spline
fbsp
5
1-1-1.5 1-1-1 1-1-0.5 2-1-1 2-1-0.5 2-1-0.1 **
string
fbspwavf
-20 20
------------------------
Family Name : Complex Morlet
cmor
5
1-1.5 1-1 1-0.5 1-1 1-0.5 1-0.1 **
string
cmorwavf
-8 8
------------------------
```

See Also

wavemngr

Introduced in R2008a

wavemenu

Wavelet Toolbox GUI tools

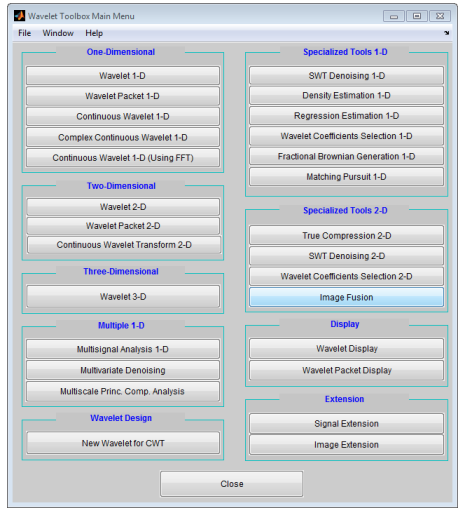
Syntax

Description

`wavemenu` opens a menu for accessing the various graphical tools provided in the Wavelet Toolbox software. All file formats supported by `audioread` are also supported in the Wavelet Toolbox GUI.

Examples

```
wavemenu
```



Introduced before R2006a

wavemngr

Wavelet manager

Syntax

```
wavemngr('add',FN,FSN,WT,NUMS,FILE)
wavemngr('add',FN,FSN,WT,NUMS,FILE,B)
wavemngr('add',FN,FSN,WT,{NUMS,TYPNUMS},FILE)
wavemngr('add',FN,FSN,WT,{NUMS,TYPNUMS},FILE,B)
```

Description

`wavemngr` is a type of wavelets manager. It allows you to add, delete, restore, or read wavelets.

`wavemngr('add',FN,FSN,WT,NUMS,FILE)` or `wavemngr('add',FN,FSN,WT,NUMS,FILE,B)` or `wavemngr('add',FN,FSN,WT,{NUMS,TYPNUMS},FILE)` or `wavemngr('add',FN,FSN,WT,{NUMS,TYPNUMS},FILE,B)`, add a new wavelet family to the toolbox.

`FN` = Family Name (string)

`FSN` = Family Short Name (string of length equal or less than four characters)

`WT` defines the wavelet type:

- `WT` = 1, for orthogonal wavelets
- `WT` = 2, for biorthogonal wavelets
- `WT` = 3, for wavelet with scaling function
- `WT` = 4, for wavelet without scaling function
- `WT` = 5, for complex wavelet without scaling function

If the family contains a single wavelet, `NUMS = ''`. Note that for this case you specify an empty string.

Examples:

mexh	j
morl	

If the wavelet is member of a finite family of wavelets, NUMS is a string containing a blank separated list of items representing wavelet parameters.

Example:

bior	NUMS = '1.1 1.3 ... 4.4 5.5 6.8'

If the wavelet is part of an infinite family of wavelets, NUMS is a string containing a blank separated list of items representing wavelet parameters, terminated by the special sequence **.

Examples:

db	NUMS = '1 2 3 4 5 6 7 8 9 10 **'
shan	NUMS = '1-1.5 1-1 1-0.5 1-0.1 2-3 **'

In these last two cases, TYPNUMS specifies the wavelet parameter input format: 'integer' or 'real' or 'string'; the default value is 'integer'.

Examples:

db	TYPNUMS = 'integer'
bior	TYPNUMS = 'real'
shan	TYPNUMS = 'string'

FILE = MAT-file or code file name (string). See usage in the “Examples” section.

B = [lb ub] specifies lower and upper bounds of effective support for wavelets of type = 3, 4, or 5.

This option is fully documented in “Adding Your Own Wavelets” in the User's Guide.

wavemngr('del',N), deletes a wavelet or a wavelet family. N is the Family Short Name or the Wavelet Name (in the family). N is a string.

wavemngr('restore') or wavemngr('restore',IN2) restores previous or initial wavelets. If nargin = 1, the previous wavelets.asc ASCII-file is restored; otherwise the initial wavelets.asc ASCII-file is restored. Here IN2 is a dummy argument.

OUT1 = wavemngr('read') returns all wavelet family names.

`OUT1 = wavemngr('read',IN2)` returns all wavelet names, `IN2` is a dummy argument.

`OUT1 = wavemngr('read_asc')` reads `wavelets.asc` ASCII-file and returns all wavelets information.

Examples

```
% List initial wavelets families.
    wavemngr('read')

ans =
==================================
Haar                haar
Daubechies          db
Symlets             sym
Coiflets            coif
BiorSplines         bior
ReverseBior         rbio
Meyer               meyr
DMeyer              dmey
Gaussian            gaus
Mexican_hat         mexh
Morlet              morl
Complex Gaussian    cgau
Shannon             shan
Frequency B-Spline  fbsp
Complex Morlet      cmor
==================================
% List all wavelets.
wavemngr('read',1)

ans =

==================================
Haar                haar
==================================
Daubechies          db
------------------------------
db1   db2   db3   db4
db5   db6   db7   db8
db9   db10  db**
==================================
```

```
Symlets             sym
-----------------------------
sym2  sym3  sym4  sym5
sym6  sym7  sym8  sym**
==================================
Coiflets            coif
-----------------------------
coif1  coif2  coif3  coif4
coif5
==================================
BiorSplines         bior
-----------------------------
bior1.1   bior1.3   bior1.5   bior2.2
bior2.4   bior2.6   bior2.8   bior3.1
bior3.3   bior3.5   bior3.7   bior3.9
bior4.4   bior5.5   bior6.8
==================================
ReverseBior           rbio
-----------------------------
rbio1.1   rbio1.3   rbio1.5   rbio2.2
rbio2.4   rbio2.6   rbio2.8   rbio3.1
rbio3.3   rbio3.5   rbio3.7   rbio3.9
rbio4.4   rbio5.5   rbio6.8
==================================
Meyer                 meyr
==================================
DMeyer                dmey
==================================
Gaussian              gaus
-----------------------------
gaus1   gaus2   gaus3   gaus4
gaus5   gaus6   gaus7   gaus8
gaus**
==================================
Mexican_hat           mexh
==================================
Morlet                morl
==================================
Complex Gaussian      cgau
-----------------------------
cgau1   cgau2   cgau3   cgau4
cgau5   cgau**
==================================
Shannon               shan
```

```
------------------------------
shan1-1.5   shan1-1   shan1-0.5   shan1-0.1
shan2-3   shan**
==================================
Frequency B-Spline    fbsp
------------------------------
fbsp1-1-1.5   fbsp1-1-1   fbsp1-1-0.5   fbsp2-1-1
fbsp2-1-0.5   fbsp2-1-0.1   fbsp**
==================================
Complex Morlet        cmor
------------------------------
cmor1-1.5   cmor1-1   cmor1-0.5   cmor1-1
cmor1-0.5   cmor1-0.1   cmor**
==================================
```

In the following example, new compactly supported orthogonal wavelets are added to the toolbox. These wavelets, which are a slight generalization of the Daubechies wavelets, are based on the use of Bernstein polynomials and are due to Kateb and Lemarié in an unpublished work.

Note The files used in this example can be found in the `wavedemo` folder.

```
% Add new family of orthogonal wavelets.
% You must define:
%
%     Family Name:          Lemarie
%     Family Short Name:    lem
%     Type of wavelet:      1 (orth)
%     Wavelets numbers:     1 2 3 4 5
%     File driver:          lemwavf
%
%     The function lemwavf.m must be as follows:
%     function w = lemwavf(wname)
%     where the input argument wname is a string:
%     wname = 'lem1' or 'lem2' ... i.e.,
%     wname = sh.name + number
%     and w the corresponding scaling filter.
%     The addition is obtained using:

wavemngr('add','Lemarie','lem',1,'1 2 3 4 5','lemwavf');

% The ascii file 'wavelets.asc' is saved as
```

```
% 'wavelets.prv', then it is modified and
% the MAT file 'wavelets.inf' is generated.

% List wavelets families.
wavemngr('read')

ans =
===================================
Haar                haar
Daubechies          db
Symlets             sym
Coiflets            coif
BiorSplines         bior
ReverseBior         rbio
Meyer               meyr
DMeyer              dmey
Gaussian            gaus
Mexican_hat         mexh
Morlet              morl
Complex Gaussian    cgau
Shannon             shan
Frequency B-Spline  fbsp
Complex Morlet      cmor
Lemarie             lem
===================================
% Remove the added family.
wavemngr('del','Lemarie');

% List wavelets families.
wavemngr('read')

ans =
===================================
Haar                haar
Daubechies          db
Symlets             sym
Coiflets            coif
BiorSplines         bior
ReverseBior         rbio
Meyer               meyr
DMeyer              dmey
Gaussian            gaus
Mexican_hat         mexh
Morlet              morl
```

```
Complex Gaussian    cgau
Shannon             shan
Frequency B-Spline  fbsp
Complex Morlet      cmor
==================================
% Restore the previous ascii file
% 'wavelets.prv', then build
% the MAT-file 'wavelets.inf'.
wavemngr('restore');

% List restored wavelets.
wavemngr('read',1)

ans =
==================================
Haar                  haar
==================================
Daubechies            db
------------------------------
db1   db2   db3   db4
db5   db6   db7   db8
db9   db10   db**
==================================
Symlets               sym
------------------------------
sym2   sym3   sym4   sym5
sym6   sym7   sym8   sym**
==================================
Coiflets              coif
------------------------------
coif1   coif2   coif3   coif4
coif5
==================================
BiorSplines           bior
------------------------------
bior1.1   bior1.3   bior1.5   bior2.2
bior2.4   bior2.6   bior2.8   bior3.1
bior3.3   bior3.5   bior3.7   bior3.9
bior4.4   bior5.5   bior6.8
==================================
ReverseBior           rbio
------------------------------
rbio1.1   rbio1.3   rbio1.5   rbio2.2
rbio2.4   rbio2.6   rbio2.8   rbio3.1
```

```
rbio3.3   rbio3.5   rbio3.7   rbio3.9
rbio4.4   rbio5.5   rbio6.8
==================================
Meyer                meyr
==================================
DMeyer               dmey
==================================
Gaussian             gaus
------------------------------
gaus1   gaus2   gaus3   gaus4
gaus5   gaus6   gaus7   gaus8
gaus**
==================================
Mexican_hat          mexh
==================================
Morlet               morl
==================================
Complex Gaussian     cgau
------------------------------
cgau1   cgau2   cgau3   cgau4
cgau5   cgau**
==================================
Shannon              shan
------------------------------
shan1-1.5   shan1-1   shan1-0.5   shan1-0.1
shan2-3   shan**
==================================
Frequency B-Spline   fbsp
------------------------------
fbsp1-1-1.5   fbsp1-1-1   fbsp1-1-0.5   fbsp2-1-1
fbsp2-1-0.5   fbsp2-1-0.1   fbsp**
==================================
Complex Morlet       cmor
------------------------------
cmor1-1.5   cmor1-1   cmor1-0.5   cmor1-1
cmor1-0.5   cmor1-0.1   cmor**
==================================
Lemarie     lem
------------------------------
lem1    lem2    lem3    lem4    lem5
==================================
% Restore initial wavelets.
%
% Restore the initial ascii file
```

```
% 'wavelets.ini' and initial
% MAT-file 'wavelets.bin'.
wavemngr('restore',0);

% List wavelets families.
wavemngr('read')

ans =
==================================
Haar                haar
Daubechies          db
Symlets             sym
Coiflets            coif
BiorSplines         bior
ReverseBior         rbio
Meyer               meyr
DMeyer              dmey
Gaussian            gaus
Mexican_hat         mexh
Morlet              morl
Complex Gaussian    cgau
Shannon             shan
Frequency B-Spline  fbsp
Complex Morlet      cmor
==================================
% Add new family of orthogonal wavelets.
wavemngr('add','Lemarie','lem',1,'1 2 3','lemwavf');

% All command line capabilities are available for
% the new wavelets.
%
% Example 1: compute the four associated filters.
[Lo_D,Hi_D,Lo_R,Hi_R] = wfilters('lem3');

% Example 2: compute scale and wavelet functions.
[phi,psi,xval] = wavefun('lem3');

% Add a new family of orthogonal wavelets: special form
% for the GUI mode.
%
% The file lemwavf allows you to compute the filter for
% any order. If you want to get a popup of the form
% 1 2 3 **, associated with the family, then wavelets are
% appended for GUI mode using:
```

```
wavemngr('restore',0);
wavemngr('add','Lemarie','lem',1,'1 2 3 **','lemwavf');

% After this sequence, all GUI capabilities are available for
% the new wavelets.
% Note that the last command allows a short cut in the
% order definition only if possible orders are integers.
```

Caution `wavemngr` works on the current folder. If you add a new wavelet family, it is available in this folder only. Refer to, “Adding Your Own Wavelets”, in the User's Guide.

Limitations

`wavemngr` allows you to add a new wavelet. You must verify that it is truly a wavelet. No check is performed either about this point or about the type of the new wavelet.

Introduced before R2006a

wavenames

Wavelet names for LWT

Syntax

```
W = wavenames(T)
```

Description

`W = wavenames(T)` returns a cell array that contains the name of all wavelets of type *T*. The valid values for *T* are

- `'all'` — all wavelets
- `'lazy'` — "lazy" wavelet
- `'orth'` — orthogonal wavelets
- `'bior'` — biorthogonal wavelets

`W = wavenames` is equivalent to `W = wavenames('all')`.

Introduced before R2006a

waverec

Multilevel 1-D wavelet reconstruction

Syntax

```
X = waverec(C,L,Lo_R,Hi_R)
X = waverec(C,L,'wname')
X = appcoef(C,L,'wname',0)
```

Description

`waverec` performs a multilevel one-dimensional wavelet reconstruction using either a specific wavelet (`'wname'`, see `wfilters`) or specific reconstruction filters (`Lo_R` and `Hi_R`). .

Note `waverec` supports only Type 1 (orthogonal) or Type 2 (biorthogonal) wavelets.

`X = waverec(C,L,'wname')` reconstructs the signal `X` based on the multilevel wavelet decomposition structure `[C,L]` and wavelet `'wname'`. (For information about the decomposition structure, see `wavedec`.)

`X = waverec(C,L,Lo_R,Hi_R)` reconstructs the signal `X` as above, using the reconstruction filters you specify. `Lo_R` is the reconstruction low-pass filter and `Hi_R` is the reconstruction high-pass filter.

Note that `X = waverec(C,L,'wname')` is equivalent to `X = appcoef(C,L,'wname',0)`.

Examples

```
% The current extension mode is zero-padding (see dwtmode).

% Load original one-dimensional signal.
load leleccum; s = leleccum(1:3920); ls = length(s);
```

```
% Perform decomposition of signal at level 3 using db5.
[c,l] = wavedec(s,3,'db5');

% Reconstruct s from the wavelet decomposition structure [c,l].
a0 = waverec(c,l,'db5');

% Check for perfect reconstruction.
err = norm(s-a0)
err =
    3.2079e-09
```

See Also

appcoef | idwt | wavedec

Introduced before R2006a

waverec2

Multilevel 2-D wavelet reconstruction

Syntax

```
X = waverec2(C,S,'wname')
X = waverec2(C,S,Lo_R,Hi_R)
waverec2(wavedec2(X,N,'wname'),'wname')
X = waverec2(C,S,'wname')
X = appcoef2(C,S,'wname',0)
```

Description

`X = waverec2(C,S,'wname')` performs a multilevel wavelet reconstruction of the matrix `X` based on the wavelet decomposition structure `[C,S]`. For detailed storage information, see `wavedec2`. '*wname*' is a string containing the name of the wavelet. See `wfilters` for more information.

Instead of specifying the wavelet name, you can specify the filters.

- `X = waverec2(C,S,Lo_R,Hi_R)`, `Lo_R` is the reconstruction low-pass filter
- `Hi_R` is the reconstruction high-pass filter.

`waverec2` is the inverse function of `wavedec2` in the sense that the abstract statement `waverec2(wavedec2(X,N,'wname'),'wname')` returns `X`.

`X = waverec2(C,S,'wname')` is equivalent to `X = appcoef2(C,S,'wname',0)`.

Examples

```
% The current extension mode is zero-padding (see dwtmode).
% Load original image.
load woman;
% X contains the loaded image.
% Perform decomposition at level 2
% of X using sym4.
```

```
[c,s] = wavedec2(X,2,'sym4');
% Reconstruct X from the wavelet
% decomposition structure [c,s].
a0 = waverec2(c,s,'sym4');
% Check for perfect reconstruction.
max(max(abs(X-a0)))
ans =
    2.5565e-10
```

More About

Tips

If `C` and `S` are obtained from an indexed image analysis or a truecolor image analysis, `X` is an `m`-by-`n` matrix or an `m`-by-`n`-by-3 array, respectively.

For more information on image formats, see the `image` and `imfinfo` reference pages.

See Also

`appcoef2` | `idwt2` | `wavedec2`

Introduced before R2006a

waverec3

Multilevel 3-D wavelet reconstruction

Syntax

```
X = waverec3(WDEC)
C = waverec3(WDEC,TYPE,N)
X = waverec3(WDEC,'a',0)
X = waverec3(WDEC,'ca',0)
C = waverec3(WDEC,TYPE)
C = waverec3(WDEC,TYPE,N)
```

Description

`waverec3` performs a multilevel 3-D wavelet reconstruction starting from a multilevel 3-D wavelet decomposition.

`X = waverec3(WDEC)` reconstructs the 3-D array `X` based on the multilevel wavelet decomposition structure `WDEC`. You can also use `waverec3` to extract coefficients from a 3-D wavelet decomposition.

`WDEC` is a structure with the fields shown in the table.

`C = waverec3(WDEC,TYPE,N)` reconstructs the multilevel components at level `N` of a 3-D wavelet decomposition. `N` must be a positive integer less than or equal to the level of the decomposition.

Valid values for `TYPE` are:

- A group of three characters `'xyz'`, one per direction, with `'x'`,`'y'` and `'z'` selected in the set {`'a'`, `'d'`, `'l'`, `'h'`} or in the corresponding uppercase set {`'A'`,`'D'`, `'L'`, `'H'`}), where `'A'` (or `'L'`) is a low-pass filter and `'D'` (or `'H'`) is a high-pass filter.
- The char `'d'` (or `'h'` or `'D'` or `'H'`) gives the sum of all the components different from the low-pass.
- The char `'a'` (or `'l'` or `'A'` or `'L'`) gives the low-pass component (the approximation at level `N`).

For extraction, the valid values for TYPE are the same but prefixed by `'c'` or `'C'`.

`X = waverec3(WDEC,'a',0)` or `X = waverec3(WDEC,'ca',0)` is equivalent to `X = waverec3(WDEC)`. `X` is a reconstruction of the coefficients in `WDEC` at level 0.

`C = waverec3(WDEC,TYPE)` is equivalent to `C = waverec3(WDEC,TYPE,N)` with `N` equal to the level of the decomposition.

`sizeINI`	Size of the three-dimensional array `X`
`level`	Level of the decomposition
`mode`	Name of the wavelet transform extension mode
`filters`	Structure with 4 fields, `LoD`, `HiD`, `LoR`, and `HiR`, which contain the filters used for DWT
`dec`	`N` x 1 cell array containing the coefficients of the decomposition. `N` is equal to `7*WDEC.level+1`. `dec{1}` contains the lowpass component (approximation) at the level of the decomposition. The approximation is equivalent to the filtering operations `'LLL'`. `dec{k+2},...,dec{k+8}` with `k = 0,7,14,...,7*(WDEC.level-1)` contain the 3-D wavelet coefficients for the multiresolution starting with the coarsest level when `k=0`. For example, if `WDEC.level=3`, `dec{2},...,dec{8}` contain the wavelet coefficients for level 3 (`k=0`), `dec{9},...,dec{15}` contain the wavelet coefficients for level 2 (`k=7`), and `dec{16},...,dec{22}` contain the wavelet coefficients for level 1 (`k=7*(WDEC.level-1)`). At each level, the wavelet coefficients in `dec{k+2},...,dec{k+8}` are in the following order: `'HLL','LHL','HHL','LLH','HLH','LHH','HHH'`. The strings give the order in which the separable filtering operations are applied from left to right. For example, `'LHH'` means that the lowpass (scaling) filter with downsampling is applied to the rows of `X`, followed by the highpass (wavelet)

	filter with downsampling applied to the columns of X. Finally, the highpass filter with downsampling is applied to the 3rd dimension of X.
sizes	Successive sizes of the decomposition components

Examples

Perfect Reconstruction with 3-D Discrete Wavelet Transform

Construct a 3-D matrix, obtain the wavelet transform down to level 2 using the db2 wavelet, and reconstruct the matrix to verify perfect reconstruction.

Create 3-D matrix.

```
M = magic(8);
X = repmat(M,[1 1 8]);
```

Obtain the 3-D discrete wavelet transform of the matrix and reconstruct the input based on the 3-D approximation and detail coefficients.

```
wd = wavedec3(X,2,'db2');
XR = waverec3(wd);
```

Verify perfect reconstruction using the wavelet decomposition down to level 2.

```
err1 = max(abs(X(:)-XR(:)))
```

Verify that the data matrix is the sum of the approximation and the details from levels 2 and 1.

```
A = waverec3(wd,'LLL');
% Reconstruct the sum of components different from
% the lowpass component.
D = waverec3(wd,'d');
% Check that X = A + D.
err2 = max(abs(X(:)-A(:)-D(:)))
```

Compare waverec3 and idwt3

Compare level-1 reconstructions based on the filtering operations 'LLH' using idwt3 and waverec3.

```
dwtOut = dwt3(X,'db2');
Xr = idwt3(dwtOut,'LLH');
Xrec = waverec3(wd,'LLH',1);
norm(Xr(:)-Xrec(:))
```

See Also

idwt3 | waveinfo | wavedec3

Introduced in R2010a

wavsupport

Wavelet support

Syntax

```
[LB,UB] = wavsupport(wname)
```

Description

`[LB,UB] = wavsupport(wname)` returns the lower bound, `LB`, and upper bound, `UB`, of the support for the wavelet specified by `wname`. `wname` is any valid wavelet. For real-valued wavelets with and without scaling functions and complex-valued wavelets without scaling functions (wavelets type 3,4, and 5), the bounds indicate the effective support of the wavelet. For orthogonal and biorthogonal wavelets (type 1 and type 2), the lower and upper bounds are `-0.5*(LF-1)` and `0.5*(LF-1)`, where `LF` is the length of the wavelet filter.

Examples

Support of Haar wavelet:

```
[LB, UB] = wavsupport('haar');
LowerBound = -0.5*(2-1);
UpperBound = 0.5*(2-1);
% Compare [LB,UB] and [LowerBound, UpperBound]
```

Effective support of complex-valued Gaussian wavelet:

```
[LB,UB] = wavsupport('cgau3');
```

See Also

`wavemngr`

Introduced in R2010b

wbmpen

Penalized threshold for wavelet 1-D or 2-D de-noising

Syntax

```
THR = wbmpen(C,L,SIGMA,ALPHA)
wbmpen(C,L,SIGMA,ALPHA,ARG)
```

Description

`THR = wbmpen(C,L,SIGMA,ALPHA)` returns global threshold `THR` for de-noising. `THR` is obtained by a wavelet coefficients selection rule using a penalization method provided by Birgé-Massart.

`[C,L]` is the wavelet decomposition structure of the signal or image to be de-noised.

`SIGMA` is the standard deviation of the zero mean Gaussian white noise in de-noising model (see `wnoisest` for more information).

`ALPHA` is a tuning parameter for the penalty term. It must be a real number greater than 1. The sparsity of the wavelet representation of the de-noised signal or image grows with `ALPHA`. Typically `ALPHA` = 2.

`THR` minimizes the penalized criterion given by

let t^* be the minimizer of

```
crit(t) = -sum(c(k)^2,k≤t) + 2*SIGMA^2*t*(ALPHA + log(n/t))
```

where `c(k)` are the wavelet coefficients sorted in decreasing order of their absolute value and `n` is the number of coefficients; then THR=$|c(t^*)|$.

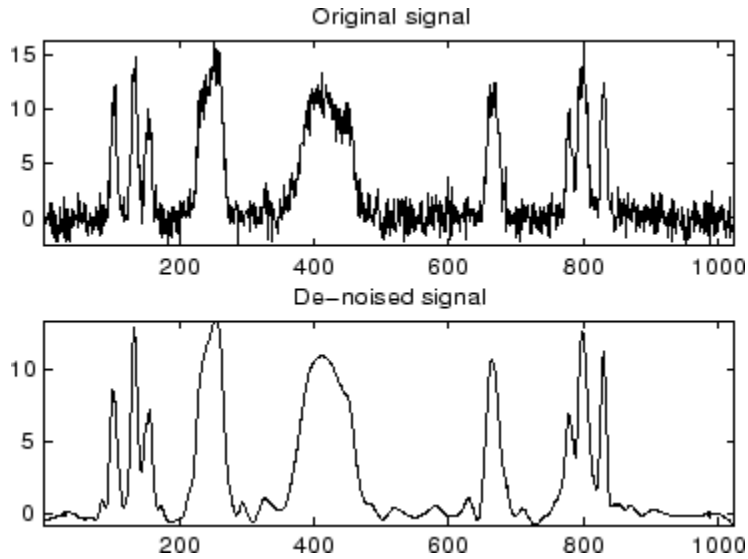
`wbmpen(C,L,SIGMA,ALPHA,ARG)` computes the global threshold and, in addition, plots three curves:

- `2*SIGMA^2*t*(ALPHA + log(n/t))`

- sum(c(k)^2,k¬≤t)
- crit(t)

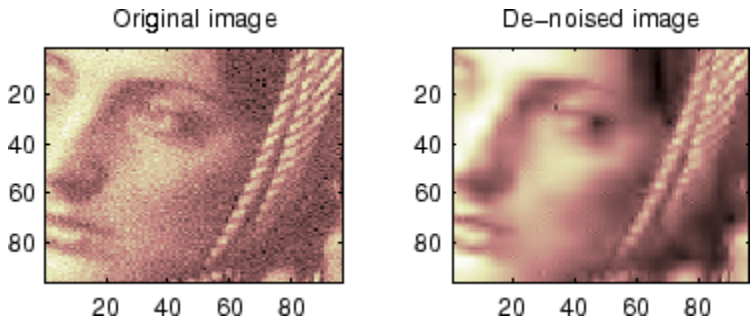
Examples

```
% Example 1: Signal de-noising.
% Load noisy bumps signal.
load noisbump; x = noisbump;

% Perform a wavelet decomposition of the signal
% at level 5 using sym6.
wname = 'sym6'; lev = 5;
[c,l] = wavedec(x,lev,wname);
% Estimate the noise standard deviation from the
% detail coefficients at level 1, using wnoisest.
sigma = wnoisest(c,l,1);

% Use wbmpen for selecting global threshold
% for signal de-noising, using the tuning parameter.
alpha = 2;
thr = wbmpen(c,l,sigma,alpha)
thr =

    2.7681

% Use wdencmp for de-noising the signal using the above
% threshold with soft thresholding and approximation kept.
keepapp = 1;
xd = wdencmp('gbl',c,l,wname,lev,thr,'s',keepapp);

% Plot original and de-noised signals.
figure(1)
subplot(211), plot(x), title('Original signal')
subplot(212), plot(xd), title('De-noised signal')
```




```
% Example 2: Image de-noising.
% Load original image.
load noiswom;
nbc = size(map,1);

% Perform a wavelet decomposition of the image
% at level 3 using coif2.
wname = 'coif2'; lev = 3;
[c,s] = wavedec2(X,lev,wname);

% Estimate the noise standard deviation from the
% detail coefficients at level 1.
det1 = detcoef2('compact',c,s,1);
sigma = median(abs(det1))/0.6745;

% Use wbmpen for selecting global threshold
% for image de-noising.
alpha = 1.2;
thr = wbmpen(c,l,sigma,alpha)

thr =

   36.0621
```

```
% Use wdencmp for de-noising the image using the above
% thresholds with soft thresholding and approximation kept.
keepapp = 1;
xd = wdencmp('gbl',c,s,wname,lev,thr,'s',keepapp);

% Plot original and de-noised images.
figure(2)
colormap(pink(nbc));
subplot(221), image(wcodemat(X,nbc))
title('Original image')
subplot(222), image(wcodemat(xd,nbc))
title('De-noised image')
```



See Also

wden | wdencmp | wpbmpen | wpdencmp

Introduced before R2006a

wcodemat

Extended pseudocolor matrix scaling

Syntax

```
Y = wcodemat(X)
Y = wcodemat(X,NBCODES)
Y = wcodemat(X,NBCODES,OPT)
Y = wcodemat(X,NBCODES,OPT,ABSOL)
```

Description

`wcodemat` rescales an input matrix to a specified range for display. If the specified range is the full range of the current colormap, `wcodemat` is similar in behavior to `imagesc`.

`Y = wcodemat(X)` rescales the matrix `X` to integers in the range [1,16].

`Y = wcodemat(X,NBCODES)` rescales the input `X` as integers in the range [1,`NBCODES`] . The default value of `NBCODES` is 16.

`Y = wcodemat(X,NBCODES,OPT)` rescales the matrix along the dimension specified by `OPT`. Valid strings for `OPT` are: `'column'` (or `'c'`), `'row'` (or `'r'`), and `'mat'` (or `'m'`). `'rows'` scales `X` row-wise, `'column'` scales `X` column-wise, and `'mat'` scales `X` globally. The default value of `OPT` is `'mat'`.

`Y = wcodemat(X,NBCODES,OPT,ABSOL)` rescales the input matrix `X` based on the absolute values of the entries in `X` if `ABSOL` is nonzero, or based on the signed values of `X` if `ABSOL` is equal to zero. The default value of `ABSOL` is 1.

Examples

Scale level-one approximation coefficients globally to the full range of the colormap.

```
load woman;
% Get the range of the colormap
NBCOL = size(map,1);
```

```
% Obtain the 2D dwt using the Haar wavelet
[cA1,cH1,cV1,cD1] = dwt2(X,'db1');
% Display without scaling
image(cA1);
colormap(map);
title('Unscaled Image');
figure;
% Display with scaling
image(wcodemat(cA1,NBCOL));
colormap(map);
title('Scaled Image');
```

Introduced before R2006a

wcoher

(Not recommended) Wavelet coherence

Compatibility

`wcoher` in not recommended. Use `wcoherence` instead.

Syntax

```
WCOH = wcoher(Sig1,Sig2,Scales,wname)
WCOH = wcoher(...,Name,Value)
[WCOH,WCS] = wcoher(...)
[WCOH,WCS,CWT_S1,CWT_S2] = wcoher(...)
[...] = wcoh(...,'plot')
```

Description

`WCOH = wcoher(Sig1,Sig2,Scales,wname)` returns the wavelet coherence for the input signals `Sig1` and `Sig2` using the wavelet specified in `wname` at the scales in `Scales`. The input signals must be real-valued and equal in length.

`WCOH = wcoher(...,Name,Value)` returns the wavelet coherence with additional options specified by one or more `Name,Value` pair arguments.

`[WCOH,WCS] = wcoher(...)` returns the wavelet cross spectrum.

`[WCOH,WCS,CWT_S1,CWT_S2] = wcoher(...)` returns the continuous wavelet transforms of `Sig1` and `Sig2`.

`[...] = wcoh(...,'plot')` displays the modulus and phase of the wavelet cross spectrum.

Input Arguments

`Sig1`

A real-valued one-dimensional input signal. `Sig1` is a row or column vector.

`Sig2`

A real-valued one-dimensional input signal. `Sig2` is a row or column vector.

`Scales`

`Scales` is a vector of real-valued, positive scales at which to compute the wavelet coherence.

`wname`

Wavelet used in the wavelet coherence. `wname` is any valid wavelet name.

Name-Value Pair Arguments

`'asc'`

Scale factor for arrows in quiver plot. `wcoher` represents the phase using `quiver`. `asc` corresponds to the `scale` input argument in `quiver`.

Default: 1

`'nas'`

Number of arrows in scale. Together with the number of scales, `nas` determines the spacing between the `y` coordinates in the input to `quiver`. The `y` input to `quiver` is `1:length(Scales)/(nas-1):Scales(end)`

Default: 20

`'nsw'`

Length of smoothing window in scale. `nsw` is a positive integer that specifies the length of a moving average filter in scale.

Default: 1

`'ntw'`

Length of smoothing window in time. `ntw` is a positive integer that specifies the length of a moving average filter in time.

Default: `min[20,0.05*length(Sig1)]`

`'plot'`

Type of plot. `plot` is one of the following strings:

- `'cwt'`

  Displays the continuous wavelet transforms of signals 1 and 2.

- `'wcs'`

  Displays the wavelet cross spectrum.

- `'wcoh'`

  Displays the phase of the wavelet cross spectrum.

- `'all'`

  Displays all plots in separate figures.

Output Arguments

`WCOH`

Wavelet coherence.

`WCS`

Wavelet cross spectrum.

`CWT_S1`

Continuous wavelet transform of signal 1.

`CWT_S2`

Continuous wavelet transform of signal 2.

Examples

Wavelet coherence of sine waves in noise with delay:

```
t  = linspace(0,1,2048);
x = sin(16*pi*t)+0.5*randn(1,2048);
y = sin(16*pi*t+pi/4)+0.5*randn(1,2048);
wname  = 'cgau3';
scales = 1:512;
ntw = 21; % smoothing parameter
% Display the modulus and phased of the wavelet cross spectrum.
wcoher(x,y,scales,wname,'ntw',ntw,'plot');
```

Sine wave and Doppler signal:

```
t  = linspace(0,1,1024);
x = -sin(8*pi*t) + 0.4*randn(1,1024);
x = x/max(abs(x));
y = wnoise('doppler',10);
wname  = 'cgau3';
scales = 1:512;
ntw = 21; % smoothing parameter
% Display of the CWT of the two signals.
wcoher(x,y,scales,wname,'ntw',ntw,'plot','cwt');
% Display of the wavelet cross spectrum.
wcoher(x,y,scales,wname,'ntw',ntw,'nsw',1,'plot','wcs');
% Display of the modulus and phased of the wavelet cross spectrum.
wcoher(x,y,scales,wname,'ntw',ntw,'plot');
```

More About

Wavelet Cross Spectrum

The wavelet cross spectrum of two time series, x and y is:

$$C_{xy}(a,b) = S(C_x^*(a,b)C_y(a,b))$$

where $C_x(a,b)$ and $C_y(a,b)$ denote the continuous wavelet transforms of x and y at scales a and positions b. The superscript * is the complex conjugate and S is a smoothing operator in time and scale.

For real-valued time series, the wavelet cross spectrum is real-valued if you use a real-valued analyzing wavelet, and complex-valued if you use a complex-valued analyzing wavelet.

Wavelet Coherence

The wavelet coherence of two time series x and y is:

$$\frac{\left|S(C_x^*(a,b)C_y(a,b))\right|^2}{S(|C_x(a,b)|^2)\ S(|C_y(a,b)|^2)}$$

where $C_x(a,b)$ and $C_y(a,b)$ denote the continuous wavelet transforms of x and y at scales a and positions b. The superscript * is the complex conjugate and S is a smoothing operator in time and scale.

For real-valued time series, the wavelet coherence is real-valued if you use a real-valued analyzing wavelet, and complex-valued if you use a complex-valued analyzing wavelet.

- Wavelet Coherence

References

Grinsted, A, J.C. Moore, and S. Jevrejeva. “Application of the cross wavelet transform and wavelet coherence to geophysical time series. *Nonlinear Processes in Geophysics*. 11, 2004, pp. 561-566.

Torrence. C., and G. Compo. “A Practical Guide to Wavelet Analysis”. *Bulletin of the American Meteorological Society*, 79, pp. 61-78.

See Also

`wcoherence` | `cwt`

Introduced in R2010b

wcoherence

Wavelet coherence and cross-spectrum

Syntax

```
wcoh = wcoherence(x,y)
[wcoh,wcs] = wcoherence(x,y)
[wcoh,wcs,period] = wcoherence(x,y,ts)
[wcoh,wcs,f] = wcoherence(x,y,fs)
[wcoh,wcs,f,coi] = wcoherence( ___ )
[wcoh,wcs,period,coi] = wcoherence( ___ ,ts)
wcoherence( ___ )
```

Description

`wcoh = wcoherence(x,y)` returns the magnitude-squared wavelet coherence, which is a measure of the correlation between signals `x` and `y` in the time-frequency plane. Wavelet coherence is useful for analyzing nonstationary signals. The inputs `x` and `y` must be equal length, 1-D, real-valued signals. The coherence is computed using the analytic Morlet wavelet.

`[wcoh,wcs] = wcoherence(x,y)` returns the wavelet cross-spectrum of `x` and `y`. You can use the phase of the wavelet cross-spectrum values to identify the relative lag between the input signals.

`[wcoh,wcs,period] = wcoherence(x,y,ts)` uses a `duration` `ts` with a positive, scalar input, as the sampling interval. The duration can be in years, days, hours, minutes, or seconds. `ts` is used to compute the scale-to period conversion, `period`. The `period` in an array of durations with the same time unit as specified in `ts`

`[wcoh,wcs,f] = wcoherence(x,y,fs)` uses the positive sampling frequency, `fs`, to compute the scale-to-frequency conversion, `f`. The `fs` input is in Hz.

`[wcoh,wcs,f,coi] = wcoherence( ___ )` returns the cone of influence, `coi`, for the wavelet coherence in cycles per sample. If you specify the sampling frequency, `fs`, the function returns both `f` and the cone of influence in Hz.

`[wcoh,wcs,period,coi] = wcoherence( ___ ,ts)` returns the cone of influence, `coi`, in cycles per unit time.

`wcoherence( ___ )` with no output arguments plots the wavelet coherence and cone of influence in the current figure. Due to the inverse relationship between frequency and period, a plot that uses the sampling interval is the inverse of a plot the uses the sampling frequency. For areas where the coherence exceeds 0.5, plots that use the sampling frequency display arrows to show the phase lag of `y` with respect to `x`. The arrows are spaced in time and scale. The direction of the arrows corresponds to the phase lag on the unit circle. For example, a vertical arrow indicates a π/2 or quarter-cycle phase lag. The corresponding lag in time depends on the duration of the cycle.

Examples

Wavelet Coherence of Two Sine Waves

Use default `wcoherence` settings to obtain the wavelet coherence between a sine wave with random noise and a frequency-modulated signal with decreasing frequency over time.

```
t  = linspace(0,1,1024);
x = -sin(8*pi*t) + 0.4*randn(1,1024);
x = x/max(abs(x));
y = wnoise('doppler',10);
wcoh = wcoherence(x,y);
```

The default coherence computation uses the Morlet wavelet, 12 voices per octave and smooths 12 scales. The default number of octaves is equal to `floor(log2(numel(x)))-1`, which in this case is 9.

Effect of Sampling Interval on Wavelet Coherence

Obtain the wavelet coherence data for two signals, specifying a sampling interval of 0.001 seconds. Both signals consist of two sine waves (10 Hz and 50 Hz) in white noise. The sine waves have different time supports.

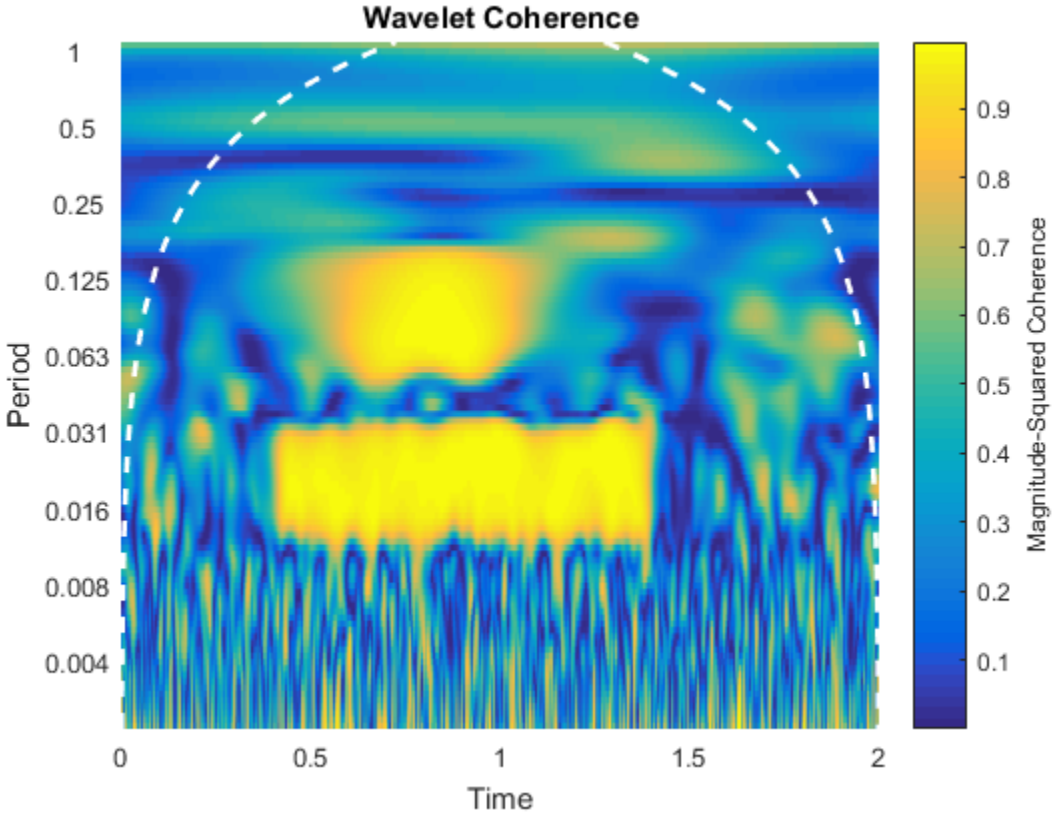
Set the random number generator to its default settings for reproducibility. Then, create the two signals and obtain the coherence.

```
rng default;
```

```
t = 0:0.001:2;
x = cos(2*pi*10*t).*(t>=0.5 & t<1.1)+ ...
cos(2*pi*50*t).*(t>= 0.2 & t< 1.4)+0.25*randn(size(t));
y = sin(2*pi*10*t).*(t>=0.6 & t<1.2)+...
sin(2*pi*50*t).*(t>= 0.4 & t<1.6)+ 0.35*randn(size(t));
[wcoh,~,period,coi] = wcoherence(x,y,seconds(0.001));
```

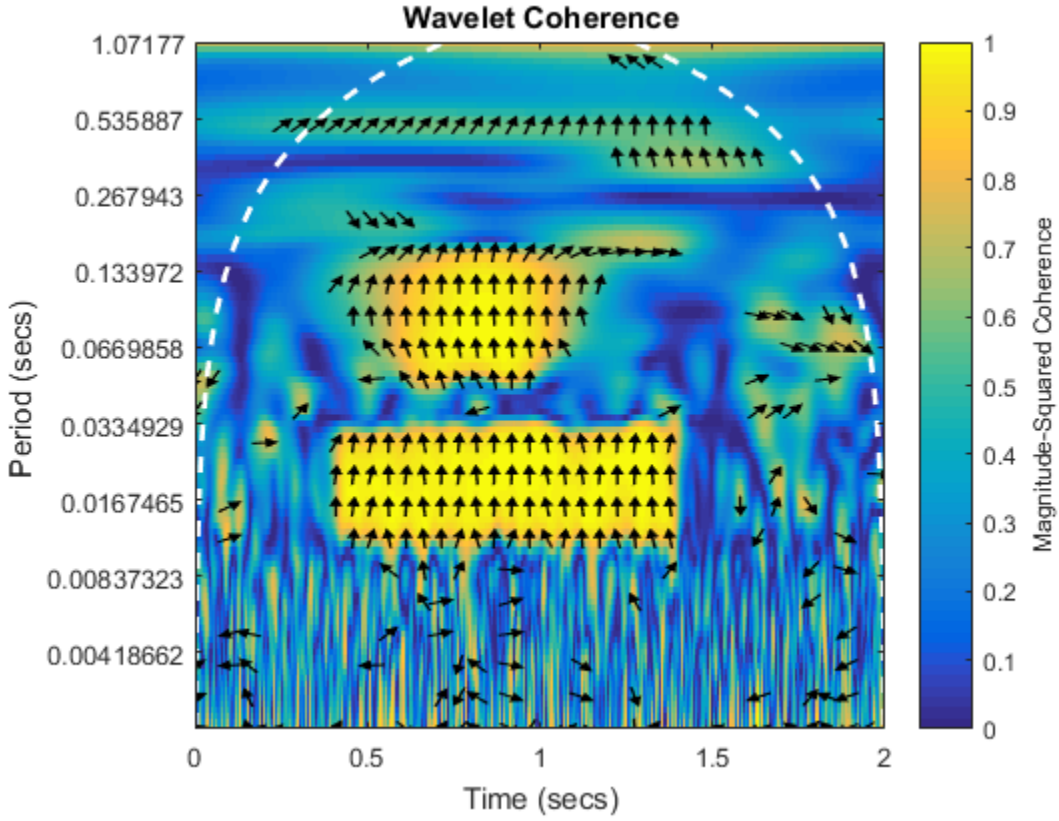
Use the `pcolor` command to plot the coherence and cone of influence.

```
period = seconds(period);
coi = seconds(coi);
h = pcolor(t,log2(period),wcoh);
h.EdgeColor = 'none';
ax = gca;
ytick=round(pow2(ax.YTick),3);
ax.YTickLabel=ytick;
ax.XLabel.String='Time';
ax.YLabel.String='Period';
ax.Title.String = 'Wavelet Coherence';
hcol = colorbar;
hcol.Label.String = 'Magnitude-Squared Coherence';
hold on;
plot(ax,t,log2(coi),'w--','linewidth',2);
```



Use `wcoherence(x,y,seconds(0.001))` without any outputs arguments. This plot includes the phase arrows and the cone of influence.

```
wcoherence(x,y,seconds(0.001));
```



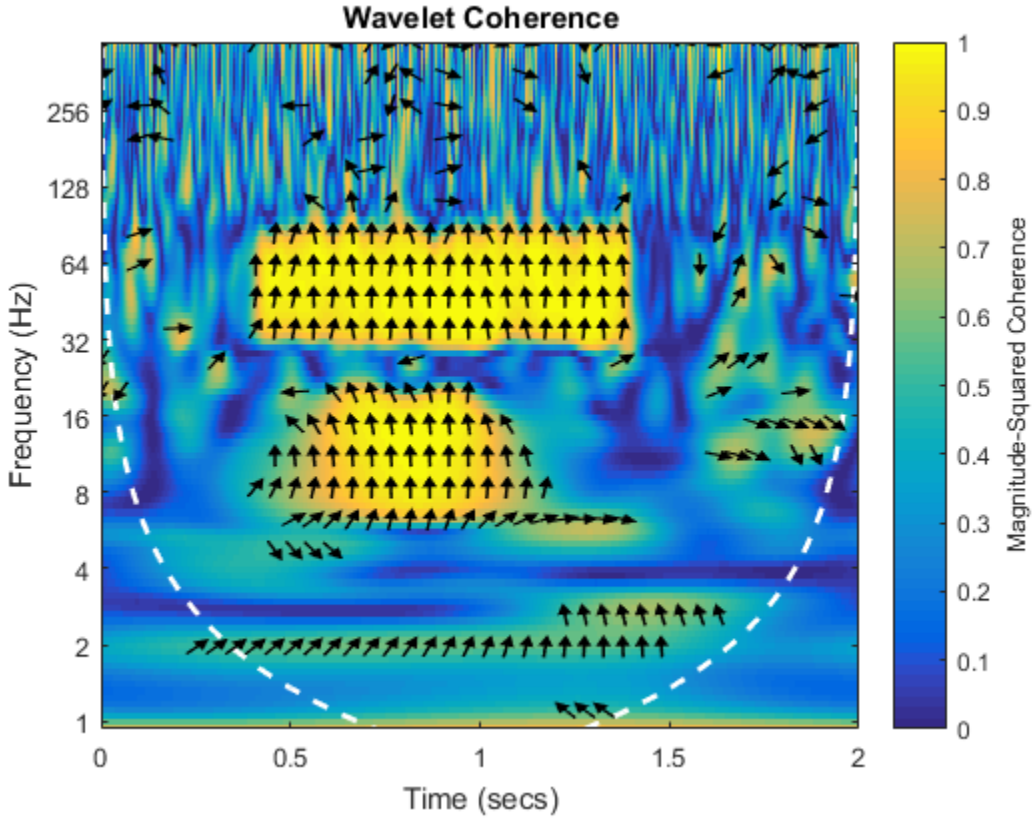
Effect of Sampling Frequency on Wavelet Coherence

Obtain the wavelet coherence for two signals, specifying a sampling frequency of 1000 Hz. Both signals consist of two sine waves (10 Hz and 50 Hz) in white noise. The sine waves have different time supports.

Set the random number generator to its default settings for reproducibility. Then, create the two signals and obtain the coherence.

```
rng default;
t = 0:0.001:2;
x = cos(2*pi*10*t).*(t>=0.5 & t<1.1)+...
cos(2*pi*50*t).*(t>= 0.2 & t< 1.4)+0.25*randn(size(t));
y = sin(2*pi*10*t).*(t>=0.6 & t<1.2)+...
```

```
sin(2*pi*50*t).*(t>= 0.4 & t<1.6)+ 0.35*randn(size(t));
wcoherence(x,y,1000);
```



This coherence plot is flipped with respect to the coherence plot in the previous example, which specifies a sampling interval instead of a sampling frequency.

Obtain the scale-to-freqeuncy conversion output in `f`.

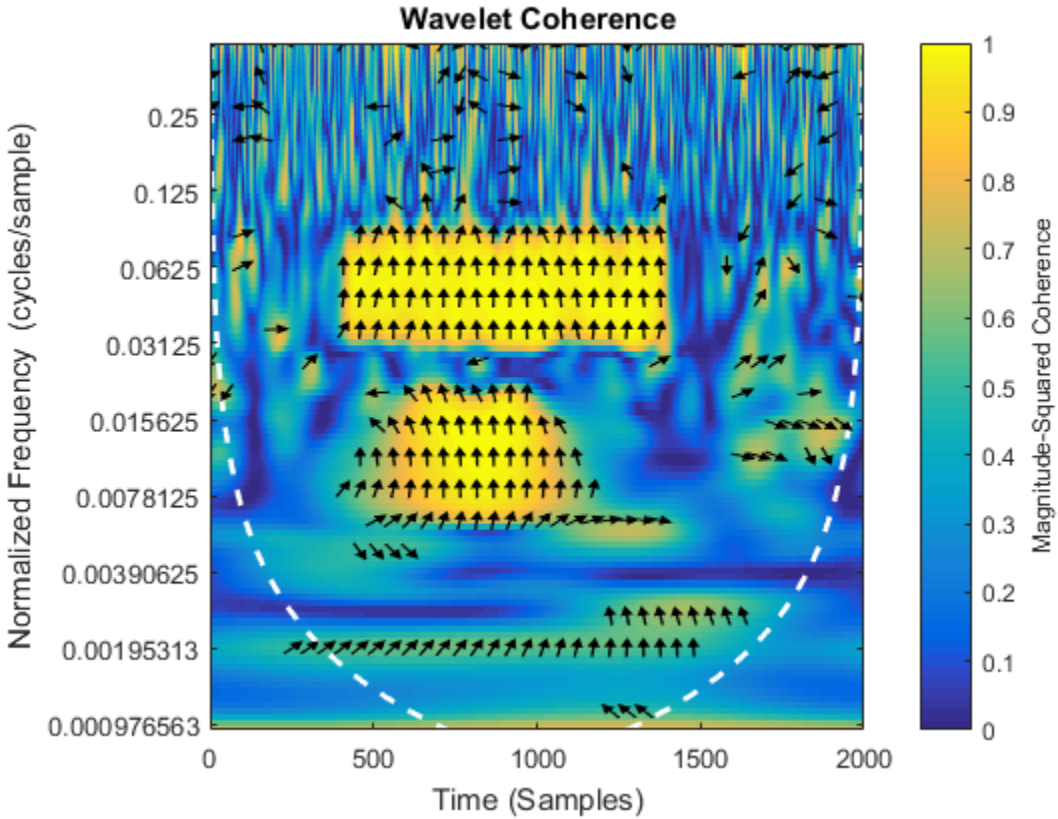
```
[wcoh,wcs,f] = wcoherence(x,y,1000);
```

Effect of Number of Smoothed Scales on Wavelet Coherence

Obtain the wavelet coherence for two signals. Both signals consist of two sine waves (10 Hz and 50 Hz) in white noise. Use the default number of scales to smooth. This value is equivalent to the number of voices per octave. Both values default to 12.

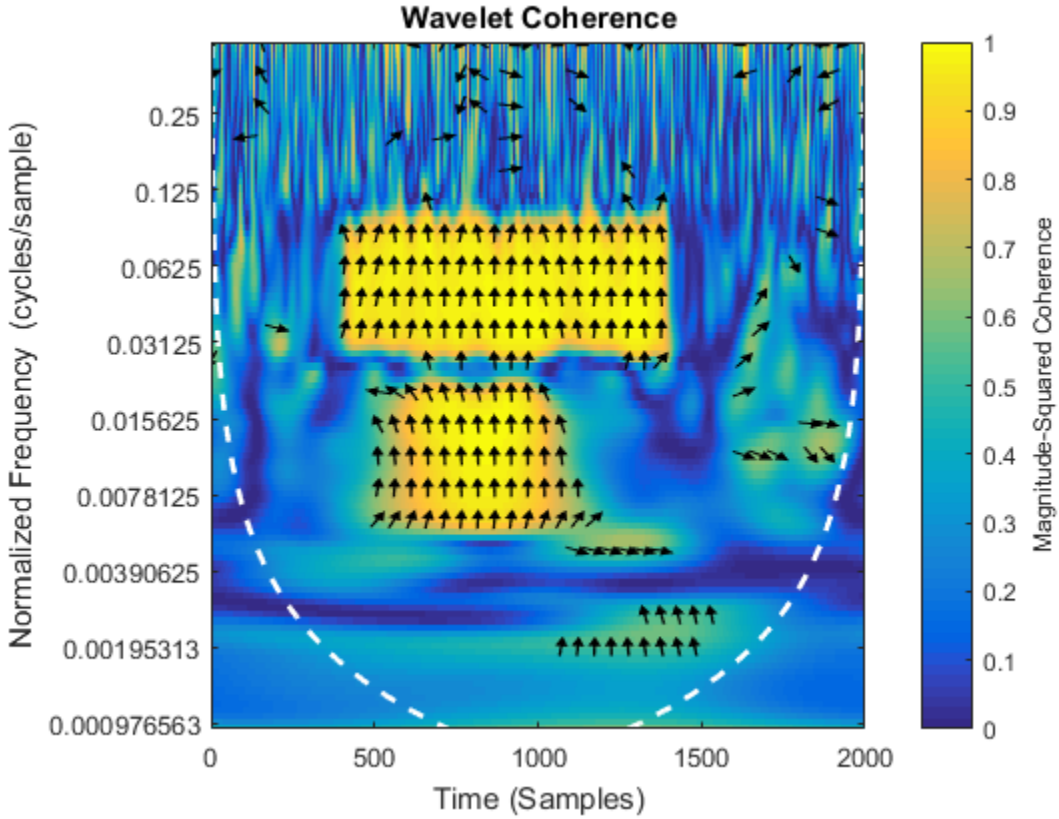
Set the random number generator to its default settings for reproducibility. Then, create the two signals and obtain the coherence.

```
rng default;
t = 0:0.001:2;
x = cos(2*pi*10*t).*(t>=0.5 & t<1.1)+ ...
cos(2*pi*50*t).*(t>= 0.2 & t< 1.4)+0.25*randn(size(t));
y = sin(2*pi*10*t).*(t>=0.6 & t<1.2)+...
sin(2*pi*50*t).*(t>= 0.4 & t<1.6)+ 0.35*randn(size(t));
wcoherence(x,y);
```



Set the number of scales to smooth to 16. The increased smoothing causes reduced low frequency resolution.

```
wcoherence(x,y,'NumScalesToSmooth',16);
```

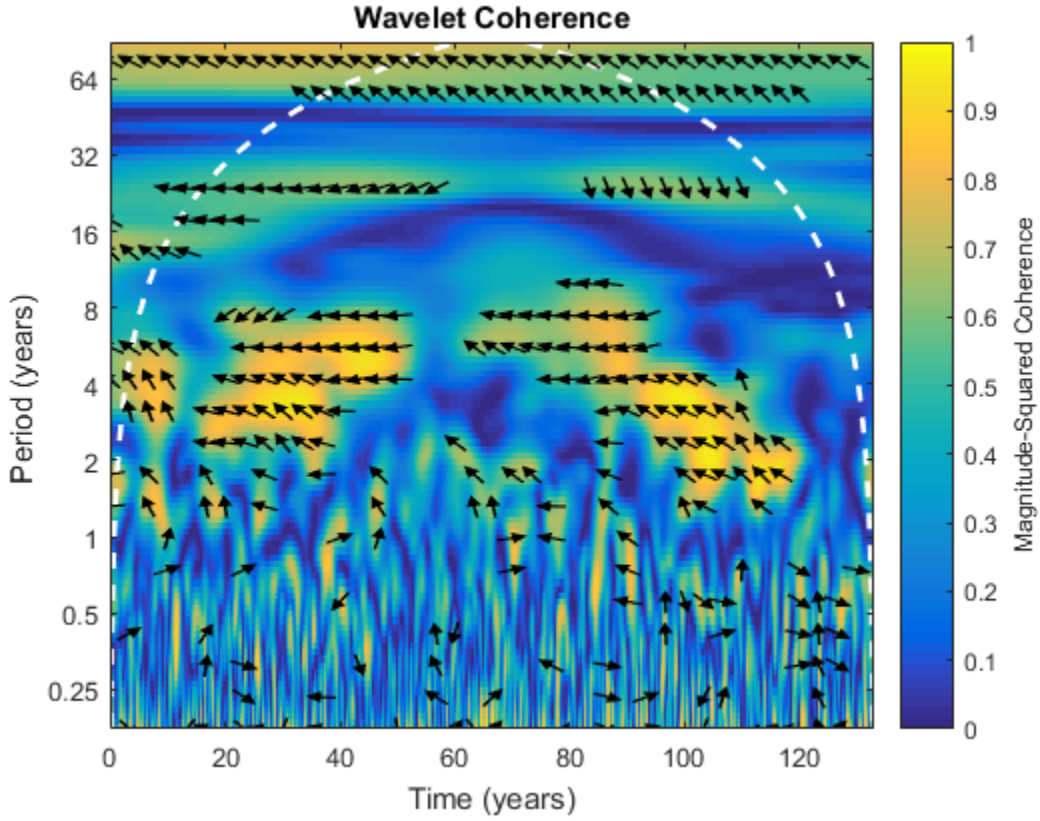


Effect of Phase Display Threshold on Wavelet Coherence of Weather Data

Compare the effects of using different phase display thresholds on the wavelet coherence.

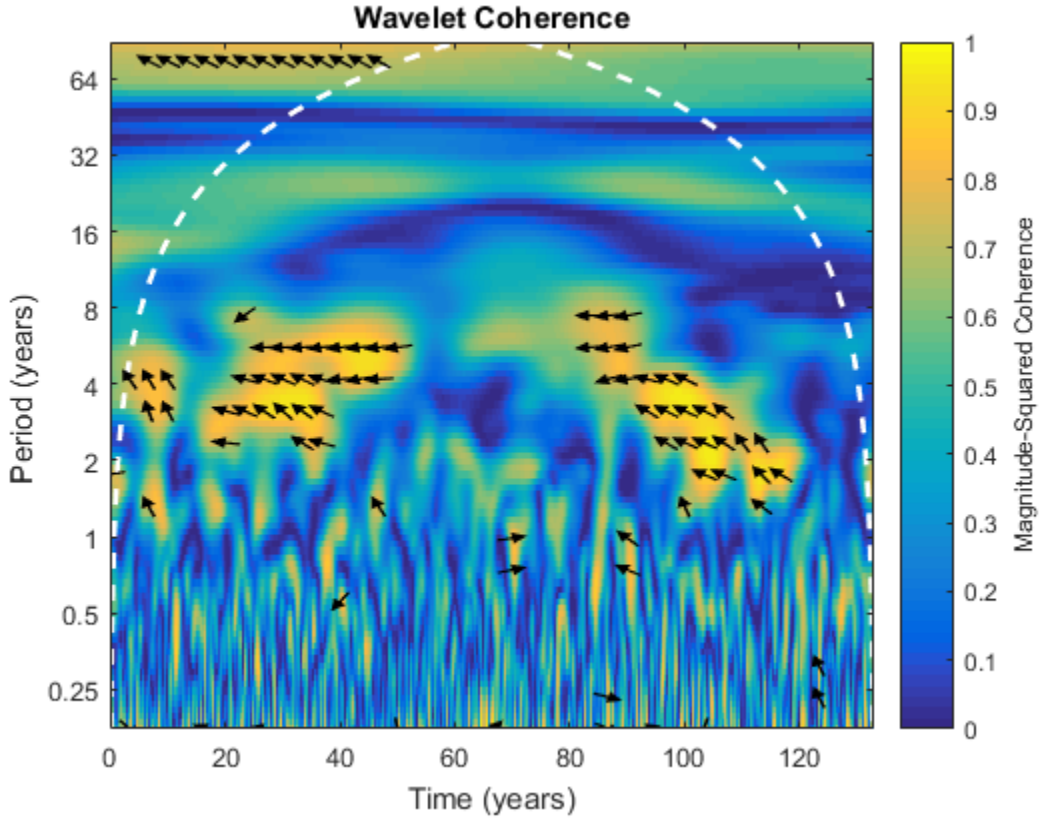
Plot the wavelet coherence between the El Nino time series and the All India Average Rainfall Index. The data are sampled monthly. Specify the sampling interval as 1/12 of a year to display the periods in years. Use the default phase display threshold of 0.5, which shows phase arrows only where the coherence is greater than or equal to 0.5.

```
load ninoairdata;
wcoherence(nino,air,years(1/12));
```



Set the phase display threshold to 0.7. The number of phase arrows decreases.

```
wcoherence(nino,air,years(1/12),'PhaseDisplayThreshold',0.7);
```



Input Arguments

x — Input signal

vector of real values

Input signal, specified as a vector of real values. `x` must be a 1-D, real-valued signal. The two input signals, `x` and `y`, must be the same length and must have at least four samples.

y — Input signal

vector of real values

Input signal, specified as vector of real values. `y` must be a 1-D, real-valued signal. The two input signals, `x` and `y`, must be the same length and must have at least four samples.

`ts` — Sampling interval

duration with positive scalar input

Sampling interval, also known as the sampling period, specified as a `duration` with positive scalar input. Valid durations are `years`, `days`, `hours`, `seconds`, and `minutes`. You can also use the `duration` function to specify `ts`. You cannot use calendar durations (`caldays`, `calweeks`, `calmonths`, `calquarters`, or `calyears`). You cannot specify both `ts` and `fs`.

`fs` — Sampling frequency

positive scalar

Sampling frequency, specified as a positive scalar.

Name-Value Pair Arguments

Specify optional comma-separated pairs of `Name,Value` arguments. `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside single quotes (`' '`). You can specify several name and value pair arguments in any order as `Name1,Value1,...,NameN,ValueN`.

Example: `'PhaseDisplayThreshold',0.7;` specifies the threshold for displaying phase vectors.

`'VoicesPerOctave'` — Number of voices per octave

12 (default) | integer from 10 to 32

Number of voices per octave to use in the wavelet coherence, specified as an integer from 10 to 32.

`'NumScalesToSmooth'` — Number of scales to smooth

number of voices per octave (default) | positive integer

Number of scales to smooth in time and scale, specified as a positive integer. This value must be less than one half the total number of scales. The function uses a moving average filter to smooth across scale. If you do not specify the number of scales to smooth, the value of `NumScalesToSmooth` defaults to the number of voices per octave. If your coherence is noisy, you can specify a larger `NumScalesToSmooth` value to smooth the coherence more.

`'NumOctaves'` — Number of octaves

positive integer

Number of octaves to use in the wavelet coherence, specified as a positive integer between 1 and `floor(log2(numel(x)))-1`. If you do not need to examine lower frequency values, use a smaller `NumOctaves` value.

`'PhaseDisplayThreshold'` — Threshold for displaying phase vectors

0.5 (default) | real scalar between 0 and 1

Threshold for displaying phase vectors, specified as a real scalar between 0 and 1. This function displays phase vectors for regions with coherence greater than or equal to the specified threshold value. Lowering the threshold value displays more phase vectors. If you use `wcoherence` with any output arguments, the `PhaseDisplayThreshold` value is ignored.

Output Arguments

`wcoh` — Wavelet coherence

matrix

Wavelet coherence, returned as a matrix. The coherence is computed using the analytic Morlet wavelet over logarithmic scales, with a default value of 12 voices per octave. The default number of octaves is equal to `floor(log2(numel(x)))-1`. If you do not specify a sampling interval, sampling frequency is assumed.

`wcs` — Wavelet cross spectrum

matrix of complex values

Wavelet cross-spectrum, returned as a matrix of complex values. You can use the phase of the wavelet cross-spectrum values to identify the relative lag between the input signals.

Data Types: `double`
Complex Number Support: Yes

`period` — Scale-to-period conversion

array of durations

Scale-to-period conversion, returned as an array of durations. The conversion values are computed from the sampling period specified in `ts`. Each `period` element has the same format as `ts`.

`f` — Scale-to-frequency conversion

vector

Scale-to-frequency conversion, returned as a vector. The vector contains the peak frequency values for the wavelets used to compute the coherence. If you want to output `f`, but do not specify a sampling frequency input, `fs`, the returned wavelet coherence is in cycles per sample.

`coi` — Cone of influence

array of doubles or array of durations

Cone of influence for the wavelet coherence, returned as either an array of doubles or array of durations. The cone of influence indicates where edge effects occur in the coherence data. If you specify a sampling frequency, `fs`, the cone of influence is in Hz. If you specify a sampling interval or period, `ts`, the cone of influence is in periods. Due to the edge effects, give less credence to areas of apparent high coherence that are outside or overlap the cone of influence. The cone of influence is indicated by a dashed line.

More About

Wavelet Cross Spectrum

The wavelet cross- spectrum is a measure of the distribution of power of two signals.

The wavelet cross spectrum of two time series, x and y, is:

$$C_{xy}(a,b) = S(C_x^*(a,b)C_y(a,b))$$

$C_x(a,b)$ and $C_y(a,b)$ denote the continuous wavelet transforms of x and y at scales a and positions b. The superscript * is the complex conjugate, and S is a smoothing operator in time and scale.

For real-valued time series, the wavelet cross-spectrum is real-valued if you use a real-valued analyzing wavelet, and complex-valued if you use a complex-valued analyzing wavelet.

Wavelet Coherence

Wavelet coherence is a measure of the correlation between two signals.

The wavelet coherence of two time series x and y is:

$$\frac{\left|S(C_x^*(a,b)C_y(a,b))\right|^2}{S(|C_x(a,b)|^2)\ S(|C_y(a,b)|^2)}$$

$C_x(a,b)$ and $C_y(a,b)$ denote the continuous wavelet transforms of x and y at scales a and positions b. The superscript * is the complex conjugate and S is a smoothing operator in time and scale.

For real-valued time series, the wavelet coherence is real-valued if you use a real-valued analyzing wavelet, and complex-valued if you use a complex-valued analyzing wavelet.

References

[1] Grinsted, A, J., C. Moore, and S. Jevrejeva. “Application of the cross wavelet transform and wavelet coherence to geophysical time series.” *Nonlinear Processes in Geophysics*. Vol. 11, Issue 5/6, 2004, pp. 561–566.

[2] Maraun, D., J. Kurths, and M. Holschneider. "Nonstationary Gaussian processes in wavelet domain: Synthesis, estimation and significance testing.” *Physical Review E 75*. 2007, pp. 016707-1–016707-14.

[3] Torrence, C., and P. Webster. "Interdecadal changes in the ESNO-Monsoon System." *Journal of Climate*. Vol. 12, 1999, pp. 2679–2690.

See Also

`cwtft` | `days` | `duration` | `hours` | `minutes` | `seconds` | `years`

Introduced in R2016a

wcompress

True compression of images using wavelets

Syntax

```
wcompress('c',X,SAV_FILENAME,COMP_METHOD)
wcompress(...,'ParName1',ParVal1,'ParName2',ParVal2,...)
[COMPRAT,BPP] = wcompress('c',...)
XC = wcompress('u',SAV_FILENAME)
XC = wcompress('u',SAV_FILENAME,'plot')
XC = wcompress('u',SAV_FILENAME,'step')
```

Description

The `wcompress` command performs either compression or uncompression of grayscale or truecolor images.

More theoretical information on true compression is in "Wavelet Compression for Images" of the Wavelet Toolbox User's Guide.

Compression

`wcompress('c',X,SAV_FILENAME,COMP_METHOD)` compresses the image `X` using the compression method `COMP_METHOD`.

The compressed image is saved in the file `SAV_FILENAME`. `X` can be either a 2-D array containing an indexed image or a 3-D array of `uint8` containing a truecolor image.

wcompress`('c',FILENAME,...)` loads the image `X` from the file `FILENAME` which is a MATLAB Supported Format (MSF) file: MAT-file or other image files (see `imread`).

wcompress`('c',I,...)` converts the indexed image `X = I{1}` to a truecolor image `Y` using the colormap `map = I{2}` and then compresses `Y`.

The valid compression methods are divided in three categories.

1 Progressive Coefficients Significance Methods (**PCSM**):

MATLAB Name	Compression Method Name
'ezw'	Embedded Zerotree Wavelet
'spiht'	Set Partitioning In Hierarchical Trees
'stw'	Spatial-orientation Tree Wavelet
'wdr'	Wavelet Difference Reduction
'aswdr'	Adaptively Scanned Wavelet Difference Reduction
'spiht_3d'	Set Partitioning In Hierarchical Trees 3D for truecolor images

For more details on these methods, see the references and especially Walker and also Said and Pearlman.

- Coefficients Thresholding Methods (**CTM-1**):

MATLAB Name	Compression Method Name
'lvl_mmc'	Subband thresholding of coefficients and Huffman encoding

For more details on this method, see the Strang and Nguyen reference.

- Coefficients Thresholding Methods (**CTM-2**):

MATLAB Name	Compression Method Name
'gbl_mmc_f'	Global thresholding of coefficients and fixed encoding
'gbl_mmc_h'	Global thresholding of coefficients and Huffman encoding

Note The Discrete Wavelet Transform uses the periodized extension mode. Each of the two dimensions of the image must be a power of 2.

All the compression methods use parameters which have default values. You can change these values using the following syntax:

```
wcompress(...,'ParName1',ParVal1,'ParName2',ParVal2,...)
```

Some of the parameters are related to display or to data transform functionalities. The others are linked to the compression process itself.

Data transform parameters

- 'ParName' = 'wname' or 'WNAME' sets the wavelet name.

  ParVal is a string (see waveletfamilies). The default for is *bior4.4*

- 'ParName' = 'level' or 'LEVEL' sets the level of decomposition.

  ParVal is an integer such that: 1 ≤ level ≤ levmax which is the maximum possible level (see wmaxlev).

  The default level depends on the method:

  - for **PCSM** methods level is equal to levmax.

  - for **CTM** methods level is equal to fix(levmax/2)

- ParName' = 'it' or 'IT' sets Image type Transform.

  ParVal must be one of the following strings:

  'n' : no transformation (default), image type (truecolor or grayscale) is automatically detected.

  'g' : grayscale transformation type.

  'c' : color transformation type (RGB uint8).

- 'ParName' = 'cc' or 'CC' sets Color Conversion parameter if X is a truecolor image.

  ParVal must be one of the following strings:

  'rgb' or 'none' : No conversion (default).

  'yuv' : YUV color space transform.

  'klt' : Karhunen-Loeve transform.

  'yiq' : YIQ color space transform.

  'xyz' : CIEXYZ color space transform.

Parameter for Progressive Coefficients Significance Methods (PCSM)

- `'ParName'` = `'maxloop'` or `'MAXLOOP'` sets the maximum number of steps for the compression algorithm.

  `ParVal` must be a positive integer or `Inf` (default is 10).

Parameters for Coefficients Thresholding Methods (CTM-1)

Either of the following parameters may be used:

- `'ParName'` = `'bpp'` or `'BPP'` sets the bit-per-pixel ratio.

  `ParVal` must be such that $0 \leq$ `ParVal` ≤ 8 (grayscale) or 24 (truecolor).
- `'ParName'` = `'comprat'` or `'COMPRAT'` sets the compression ratio.

  `ParVal` must be such that $0 \leq$ `ParVal` ≤ 100.

Parameters for Coefficients Thresholding Methods (CTM-2)

Two parameters may be used. The first is related to the threshold and the second is the number of classes for quantization.

The first one may be chosen among the five following parameters:

- `'ParName'` = `'threshold'` or `'THRESHOLD'` sets the threshold value for compression.

  `ParVal` must be a positive (or zero) real number.
- `'ParName'` = `'nbcfs'` or `'NBCFS'` sets the number of preserved coefficients in the wavelet decomposition.

  `ParVal` must be an integer such that: $0 \leq$ `ParVal` $\leq$ total number of coefficients of wavelet decomposition.
- `'ParName'` = `'percfs'` or `'PERCFS'` sets the percentage of preserved coefficients in the wavelet decomposition.

  `ParVal` must be a real number such that: $0 \leq$ `ParVal` ≤ 100.
- `'ParName'` = `'bpp'` or `'BPP'` sets the bit-per-pixel ratio.

  `ParVal` must be such that: $0 \leq$ `ParVal` ≤ 8 (grayscale) or 24 (truecolor)

- `'ParName' = 'comprat'` or `'COMPRAT'` sets the compression ratio.

  `ParVal` must be such that: `0 ≤ ParVal ≤ 100`.

The second parameter sets the number of classes for quantization:

- `'ParName' = 'nbclas'` or `'NBCLAS'` sets the number of classes.

  `ParVal` must be a real number such that: `2 ≤ ParVal ≤ 200`.

Display parameter

- `'ParName' = 'plotpar'` or `'PLOTPAR'` sets the plot parameter.

  `ParVal` must be one of the following strings or numbers:

  `'plot'` or `0`: plots only the compressed image.

  `'step'` or `1`: displays each step of the encoding process (only for **PCSM** methods).

`[COMPRAT,BPP] = wcompress('c',...)` returns the compression ratio `COMPRAT` and the bit_per_pixel ratio `BPP`.

Uncompression

`XC = wcompress('u',SAV_FILENAME)` uncompresses the file `SAV_FILENAME` and returns the image `XC`. Depending on the initial compressed image, `XC` can be a 2-D array containing either an indexed image or a 3-D array of `uint8` containing a truecolor image.

`XC = wcompress('u',SAV_FILENAME,'plot')` plots the uncompressed image.

`XC = wcompress('u',SAV_FILENAME,'step')` shows the step-by-step uncompression, only for **PCSM** methods.

Examples

Image Compression Using Basic Parameters

This example shows how to compress and uncompress the jpeg image arms.jpg.

Use the spatial orientation tree wavelet (`'stw'`) compression method and save the compressed image to a file.

```
wcompress('c','arms.jpg','comp_arms.wtc','stw');
```

Load the stored image and display the step-by-step uncompression to produce the uncompressed image.

```
wcompress('u','comp_arms.wtc','step');
```

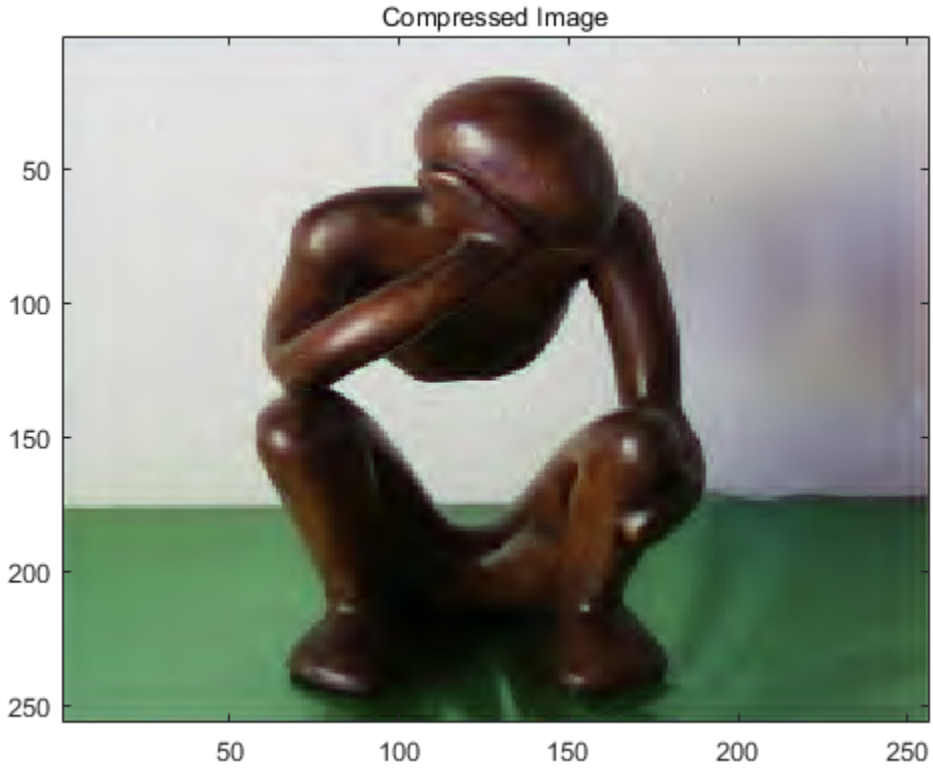
Image Compression and Uncompression Using Advanced Parameters.

This example show how to compress a jpeg image using the adaptively scanned wavelet difference reduction compression method (`'aswdr'`). The conversion color (`'cc'`) uses the Karhunen-Loeve transform (`'kit'`). The maximum number of loops (`'maxloop'`) is set to 11 and the plot type (`'plotpar'`) is set to step through the compression. Show the

compression ratio (`cratio`) and the bit-per-pixel ratio (`bpp`), which indicate the quality of the compression.

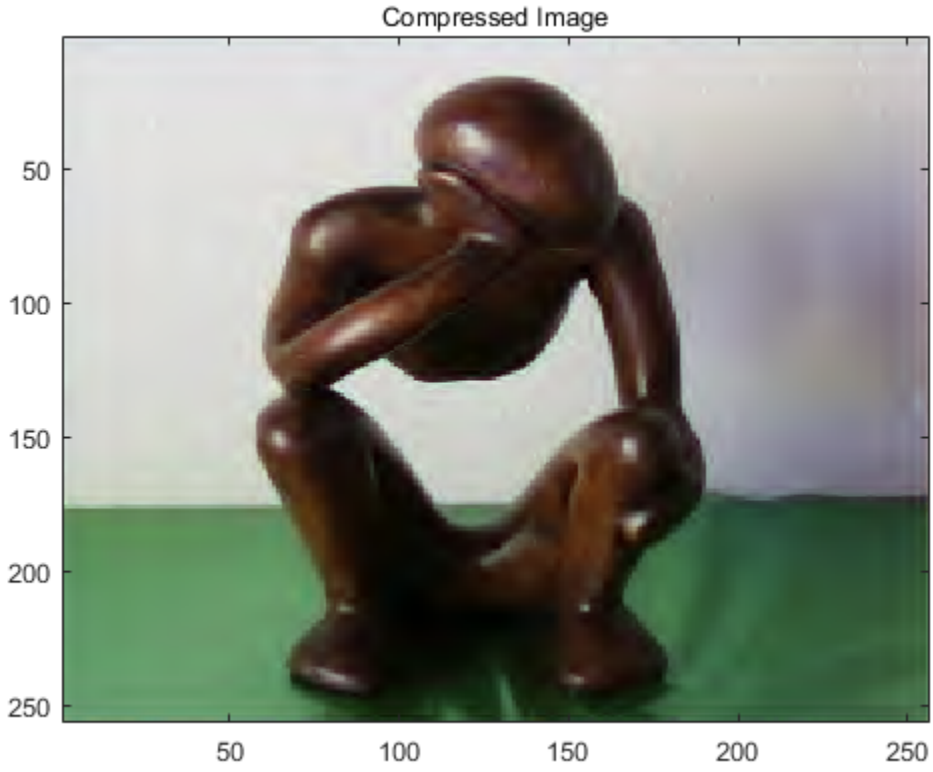
```
[cratio,bpp] = wcompress('c','woodstatue.jpg','woodstatue.wtc', ...
               'aswdr','cc','klt','maxloop',11,'plotpar','step');
cratio
bpp
```

```
cratio =

    3.0701


bpp =

    0.7368
```



Load the compressed image and step through the uncompression process.

```
wcompress('u','woodstatue.wtc','step');
```



Compression and Uncompression of a Grayscale Image

This example shows how to compress a grayscale image using the set partitioning in hierarchical trees (`'spiht'`) compression method. It also computes the mean square error (MSE) and the peak signal to noise ratio (PSNR) error values. You use these two measures to quantify the error between two images. The PSNR is expressed in decibels.

Load the image and store it in a file.

```
load mask;
[cr,bpp] = wcompress('c',X,'mask.wtc','spiht','maxloop',12)


cr =
```

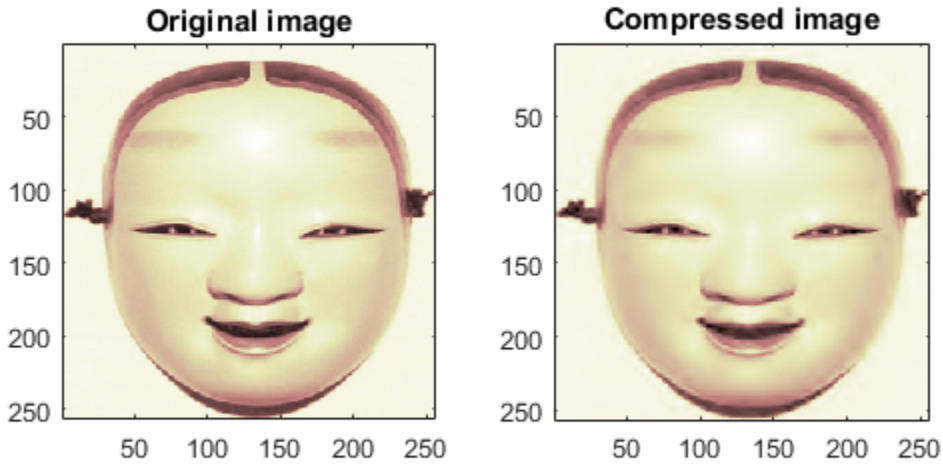
```
    2.8336

bpp =

    0.2267
```

Load the stored image from the file, uncompress it, and delete the file.

```
Xc = wcompress('u','mask.wtc');
delete('mask.wtc')
```

Display the original and compressed images.

```
colormap(pink(255))
subplot(1,2,1); image(X);  title('Original image')
axis square
subplot(1,2,2); image(Xc); title('Compressed image')
axis square
```



Compute the MSE and PSNR.

```
D = abs(X-Xc).^2;
mse  = sum(D(:))/numel(X)
psnr = 10*log10(255*255/mse)
```

```
mse =

   33.6564


psnr =
```

```
    32.8601
```

Compression and Uncompression of a Truecolor Image

is example shows how to compress a trucolor image using the set partitioning in hierarchical trees - 3D (`'spiht_3D'`) compression method.

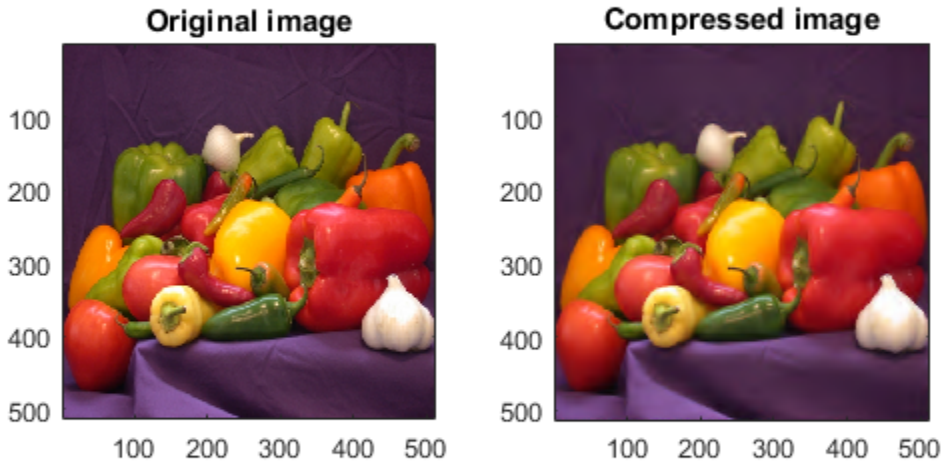
Load, compress, and store the image in a file. Plot the original and compressed images. Display the compression ratio (`'cratio'`) and the bits-per-pixel (`'bpp'`), which indicate the quality of the compression.

```
load mask;
X = imread('wpeppers.jpg');
[cratio,bpp] = wcompress('c',X,'wpeppers.wtc','spiht','maxloop',12)
Xc = wcompress('u','wpeppers.wtc');
delete('wpeppers.wtc')
```

```
cratio =

    1.6527


bpp =

    0.3966
```

Display the original and compressed images.

```
subplot(1,2,1); image(X);  title('Original image'), axis square
subplot(1,2,2); image(Xc); title('Compressed image'), axis square
```



Compute the mean square error (MSE) and the peak signal-to-noise ratio (PSNR) error values. You use these two measures to quantify the error between two images. The PSNR is expressed in decibels.

```
D = abs(double(X)-double(Xc)).^2;
mse  = sum(D(:))/numel(X)
psnr = 10*log10(255*255/mse)
```

```
mse =

   26.7808
```

```
psnr =

   33.8526
```

References

Christophe, E., C. Mailhes, P. Duhamel (2006), "Adaptation of zerotrees using signed binary digit representations for 3 dimensional image coding," EURASIP *Journal on Image and Video Processing*, 2007, to appear in the special issue on Wavelets in Source Coding, Communications, and Networks, Paper ID 54679.

Misiti, M., Y. Misiti, G. Oppenheim, J.-M. Poggi (2007), *Wavelets and their applications*, ISTE DSP Series.

Said A., W.A. Pearlman (1996), "A new, fast, and efficient image codec based on set partitioning in hierarchical trees," *IEEE Trans. on Circuits and Systems for Video Technology*, Vol. 6, No. 3, pp. 243–250.

Shapiro J.M. (1993), "Embedded image coding using zerotrees of wavelet coefficients",P *IEEE Trans. Signal Proc.*, Vol. 41, No. 12, pp. 3445–3462.

Strang, G.; T. Nguyen (1996), *Wavelets and Filter Banks*, Wellesley-Cambridge Press.

Walker J.S. (1999), "Wavelet-Based Image Compression," University of Wisconsin, Eau Claire, Wisconsin, USA, , Sub-chapter of CRC Press book: *Transform and Data Compression. A Primer on Wavelets and Their Scientific Applications*.

See Also

`imread` | `imwrite` | `wmaxlev`

Introduced in R2008b

wdcbm

Thresholds for wavelet 1-D using Birgé-Massart strategy

Syntax

```
[THR,NKEEP] = wdcbm(C,L,ALPHA,M)
wdcbm(C,L,ALPHA)
wdcbm(C,L,ALPHA,L(1))
```

Description

`[THR,NKEEP] = wdcbm(C,L,ALPHA,M)` returns level-dependent thresholds `THR` and numbers of coefficients to be kept `NKEEP`, for de-noising or compression. `THR` is obtained using a wavelet coefficients selection rule based on the Birgé-Massart strategy.

`[C,L]` is the wavelet decomposition structure of the signal to be de-noised or compressed, at level `j = length(L)-2`. `ALPHA` and `M` must be real numbers greater than 1.

`THR` is a vector of length `j`; `THR(i)` contains the threshold for level i.

`NKEEP` is a vector of length `j`; `NKEEP(i)` contains the number of coefficients to be kept at level i.

j, `M` and `ALPHA` define the strategy:

- At level j+1 (and coarser levels), everything is kept.
- For level i from 1 to j, the n_i largest coefficients are kept with n_i = `M` / (j+2-i)$^{\mathrm{ALPHA}}$.

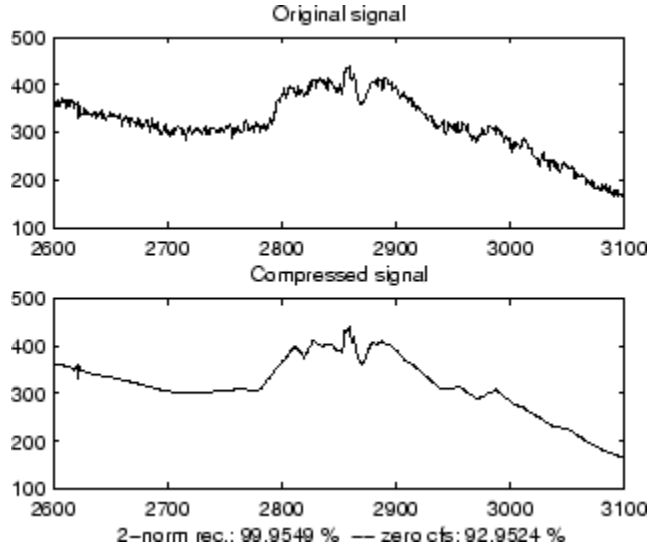
Typically `ALPHA` = 1.5 for compression and `ALPHA` = 3 for de-noising.

A default value for `M` is `M` = `L`(1), the number of the coarsest approximation coefficients, since the previous formula leads for i = j+1, to n_{j+1} = `M` = `L`(1). Recommended values for `M` are from `L`(1) to 2*`L`(1).

`wdcbm(C,L,ALPHA)` is equivalent to `wdcbm(C,L,ALPHA,L(1))`.

Examples

```
% Load electrical signal and select a part of it.
load leleccum; indx = 2600:3100;
x = leleccum(indx);

% Perform a wavelet decomposition of the signal
% at level 5 using db3.
wname = 'db3'; lev = 5;
[c,l] = wavedec(x,lev,wname);

% Use wdcbm for selecting level dependent thresholds
% for signal compression using the adviced parameters.
alpha = 1.5; m = l(1);
[thr,nkeep] = wdcbm(c,l,alpha,m)

thr =
   19.5569   17.1415   20.2599   42.8959   15.0049

nkeep =
     1     2     3     4     7

% Use wdencmp for compressing the signal using the above
% thresholds with hard thresholding.
[xd,cxd,lxd,perf0,perfl2] = ...
                 wdencmp('lvd',c,l,wname,lev,thr,'h');

% Plot original and compressed signals.
subplot(211), plot(indx,x), title('Original signal');
subplot(212), plot(indx,xd), title('Compressed signal');
xlab1 = ['2-norm rec.: ',num2str(perfl2)];
xlab2 = [' %  -- zero cfs: ',num2str(perf0), ' %'];
xlabel([xlab1 xlab2]);
```



References

Birgé, L.; P. Massart (1997), "From model selection to adaptive estimation," in D. Pollard (ed), *Festchrift for L. Le Cam*, Springer, pp. 55–88.

See Also

`wden` | `wdencmp` | `wpdencmp`

Introduced before R2006a

wdcbm2

Thresholds for wavelet 2-D using Birgé-Massart strategy

Syntax

```
[THR,NKEEP] = wdcbm2(C,S,ALPHA,M)
wdcbm2(C,S,ALPHA)
wdcbm2(C,S,ALPHA,prod(S(1,:)))
```

Description

`[THR,NKEEP] = wdcbm2(C,S,ALPHA,M)` returns level-dependent thresholds `THR` and numbers of coefficients to be kept `NKEEP`, for de-noising or compression. `THR` is obtained using a wavelet coefficients selection rule based on the Birgé-Massart strategy.

`[C,S]` is the wavelet decomposition structure of the image to be de-noised or compressed, at level `j = size(S,1)-2`.

`ALPHA` and `M` must be real numbers greater than 1.

`THR` is a matrix 3 by `j`; `THR(:,i)` contains the level dependent thresholds in the three orientations: horizontal, diagonal, and vertical, for level i.

`NKEEP` is a vector of length `j`; `NKEEP(i)` contains the number of coefficients to be kept at level i.

j, `M` and `ALPHA` define the strategy:

- At level `j`+1 (and coarser levels), everything is kept.
- For level i from 1 to `j`, the n_i largest coefficients are kept with $n_i = M \quad (j+2-i)^{ALPHA}$.

Typically `ALPHA` = 1.5 for compression and `ALPHA` = 3 for de-noising.

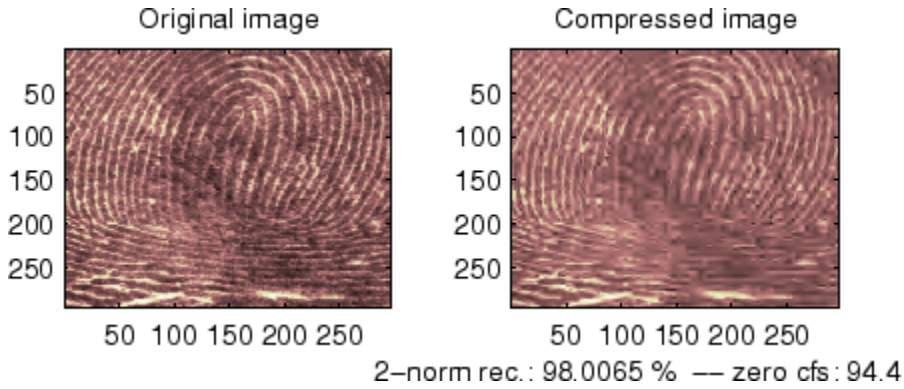
A default value for `M` is `M = prod(S(1,:))`, the length of the coarsest approximation coefficients, since the previous formula leads for i = j+1, to $n_{j+1} = M = $ `prod(S(1,:))`.

Recommended values for M are from `prod(S(1,:))` to `6*prod(S(1,:))`.

`wdcbm2(C,S,ALPHA)` is equivalent to `wdcbm2(C,S,ALPHA,prod(S(1,:)))`.

Examples

```
% Load original image.
load detfingr;
nbc = size(map,1);

% Perform a wavelet decomposition of the image
% at level 3 using sym4.
wname = 'sym4'; lev = 3;
[c,s] = wavedec2(X,lev,wname);

% Use wdcbm2 for selecting level dependent thresholds
% for image compression using the adviced parameters.
alpha = 1.5; m = 2.7*prod(s(1,:));
[thr,nkeep] = wdcbm2(c,s,alpha,m)

thr =
   21.4814   46.8354   40.7907
   21.4814   46.8354   40.7907
   21.4814   46.8354   40.7907

nkeep =
         624         961        1765

% Use wdencmp for compressing the image using the above
% thresholds with hard thresholding.
[xd,cxd,sxd,perf0,perfl2] = ...
                  wdencmp('lvd',c,s,wname,lev,thr,'h');

% Plot original and compressed images.
colormap(pink(nbc));
subplot(221), image(wcodemat(X,nbc)),
title('Original image')
subplot(222), image(wcodemat(xd,nbc)),
title('Compressed image')
xlab1 = ['2-norm rec.: ',num2str(perfl2)];
xlab2 = [' %  -- zero cfs: ',num2str(perf0), ' %'];
xlabel([xlab1 xlab2]);
```



References

Birgé, L.; P. Massart (1997). “From model selection to adaptive estimation,” in D. Pollard (ed), *Festchrift for L. Le Cam*, Springer, pp. 55–88.

See Also

`wdencmp` | `wpdencmp`

Introduced before R2006a

wdecenergy

Multisignal 1-D decomposition energy distribution

Syntax

```
[E,PEC,PECFS] = wdecenergy(DEC)
[E,PEC,PECFS,IDXSORT,LONGS] = wdecenergy(DEC,'sort')
[E,PEC,PECFS] = wdecenergy(DEC,OPTSORT,IDXSIG)
[E,PEC,PECFS,IDXSORT,LONGS] = wdecenergy(DEC,OPTSORT,IDXSIG)
```

Description

`[E,PEC,PECFS] = wdecenergy(DEC)` computes the vector E that contains the energy (L2-Norm) of each decomposed signal, the matrix PEC that contains the percentage of energy for each wavelet component (approximation and details) of each signal, and the matrix PECFS that contains the percentage of energy for each coefficient.

- E(i) is the energy (L2-norm) of the ith signal.
- PEC(i,1) is the percentage of energy for the approximation of level MAXLEV = DEC.level of the ith signal.
- PEC(i,j), j=2,...,MAXLEV+1 is the percentage of energy for the detail of level (MAXLEV+1-j) of the ith signal.
- PECFS(i,j), is the percentage of energy for jth coefficients of the ith signal.

`[E,PEC,PECFS,IDXSORT,LONGS] = wdecenergy(DEC,'sort')` returns PECFS sorted (by row) in ascending order and an index vector IDXSORT.

- Replacing 'sort' by 'ascend' returns the same result.
- Replacing 'sort' by 'descend' returns PECFS sorted in descending order.

LONGS is a vector containing the lengths of each family of coefficients.

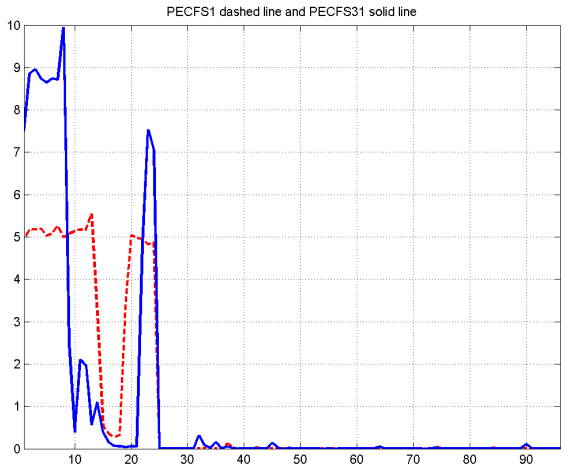
`[E,PEC,PECFS] = wdecenergy(DEC,OPTSORT,IDXSIG)` returns the values for the signals whose indices are given by the `IDXSIG` vector.

`[E,PEC,PECFS,IDXSORT,LONGS] = wdecenergy(DEC,OPTSORT,IDXSIG)` returns the values for the signals whose indices are given by the IDXSIG vector, the index

vector IDXSORT, and LONGS, which is a vector containing the lengths of each family of coefficients. Valid values for OPTSORT are 'none', 'sort', 'ascend', 'descend'.

Examples

```
% Load original 1D-multisignal.
load thinker
% Perform a decomposition at level 2 using wavelet db2.
dec = mdwtdec('r',X,2,'db2');
% Compute the energy distribution.
[E,PEC,PECFS] = wdecenergy(dec);
% Display the total energy and the distribution of energy
% for each wavelet component (A2, D2, D1).
E31 = E(31)
perA2D2D1 = PEC(31,:)
% Compare the coefficient energy distribution
% for signal 1 and signal 31.
PECFS_1  = PECFS(1,:);
PECFS_31 = PECFS(31,:);
figure;
plot(PECFS_1,'--r','linewidth',2); hold on
plot(PECFS_31,'b','linewidth',2);
grid; set(gca,'Xlim',[1,size(PECFS,2)])
title('PECFS1 dashed line and PECFS31 solid line')
```



See Also

`mdwtdec` | `mdwtrec`

Introduced in R2012a

wden

Automatic 1-D de-noising

Syntax

```
XD = wden(X,TPTR,SORH,SCAL,N,'wname')
XD = wden(C,L,TPTR,SORH,SCAL,N,'wname')
XD = wden(W,'modwtsqtwolog',SORH,'mln',N,WNAME)
[XD,CXD] = wden(...)
[XD,CXD,LXD] = wden(...)
```

Description

`wden` is a one-dimensional de-noising function.

`wden` performs an automatic de-noising process of a one-dimensional signal using wavelets.

`XD = wden(X,TPTR,SORH,SCAL,N,'wname')` returns a de-noised version `XD` of input signal `X` obtained by thresholding the wavelet coefficients.

`TPTR` string contains the threshold selection rule:

- `'rigrsure'` uses the principle of Stein's Unbiased Risk.
- `'heursure'` is an heuristic variant of the first option.
- `'sqtwolog'` for the universal threshold $\sqrt{2\ln(\bullet)}$
- `'minimaxi'` for minimax thresholding (see `thselect` for more information)

`SORH` (`'s'` or `'h'`) is for soft or hard thresholding (see `wthresh` for more information).

`SCAL` defines multiplicative threshold rescaling:

`'one'` for no rescaling

`'sln'` for rescaling using a single estimation of level noise based on first-level coefficients

`'mln'` for rescaling done using level-dependent estimation of level noise

Wavelet decomposition is performed at level N and '*wname*' is a string containing the name of the desired orthogonal wavelet (see wmaxlev and wfilters for more information).

XD = wden(C,L,TPTR,SORH,SCAL,N,'*wname*') returns the same output arguments, using the same options as above, but obtained directly from the input wavelet decomposition structure [C,L] of the signal to be de-noised, at level N and using '*wname*' orthogonal wavelet.

XD = wden(W,'modwtsqtwolog',SORH,'mln',N,WNAME) returns the denoised signal obtained by operating on the MODWT transform matrix W, where W is the output of MODWT. You must use the same wavelet in both modwt and wden.

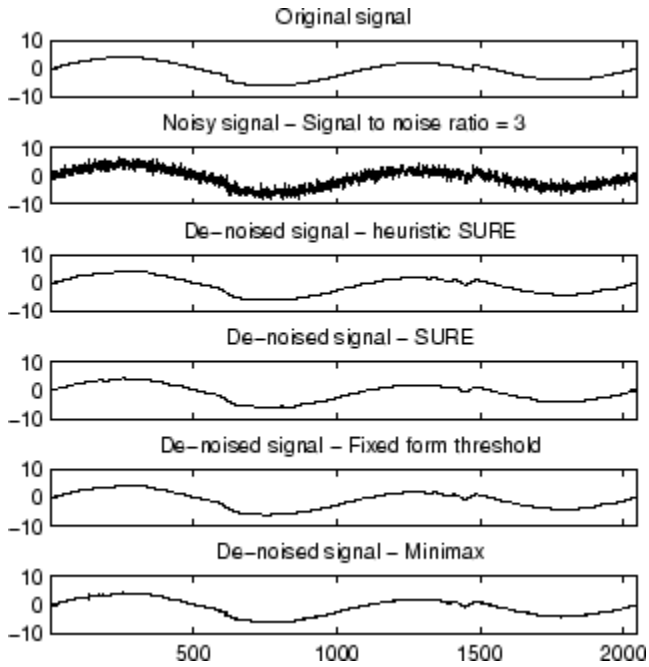
[XD,CXD] = wden(...) returns the denoised wavelet coefficients. For DWT denoising, CXD is a vector (see wavedec). For MODWT denoising, CXD is a matrix with N+1 rows (see modwt). The number of columns is equal to the length of the input signal X.

[XD,CXD,LXD] = wden(...) returns the number of coefficients by level for DWT denoising. See wavedec for details. The LXD output is not supported for MODWT denoising. The additional output arguments [CXD,LXD] are the wavelet decomposition structure (see wavedec for more information) of the de-noised signal XD.

Examples

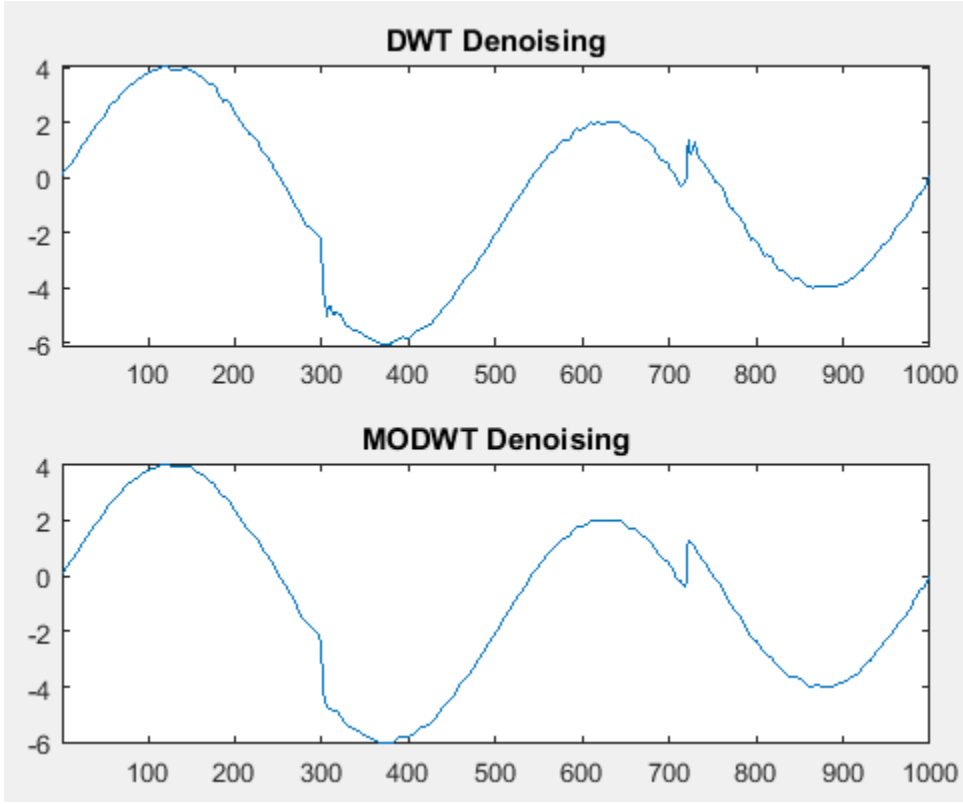
```
% The current extension mode is zero-padding (see dwtmode).

% Set signal to noise ratio and set rand seed.
snr = 3; init = 2055615866;

% Generate original signal and a noisy version adding
% a standard Gaussian white noise.
[xref,x] = wnoise(3,11,snr,init);

% De-noise noisy signal using soft heuristic SURE thresholding
% and scaled noise option, on detail coefficients obtained
% from the decomposition of x, at level 5 by sym8 wavelet.
lev = 5;
xd = wden(x,'heursure','s','one',lev,'sym8');

% Plot signals.
subplot(611), plot(xref), axis([1 2048 -10 10]);
title('Original signal');
```

```
subplot(612), plot(x), axis([1 2048 -10 10]);
title(['Noisy signal - Signal to noise ratio = ',...
num2str(fix(snr))]);
subplot(613), plot(xd), axis([1 2048 -10 10]);
title('De-noised signal - heuristic SURE');

% De-noise noisy signal using soft SURE thresholding
xd = wden(x,'heursure','s','one',lev,'sym8');

% Plot signal.
subplot(614), plot(xd), axis([1 2048 -10 10]);
title('De-noised signal - SURE');

% De-noise noisy signal using fixed form threshold with
% a single level estimation of noise standard deviation.
xd = wden(x,'sqtwolog','s','sln',lev,'sym8');

% Plot signal.
subplot(615), plot(xd), axis([1 2048 -10 10]);
title('De-noised signal - Fixed form threshold');

% De-noise noisy signal using minimax threshold with
% a multiple level estimation of noise standard deviation.
xd = wden(x,'minimaxi','s','sln',lev,'sym8');

% Plot signal.
subplot(616), plot(xd), axis([1 2048 -10 10]);
title('De-noised signal - Minimax');

% If many trials are necessary, it is better to perform
% decomposition once and threshold it many times:

% decomposition.
[c,l] = wavedec(x,lev,'sym8');

% threshold the decomposition structure [c,l].
xd = wden(c,l,'minimaxi','s','sln',lev,'sym8');

% Editing some graphical properties,
% the following figure is generated.
```



Denoise a signal consisting of a 2-Hz sine wave with transients at 0.3 and 0.72 seconds. Use Donoho and Johnstone's universal threshold with level-dependent estimation of the noise. Obtain denoised versions using the DWT and MODWT. Compare the results.

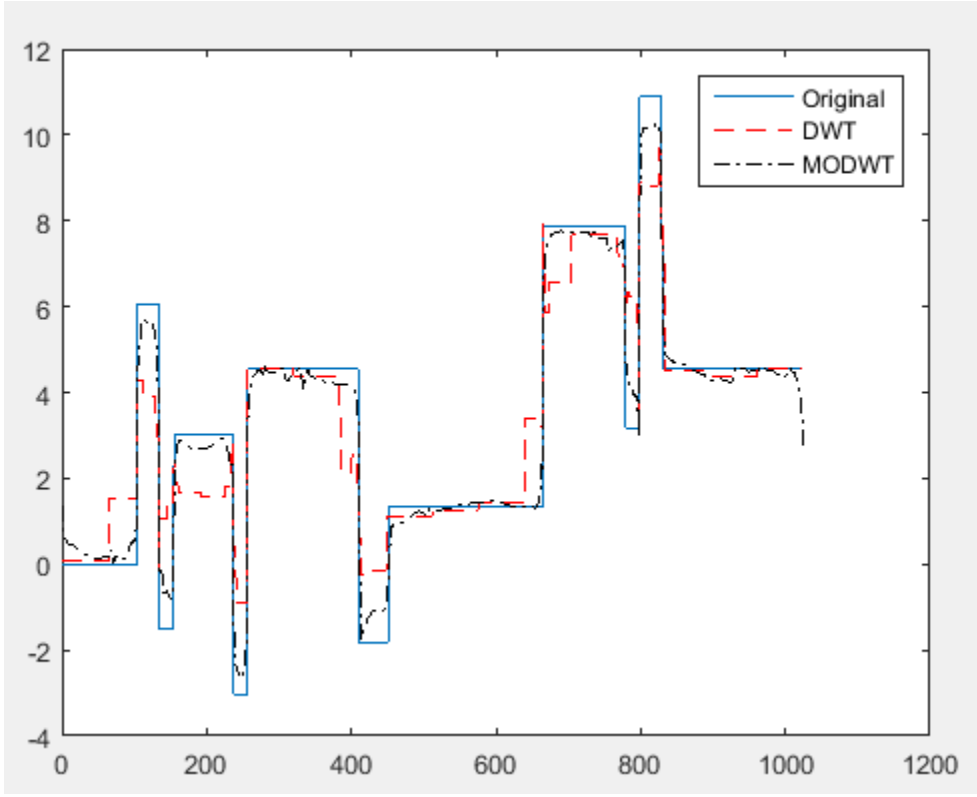
```
N = 1000;
t = linspace(0,1,N);
x = 4*sin(4*pi*t);
x = x - sign(t-.3)-sign(.72 - t);
y = x+0.15*randn(size(t));
xdDWT = wden(y,'sqtwolog','s','mln',3,'db2');
xdMODWT = wden(y,'modwtsqtwolog','s','mln',3,'db2');
subplot(2,1,1)
plot(xdDWT), title('DWT Denoising');
axis tight;
subplot(2,1,2)
plot(xdMODWT), title('MODWT Denoising');
axis tight;
```



Denoise a blocky signal using the Haar wavelet with MODWT and DWT denoising. Compare the L2 and L-infty norms of the difference between the original signal and the denoised versions.

```
[x,xn] = wnoise('blocks',10,3);
xdMODWT = wden(xn,'modwtsqtwolog','s','mln',6,'haar');
xd = wden(xn,'sqtwolog','s','mln',6,'haar');
plot(x)
hold on
plot(xd,'r--')
plot(xdMODWT,'k-.')
legend('Original','DWT','MODWT')
hold off
norm(abs(x-xd),2),
norm(abs(x-xd),Inf)
```

```
norm(abs(x-xdMODWT),2),
norm(abs(x-xdMODWT),Inf)
```



More About

Algorithms

The underlying model for the noisy signal is basically of the following form:

$$s(n) = f(n) + \sigma e(n)$$

where time n is equally spaced.

In the simplest model, suppose that $e(n)$ is a Gaussian white noise $N(0,1)$ and the noise level σ a is supposed to be equal to 1.

The de-noising objective is to suppress the noise part of the signal s and to recover f.

The de-noising procedure proceeds in three steps:

1. Decomposition. Choose a wavelet, and choose a level `N`. Compute the wavelet decomposition of the signal s at level `N`.
2. Detail coefficients thresholding. For each level from 1 to `N`, select a threshold and apply soft thresholding to the detail coefficients.
3. Reconstruction. Compute wavelet reconstruction based on the original approximation coefficients of level `N` and the modified detail coefficients of levels from 1 to `N`.

More details about threshold selection rules are in "Wavelet Denoising and Nonparametric Function Estimation", in the User's Guide, and in the help of the `thselect` function. Let us point out that

- The detail coefficients vector is the superposition of the coefficients of f and the coefficients of e, and that the decomposition of e leads to detail coefficients that are standard Gaussian white noises.
- Minimax and SURE threshold selection rules are more conservative and are more convenient when small details of function f lie in the noise range. The two other rules remove the noise more efficiently. The option `'heursure'` is a compromise.

In practice, the basic model cannot be used directly. This section examines the options available, to deal with model deviations. The remaining parameter `scal` has to be specified. It corresponds to threshold rescaling methods.

- Option `scal` = `'one'` corresponds to the basic model.
- In general, you can ignore the noise level that must be estimated. The detail coefficients CD_1 (the finest scale) are essentially noise coefficients with standard deviation equal to σ. The median absolute deviation of the coefficients is a robust estimate of σ. The use of a robust estimate is crucial because if level 1 coefficients contain f details, these details are concentrated in few coefficients to avoid signal end effects, which are pure artifacts due to computations on the edges.
- The option `scal = 'sln'` handles threshold rescaling using a single estimation of level noise based on the first-level coefficients.

- When you suspect a nonwhite noise *e*, thresholds must be rescaled by a level-dependent estimation of the level noise. The same kind of strategy is used by estimating σ_{lev} level by level. This estimation is implemented in the file `wnoisest`, which handles the wavelet decomposition structure of the original signal *s* directly.
- The option `scal = 'mln'` handles threshold rescaling using a level-dependent estimation of the level noise.

References

Antoniadis, A.; G. Oppenheim, Eds. (1995), *Wavelets and statistics*, 103, Lecture Notes in Statistics, Springer Verlag.

Donoho, D.L. (1993), "Progress in wavelet analysis and WVD: a ten minute tour," in *Progress in wavelet analysis and applications*, Y. Meyer, S. Roques, pp. 109–128. Frontières Ed.

Donoho, D.L.; I.M. Johnstone (1994), "Ideal spatial adaptation by wavelet shrinkage," *Biometrika*, Vol. 81, pp. 425–455.

Donoho, D.L. (1995), "De-noising by soft-thresholding," *IEEE Trans. on Inf. Theory*, 42 3, pp. 613– 627.

Donoho, D.L.; I.M. Johnstone, G. Kerkyacharian, D. Picard (1995), "Wavelet shrinkage: asymptotia," *Jour. Roy. Stat. Soc.*, *series B*, Vol. 57, No. 2, pp. 301–369.

See Also

`thselect` | `wavedec` | `wdencmp` | `wfilters` | `wthresh`

Introduced before R2006a

wdencmp

De-noising or compression

Syntax

```
[XC,CXC,LXC,PERF0,PERFL2] =
wdencmp('gbl',X,'wname',N,THR,SORH,KEEPAPP)
wdencmp('gbl',C,L,'wname',N,THR,SORH,KEEPAPP)
[XC,CXC,LXC,PERF0,PERFL2] = wdencmp('lvd',X,'wname',N,THR,SORH)
[XC,CXC,LXC,PERF0,PERFL2] = wdencmp('lvd',C,L,'wname',N,THR,SORH)
[XC,CXC,LXC,PERF0,PERFL2] = wdencmp('lvd',X,'wname',N,THR,SORH)
[XC,CXC,LXC,PERF0,PERFL2] = wdencmp('lvd',C,L,'wname',N,THR,SORH)
```

Description

`wdencmp` is a one- or two-dimensional de-noising and compression-oriented function.

`wdencmp` performs a de-noising or compression process of a signal or an image, using wavelets.

`[XC,CXC,LXC,PERF0,PERFL2] = wdencmp('gbl',X,'wname',N,THR,SORH,KEEPAPP)` returns a de-noised or compressed version `XC` of input signal `X` (one- or two-dimensional) obtained by wavelet coefficients thresholding using global positive threshold `THR`.

Additional output arguments `[CXC,LXC]` are the wavelet decomposition structure of `XC` (see `wavedec` or `wavedec2` for more information). `PERF0` and `PERFL2` are L^2 -norm recovery and compression score in percentage.

`PERFL2` = 100 * (vector-norm of `CXC` / vector-norm of `C`)^2 if `[C,L]` denotes the wavelet decomposition structure of `X`.

If `X` is a one-dimensional signal and `'wname'` an orthogonal wavelet, `PERFL2` is reduced to

$$\frac{100\|XC\|^2}{\|X\|^2}$$

Wavelet decomposition is performed at level N and '*wname*' is a string containing wavelet name (see wmaxlev and wfilters for more information). SORH ('s' or 'h') is for soft or hard thresholding (see wthresh for more information). If KEEPAPP = 1, approximation coefficients cannot be thresholded, otherwise it is possible.

wdencmp('gbl',C,L,'*wname*',N,THR,SORH,KEEPAPP) has the same output arguments, using the same options as above, but obtained directly from the input wavelet decomposition structure [C,L] of the signal to be de-noised or compressed, at level N and using '*wname*' wavelet.

For the one-dimensional case and 'lvd' option, [XC,CXC,LXC,PERF0,PERFL2] = wdencmp('lvd',X,'*wname*',N,THR,SORH) or [XC,CXC,LXC,PERF0,PERFL2] = wdencmp('lvd',C,L,'*wname*',N,THR,SORH) have the same output arguments, using the same options as above, but allowing level-dependent thresholds contained in vector THR (THR must be of length N). In addition, the approximation is kept. Note that, with respect to wden (automatic de-noising), wdencmp allows more flexibility and you can implement your own de-noising strategy.

For the two-dimensional case and 'lvd' option, [XC,CXC,LXC,PERF0,PERFL2] = wdencmp('lvd',X,'*wname*',N,THR,SORH) or [XC,CXC,LXC,PERF0,PERFL2] = wdencmp('lvd',C,L,'*wname*',N,THR,SORH).

THR must be a matrix 3 by N containing the level-dependent thresholds in the three orientations, horizontal, diagonal, and vertical.

Like denoising, the compression procedure contains three steps:

1. Decomposition.
2. Detail coefficient thresholding. For each level from 1 to N, a threshold is selected and hard thresholding is applied to the detail coefficients.
3. Reconstruction.

The difference with the denoising procedure is found in step 2.

Examples

Denoise Image Using Default Global Threshold

Denoise an image in additive white Gaussian noise using the Donoho-Johnstone universal threshold.

Load the image and add white Gaussian noise.

```
load sinsin;
Y = X + 18*randn(size(X));
```

Use `ddencmp` to obtain the threshold and denoise the image. Plot the original image, noisy image, and denoised result.

```
[thr,sorh,keepapp] = ddencmp('den','wv',Y);
xd = wdencmp('gbl',Y,'sym4',2,thr,sorh,keepapp);
subplot(221)
imagesc(X); title('Original Image');
subplot(222);
imagesc(Y); title('Noisy Image');
subplot(223)
imagesc(xd); title('Denoised Image');
```

Denoise 1-D Signal Using Default Global Threshold

Denoise 1-D electricity consumption data using the Donoho-Johnstone global threshold.

Load the signal and select a segment for denoising.

```
load leleccum; indx = 2600:3100;
x = leleccum(indx);
```

Use `ddencmp` to determine the default global threshold and denoise the signal. Plot the original and denoised signals.

```
[thr,sorh,keepapp] = ddencmp('den','wv',x);
xd = wdencmp('gbl',x,'db3',2,thr,sorh,keepapp);
subplot(211)
plot(x); title('Original Signal');
subplot(212)
plot(xd); title('Denoised Signal');
```

References

DeVore, R.A.; B. Jawerth, B.J. Lucier (1992), "Image compression through wavelet transform coding," *IEEE Trans. on Inf. Theory*, vol. 38, No 2, pp. 719–746.

Donoho, D.L. (1993), "Progress in wavelet analysis and WVD: a ten minute tour," in Progress in wavelet analysis and applications, Y. Meyer, S. Roques, pp. 109–128. Frontières Ed.

Donoho, D.L.; I.M. Johnstone (1994), "Ideal spatial adaptation by wavelet shrinkage," *Biometrika*, vol. 81, pp. 425–455.

Donoho, D.L.; I.M. Johnstone, G. Kerkyacharian, D. Picard (1995), "Wavelet shrinkage: asymptopia," *Jour. Roy. Stat. Soc.*, *series B*, vol. 57 no. 2, pp. 301–369.

Donoho, D.L.; I.M. Johnstone, "Ideal de-noising in an orthonormal basis chosen from a library of bases," C.R.A.S. Paris, t. 319, Ser. I, pp. 1317–1322.

Donoho, D.L. (1995), "De-noising by soft-thresholding," *IEEE Trans. on Inf. Theory*, 41, 3, pp. 613–627.

See Also

`ddencmp` | `wavedec` | `wavedec2` | `wbmpen` | `wcompress` | `wdcbm2` | `wden` | `wpdencmp` | `wthresh`

Introduced before R2006a

wenergy

Energy for 1-D wavelet or wavelet packet decomposition

Syntax

```
[Ea,Ed] = wenergy(C,L)
E = wenergy(T)
```

Description

For a one-dimensional wavelet decomposition `[C,L]` (see `wavedec` for details), `[Ea,Ed] = wenergy(C,L)` returns `Ea`, which is the percentage of energy corresponding to the approximation and `Ed`, which is the vector containing the percentages of energy corresponding to the details.

For a wavelet packet tree *T* (see `wptree`, `wpdec`, `wpdec2`), `E = wenergy(T)` returns a vector `E`, which contains the percentages of energy corresponding to the terminal nodes of the tree *T*. In this case, `wenergy` is a method of the `wptree` object *T*, which overloads the previous `wenergy` function.

Examples

```
% Example 1: 1-D wavelet decomposition
%-------------------------------------
load noisbump
[C,L] = wavedec(noisbump,4,'sym4');
[Ea,Ed] = wenergy(C,L)

Ea =

   88.2860

Ed =

    2.1560    1.2286    1.4664    6.8630
```

```
% Example 2: 1-D wavelet packet decomposition
%-------------------------------------------
load noisbump
T = wpdec(noisbump,3,'sym4');
E = wenergy(T)

E =

95.0329  1.4664  0.6100  0.6408  0.5935  0.5445  0.5154
0.5965
```

Introduced before R2006a

wenergy2

Energy for 2-D wavelet decomposition

Syntax

```
[Ea,Eh,Ev,Ed] = wenergy2(C,S)
[Ea,EDetail] = wenergy2(C,S)
```

Description

For a two-dimensional wavelet decomposition `[C,S]` (see `wavedec2` for details), `[Ea,Eh,Ev,Ed] = wenergy2(C,S)` returns `Ea`, which is the percentage of energy corresponding to the approximation, and vectors `Eh`, `Ev`, `Ed`, which contain the percentages of energy corresponding to the horizontal, vertical, and diagonal details, respectively.

`[Ea,EDetail] = wenergy2(C,S)` returns `Ea`, and `EDetail`, which is the sum of vectors `Eh`, `Ev`, and `Ed`.

Examples

```
load detail
[C,S] = wavedec2(X,2,'sym4');
[Ea,Eh,Ev,Ed] = wenergy2(C,S)

Ea =
   89.3520

Eh =
    1.8748    2.7360

Ev =
    1.5860    2.6042

Ed =
    0.7539    1.0932
```

```
[Ea,EDetails] = wenergy2(C,S)

Ea =
   89.3520

EDetails =
    4.2147    6.4334
```

Introduced before R2006a

wentropy

Entropy (wavelet packet)

Syntax

```
E = wentropy(X,T,P)
E = wentropy(X,T)
E = wentropy(X,T,0)
```

Description

`E = wentropy(X,T,P)` returns the entropy `E` of the vector or matrix input `X`. In both cases, output `E` is a real number.

`E = wentropy(X,T)` is equivalent to `E = wentropy(X,T,0)`.

T is a string containing the type of entropy and `P` is an optional parameter depending on the value of *T*.

Entropy Type Name (*T*)	Parameter (P)	Comments
`'shannon'`		`P` is not used.
`'log energy'`		`P` is not used.
`'threshold'`	`0 ≤ P`	`P` is the threshold.
`'sure'`	`0 ≤ P`	`P` is the threshold.
`'norm'`	`1 ≤ P`	`P` is the power.
`'user'`	`string`	`P` is a string containing the file name of your own entropy function, with a single input `X`.
`FunName`	`No constraints on P`	`FunName` is any other string except those used for the previous Entropy Type Names listed above. `FunName` contains the file name of your own entropy function, with `X` as input

Entropy Type Name (*T*)	Parameter (P)	Comments
		and P as additional parameter to your entropy function.

Note The `'user'` option is historical and still kept for compatibility, but it is obsoleted by the last option described in the table above. The `FunName` option do the same as the `'user'` option and in addition gives the possibility to pass a parameter to your own entropy function.

Functionals verifying an additive-type property are well suited for efficient searching of binary-tree structures and the fundamental splitting property of the wavelet packets decomposition. Classical entropy-based criteria match these conditions and describe information-related properties for an accurate representation of a given signal. Entropy is a common concept in many fields, mainly in signal processing. The following example lists different entropy criteria. Many others are available and can be easily integrated. In the following expressions, s is the signal and $(s_i)_i$ the coefficients of s in an orthonormal basis.

The entropy `E` must be an additive cost function such that $E(0) = 0$ and

$$E(s) = \sum_i E(s_i)$$

- The (nonnormalized) Shannon entropy.

  $E1(s_i) = s_i^2 \log(s_i^2)$ so $E1(s) = -\sum_i s_i^2 \log(s_i^2)$

  with the convention $0 log(0) = 0$.

- The concentration in l^p norm entropy with $1 \le p$.

  $E2(s_i) = |s_i|^p$ so $E2(s) = \sum_i |s_i|^p = \|s\|_p^p$

- The "log energy" entropy.

  $E3(s_i) = \log(s_i^2)$ so $E3(s) = \sum_i \log(s_i^2)$

with the convention $log(0) = 0$.

- The threshold entropy.

  $E4(s_i) = 1$ if $|s_i| > p$ and 0 elsewhere so $E4(s) = \#\{i$ such that $|s_i| > p\}$ is the number of time instants when the signal is greater than a threshold p.

- The "SURE" entropy.

  E5(s) = n - #{i such that $|s_i| \le p\} + \sum_i min(s_i^2, p^2)$

  For more information, see the section "Wavelet Packets for Compression and Denoising" of the User's Guide.

Examples

```
% The current extension mode is zero-padding (see dwtmode).

% Generate initial signal.
x = randn(1,200);

% Compute Shannon entropy of x.
e = wentropy(x,'shannon')

e =
    -142.7607

% Compute log energy entropy of x.
e = wentropy(x,'log energy')
e =
    -281.8975

% Compute threshold entropy of x
% with threshold equal to 0.2.
e = wentropy(x,'threshold',0.2)
e =
    162

% Compute Sure entropy of x
% with threshold equal to 3.
e = wentropy(x,'sure',3)
```

```
e =
 -0.6575

% Compute norm entropy of x with power equal to 1.1.
e = wentropy(x,'norm',1.1)
e =
 160.1583

% Compute user entropy of x with a user defined
% function: userent for example.
% This function must be a code file, with first line
% of the following form:
%
%     function e = userent(x)
%
% where x is a vector and e is a real number.
% Then a new entropy is defined and can be used typing:
%
% e = wentropy(x,'user','userent')
%
% or more directly
%
% e = wentropy(x,'userent')
```

References

Coifman, R.R.; M.V. Wickerhauser (1992), "Entropy-based Algorithms for best basis selection," *IEEE Trans. on Inf. Theory*, vol. 38, 2, pp. 713–718.

Donoho, D.L.; I.M. Johnstone, "Ideal de-noising in an orthonormal basis chosen from a library of bases," *C.R.A.S. Paris*, *Ser. I*, t. 319, pp. 1317–1322.

Introduced before R2006a

wextend

Extend vector or matrix

Syntax

```
YEXT= wextend(TYPE,MODE,X,LEN)
YEXT= wextend(TYPE,MODE,X,LEN,LOC)
```

Description

`YEXT= wextend(TYPE,MODE,X,LEN)` returns an extended version of the input matrix or vector, `X`. The `TYPE` specifies the general method to extend the input. The `MODE` is the specific extension to use. The `LEN` is the amount by which to extend the input. The tables show the valid values for `TYPE` and `MODE`.

TYPE	Description
`1`, `'1'`, `'1d'` or `'1D'`	1-D extension
`2`, `'2'`, `'2d'` or `'2D'`	2-D extension
`'ar'` or `'addrow'`	Add rows
`'ac'` or `'addcol'`	Add columns

The valid extension modes (`MODE`) are listed in the table below.

MODE	Description
`'zpd'`	Zero extension
`'sp0'`	Smooth extension of order 0
`'spd'` (or `'sp1'`)	Smooth extension of order 1
`'sym'` or `'symh'`	Symmetric-padding (half-point): boundary value symmetric replication
`'symw'`	Symmetric-padding (whole-point): boundary value symmetric replication
`'asym'` or `'asymh'`	Antisymmetric-padding (half-point): boundary value antisymmetric replication

MODE	Description
'asymw'	Antisymmetric-padding (whole-point): boundary value antisymmetric replication
'ppd'	Periodized extension (1)
'per'	Periodized extension (2): If the signal length is odd, wextend adds an extra sample, equal to the last value, on the right and performs extension using the 'ppd' mode. Otherwise, 'per' reduces to 'ppd'. The same kind of rule stands for images.

For more information on symmetric extension modes see "References" on page 1-662.

YEXT= wextend(TYPE,MODE,X,LEN,LOC) uses the specified location, LOC, for the extension. Valid and default values of LOC and description of LEN for each TYPE are described in the table.

TYPE	LOC
1, '1', 1d' or '1D'	'l' (or 'u') for left (or up) extension. 'r' (or 'd') for right (or down) extension. 'b' for extension on both sides. 'n' null extension. Default: 'b' LEN is the length of the extension.
2, '2', '2d' or '2D'	[LOCROW,LOCCOL], where LOCROW and LOCCOL are 1D extension locations or 'n' (none). Default: 'bb' LEN, expressed as [LROW,LCOL], is the number of rows LROW and number of columns LCOL to add.
'ar', or 'addrow'	'l' (or 'u') for left (or up) extension. 'r' (or 'd') for right (or down) extension. 'b' for extension on both sides. 'n' null extension. Default: 'b' LEN is the number of rows to add.
'ac', or 'addcol'	'l' (or 'u') for left (or up) extension.

TYPE	LOC
	'r' (or 'd') for right (or down) extension. 'b' for extension on both sides. 'n' null extension. Default: 'b' LEN is the number of columns to add.

Examples

```
% Original signal.
x = [1 2 3]

x =

     1     2     3

% 1-D extension length.
l = 2;

% Zero-padding extensions 1-D.
xextzpd1 = wextend('1','zpd',x,l)
xextzpd1 =

     0     0     1     2     3     0     0

xextzpd2 = wextend('1D','zpd',x,l,'b')

xextzpd2 =

     0     0     1     2     3     0     0

% Symmetric extension 1-D.
xextsym = wextend('1D','sym',x,l)

xextsym =

     2     1     1     2     3     3     2

% Periodic extension 1-D.
xextper = wextend('1D','per',x,l)

xextper =
```

```
     3     3     1     2     3     3     1     2

% Original image.
X = [1 2 3;4 5 6]

X =
     1     2     3
     4     5     6

% 2-D extension length.
l = 2;

% Zero-padding extension 2-D.
Xextzpd = wextend(2,'zpd',X,l)

Xextzpd =
     0     0     0     0     0     0     0
     0     0     0     0     0     0     0
     0     0     1     2     3     0     0
     0     0     4     5     6     0     0
     0     0     0     0     0     0     0
     0     0     0     0     0     0     0

% Symmetric extension 2-D.
Xextsym = wextend('2D','sym',X,l)

Xextsym =
     5     4     4     5     6     6     5
     2     1     1     2     3     3     2
     2     1     1     2     3     3     2
     5     4     4     5     6     6     5
     5     4     4     5     6     6     5
     2     1     1     2     3     3     2
```

References

Strang, G.; T. Nguyen (1996), *Wavelets and filter banks*, Wellesley- Cambridge Press.

Introduced before R2006a

wfbm

Fractional Brownian motion synthesis

Syntax

```
FBM = wfbm(H,L)
FBM = wfbm(H,L,'plot')
FBM = wfbm(H,L,NS,W)
FBM = wfbm(H,L,W,NS)
wfbm(H,L,'plot',NS)
wfbm(H,L,'plot',W)
wfbm(H,L,'plot',NS,W)
wfbm(H,L,'plot',W,NS)
```

Description

`FBM = wfbm(H,L)` returns a fractional Brownian motion signal `FBM` of the Hurst parameter `H` (`0 < H < 1`) and length `L`, following the algorithm proposed by Abry and Sellan.

`FBM = wfbm(H,L,'plot')` generates and plots the `FBM` signal.

`FBM = wfbm(H,L,NS,W)` or `FBM = wfbm(H,L,W,NS)` returns the `FBM` using `NS` reconstruction steps and the sufficiently regular orthogonal wavelet `W`.

`wfbm(H,L,'plot',NS)` or `wfbm(H,L,'plot',W)` or `wfbm(H,L,'plot',NS,W)` or `wfbm(H,L,'plot',W,NS)` generates and plots the `FBM` signal.

`wfbm(H,L)` is equivalent to `WFBM(H,L,6,'db10')`.

`wfbm(H,L,NS)` is equivalent to `WFBM(H,L,NS,'db10')`.

`wfbm(H,L,W)` is equivalent to `WFBM(H,L,W,6)`.

A fractional Brownian motion (`fBm`) is a continuous-time Gaussian process depending on the Hurst parameter `0 < H < 1`. It generalizes the ordinary Brownian motion

corresponding to `H = 0.5` and whose derivative is the white noise. The `fBm` is self-similar in distribution and the variance of the increments is given by

```
Var(fBm(t)-fBm(s)) = v |t-s|^(2H)
```

where `v` is a positive constant.

Examples

According to the value of `H`, the `fBm` exhibits for `H > 0.5`, long-range dependence and for `H < 0.5`, short or intermediate dependence. This example shows each situation using the `wfbm` file, which generates a sample path of this process.

```
% Generate fBm for H = 0.3 and H = 0.7

% Set the parameter H and the sample length
H = 0.3; lg = 1000;
% Generate and plot wavelet-based fBm for H = 0.3
fBm03 = wfbm(H,lg,'plot');

H = 0.7;
% Generate and plot wavelet-based fBm for H = 0.7
fBm07 = wfbm(H,lg,'plot');

% The last step is equivalent to
% Define wavelet and level of decomposition
% w = ' db10'; ns = 6;
% Generate
% fBm07 = wfbm(H,lg,'plot',w,ns);
```

`fBm07` clearly exhibits a stronger low-frequency component and has, locally, less irregular behavior.

More About

Algorithms

Starting from the expression of the `fBm` process as a fractional integral of the white noise process, the idea of the algorithm is to build a biorthogonal wavelet depending on a given orthogonal one and adapted to the parameter `H`.

Then the generated sample path is obtained by the reconstruction using the new wavelet starting from a wavelet decomposition at a given level designed as follows: details coefficients are independent random Gaussian realizations and approximation coefficients come from a fractional ARIMA process.

This method was first proposed by Meyer and Sellan and implementation issues were examined by Abry and Sellan.

Nevertheless, the samples generated following this original scheme exhibit too many high-frequency components. To circumvent this undesirable behavior Bardet et al. propose downsampling the obtained sample by a factor 10.

Two internal parameters `delta = 10` (the downsampling factor) and a threshold `prec = 1E-4`, to evaluate series by truncated sums, can be modified by the user for extreme values of `H`.

A complete overview of long-range dependence process generators is available in Bardet et al.

References

Abry, P.; F. Sellan (1996), "The wavelet-based synthesis for the fractional Brownian motion proposed by F. Sellan and Y. Meyer: Remarks and fast implementation," *Appl. and Comp. Harmonic Anal.*, 3(4), pp. 377–383.

Bardet, J.-M.; G. Lang, G. Oppenheim, A. Philippe, S. Stoev, M.S. Taqqu (2003), "Generators of long-range dependence processes: a survey," Theory and applications of long-range dependence, Birkhäuser, pp. 579–623.

See Also

`wfbmesti`

Introduced before R2006a

wfbmesti

Parameter estimation of fractional Brownian motion

Syntax

```
HEST = wfbmesti(X)
```

Description

`HEST = wfbmesti(X)` returns a one-by-three vector `HEST` which contains three estimates of the fractal index `H` of the input signal `X`. The signal `X` is assumed to be a realization of fractional Brownian motion with Hurst index `H`.

The first two elements of the vector are estimates based on the second derivative with the second computed in the wavelet domain.

The third estimate is based on the linear regression in loglog plot, of the variance of detail versus level.

A fractional Brownian motion (`fBm`) is a continuous-time Gaussian process depending on the so-called Hurst parameter `0 < H < 1`. It generalizes the ordinary Brownian motion corresponding to `H = 0.5` and whose derivative is the white noise. The `fBm` is self-similar in distribution and the variance of the increments is

```
Var(fBm(t)-fBm(s)) = v |t-s|^(2H)
```

where `v` is a positive constant.

This special form of the variance of the increments suggests various ways to estimate the parameter `H`. One can find in Bardet et al. a survey of such methods. The `wfbmesti` file provides three different estimates. The first one, due to Istas and Lang, is based on the discrete second-order derivative. The second one is a wavelet-based adaptation and has similar properties. The third one, proposed by Flandrin, estimates `H` using the slope of the loglog plot of the detail variance versus the level. A more recent extension can be found in Abry et al.

Examples

Hurst Parameter Estimation

This example shows how to estimate the Hurst index of a fractional Brownian motion. The example simulates 1,000 realizations of fractional Brownian motion with H=0.6. Each realization consists of 10,000 samples. At the end of the simulation, the three estimates of the Hurst index are compared.

Initialize the random number generator for repeatable results. Set the Hurst index equal to 0.6 and the length of the realizations to be 10,000.

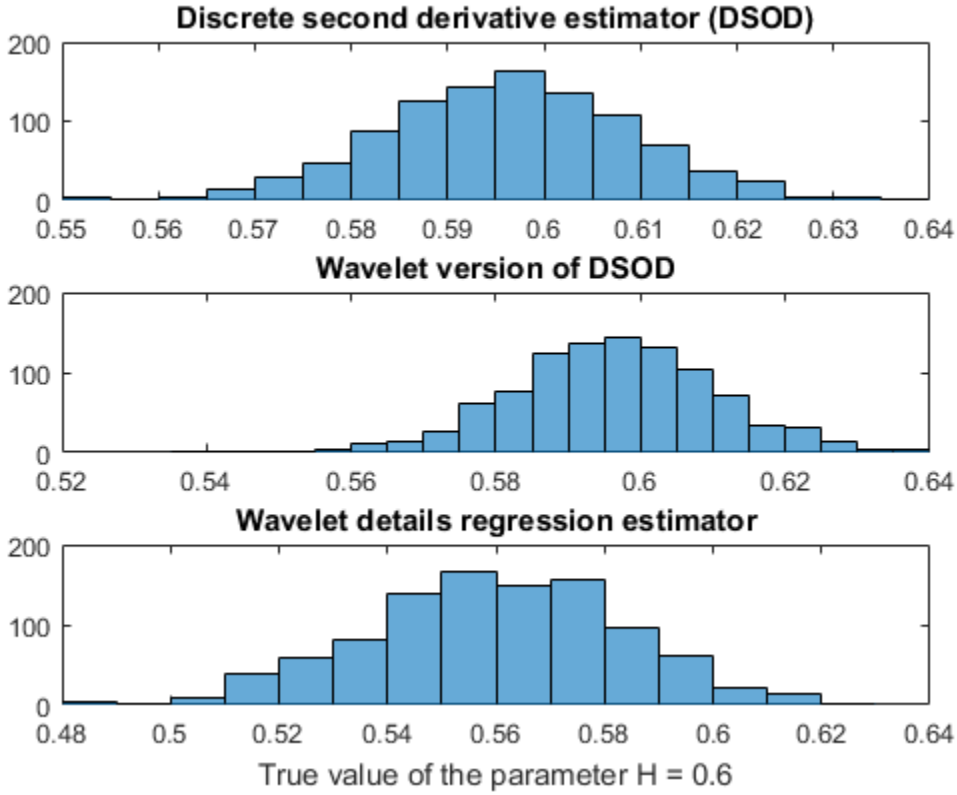
```
rng default;
H = 0.6;
len = 10000;
```

Generate 1,000 realizations of fractional Brownian motion and compute the estimates of the Hurst parameter.

```
n = 1000;
Hest = zeros(n,3);
for ii = 1:n
 fBm06 = wfbm(H,len);
 Hest(ii,:) = wfbmesti(fBm06);
end
```

Compare the estimates.

```
subplot(311), histogram(Hest(:,1));
title('Discrete second derivative estimator (DSOD)')
subplot(312), histogram(Hest(:,2));
title('Wavelet version of DSOD')
subplot(313), histogram(Hest(:,3));
title('Wavelet details regression estimator')
xlabel('True value of the parameter H = 0.6')
```



References

Abry, P.; P. Flandrin, M.S. Taqqu, D. Veitch (2003), "Self-similarity and long-range dependence through the wavelet lens," Theory and applications of long-range dependence, Birkhäuser, pp. 527–556.

Bardet, J.-M.; G. Lang, G. Oppenheim, A. Philippe, S. Stoev, M.S. Taqqu (2003), "Semi-parametric estimation of the long-range dependence parameter: a survey," Theory and applications of long-range dependence, Birkhäuser, pp. 557–577.

Flandrin, P. (1992), "Wavelet analysis and synthesis of fractional Brownian motion," *IEEE Trans. on Inf. Th.*, 38, pp. 910–917.

Istas, J.; G. Lang (1994), "Quadratic variations and estimation of the local Hölder index of a Gaussian process," *Ann. Inst. Poincaré*, 33, pp. 407–436.

See Also

`wfbm`

Introduced before R2006a

wfilters

Wavelet filters

Syntax

```
[Lo_D,Hi_D,Lo_R,Hi_R] = wfilters('wname')
[F1,F2] = wfilters('wname','type')
```

Description

`[Lo_D,Hi_D,Lo_R,Hi_R] = wfilters('wname')` computes four filters associated with the orthogonal or biorthogonal wavelet named in the string `'wname'`.

The four output filters are

- `Lo_D`, the decomposition low-pass filter
- `Hi_D`, the decomposition high-pass filter
- `Lo_R`, the reconstruction low-pass filter
- `Hi_R`, the reconstruction high-pass filter

Available orthogonal or biorthogonal wavelet names `'wname'` are listed in the table below.

Wavelet Families	Wavelets
Daubechies	`'db1'` or `'haar'`, `'db2'`, ... ,`'db10'`, ... , `'db45'`
Coiflets	`'coif1'`, ... , `'coif5'`
Symlets	`'sym2'`, ... , `'sym8'`, ... ,`'sym45'`
Fejer-Korovkin filters	`'fk4'`, `'fk6'`, `'fk8'`, `'fk14'`, `'fk22'`
Discrete Meyer	`'dmey'`
Biorthogonal	`'bior1.1'`, `'bior1.3'`, `'bior1.5'` `'bior2.2'`, `'bior2.4'`, `'bior2.6'`, `'bior2.8'`

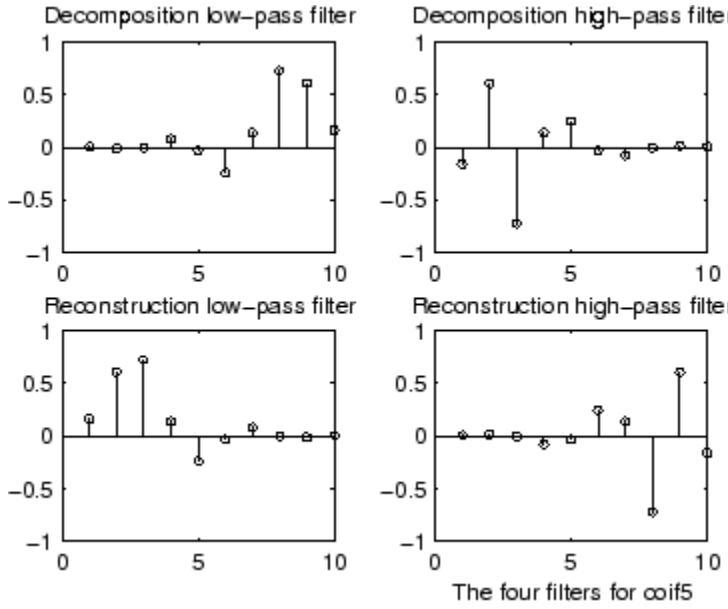
Wavelet Families	Wavelets
	'bior3.1', 'bior3.3', 'bior3.5', 'bior3.7' 'bior3.9', 'bior4.4', 'bior5.5', 'bior6.8'
Reverse Biorthogonal	'rbio1.1', 'rbio1.3', 'rbio1.5' 'rbio2.2', 'rbio2.4', 'rbio2.6', 'rbio2.8' 'rbio3.1', 'rbio3.3', 'rbio3.5', 'rbio3.7' 'rbio3.9', 'rbio4.4', 'rbio5.5', 'rbio6.8'

`[F1,F2] = wfilters('wname','type')` returns the following filters:

`Lo_D` and `Hi_D`	(Decomposition filters)	If `'type' = 'd'`
`Lo_R` and `Hi_R`	(Reconstruction filters)	If `'type' = 'r'`
`Lo_D` and `Lo_R`	(Low-pass filters)	If `'type' = 'l'`
`Hi_D` and `Hi_R`	(High-pass filters)	If `'type' = 'h'`

Examples

```
% Set wavelet name.
wname = 'db5';

% Compute the four filters associated with wavelet name given
% by the input string wname.
[Lo_D,Hi_D,Lo_R,Hi_R] = wfilters(wname);
subplot(221); stem(Lo_D);
title('Decomposition low-pass filter');
subplot(222); stem(Hi_D);
title('Decomposition high-pass filter');
subplot(223); stem(Lo_R);
title('Reconstruction low-pass filter');
subplot(224); stem(Hi_R);
title('Reconstruction high-pass filter');
xlabel('The four filters for db5')

% Editing some graphical properties,
% the following figure is generated.
```



References

Daubechies, I. (1992), *Ten lectures on wavelets*, CBMS-NSF conference series in applied mathematics. SIAM Ed.

Mallat, S. (1989), "A theory for multiresolution signal decomposition: the wavelet representation," *IEEE Pattern Anal. and Machine Intell.*, vol. 11, no. 7, pp. 674–693.

See Also

`biorfilt` | `orthfilt` | `waveinfo`

Introduced before R2006a

wfusimg

Fusion of two images

Syntax

```
XFUS = wfusimg(X1,X2,WNAME,LEVEL,AFUSMETH,DFUSMETH)
[XFUS,TXFUS,TX1,TX2] = wfusimg(X1,X2,WNAME,LEVEL,AFUSMETH,DFUSMETH)
wfusimg(X1,X2,WNAME,LEVEL,AFUSMETH,DFUSMETH,FLAGPLOT)
```

Description

The principle of image fusion using wavelets is to merge the wavelet decompositions of the two original images using fusion methods applied to approximations coefficients and details coefficients (see Zeeuw and Misiti et al.).

`XFUS = wfusimg(X1,X2,WNAME,LEVEL,AFUSMETH,DFUSMETH)` returns the fused image `XFUS` obtained by fusion of the two original images `X1` and `X2`. Each fusion method, defined by `AFUSMETH` and `DFUSMETH`, merges in a specific way detailed below, the decompositions of `X1` and `X2`, at level `LEVEL` and using wavelet `WNAME`.

`AFUSMETH` and `DFUSMETH` define the fusion method for approximations and details, respectively.

`[XFUS,TXFUS,TX1,TX2] = wfusimg(X1,X2,WNAME,LEVEL,AFUSMETH,DFUSMETH)` returns, in addition to matrix `XFUS`, three objects of the class WDECTREE associated with `XFUS`, `X1`, and `X2` respectively (see @WDECTREE). `wfusimg(X1,X2,WNAME,LEVEL,AFUSMETH,DFUSMETH,FLAGPLOT)` also plots the objects `TXFUS`, `TX1`, and `TX2`.

`Fusmeth` denotes `AFUSMETH` or `DFUSMETH`. Available fusion methods are

- Simple — `Fusmeth` can be `'max'`, `'min'`, `'mean'`, `'img1'`, `'img2'` or `'rand'`, which merges the two approximations or details structures obtained from `X1` and `X2` elementwise by taking the maximum, the minimum, the mean, the first element, the second element, or a randomly chosen element
- Parameter-dependent — `Fusmeth` is of the following form

  ```
  Fusmeth = struct('name',nameMETH,'param',paramMETH)
  ```

where nameMETH can be

'linear'	
'UD_fusion'	Up-down fusion
'DU_fusion'	Down-up fusion
'RL_fusion'	Right-left fusion
'UserDEF'	User-defined fusion

For the description of these options and the paramMETH parameter, see wfusmat.

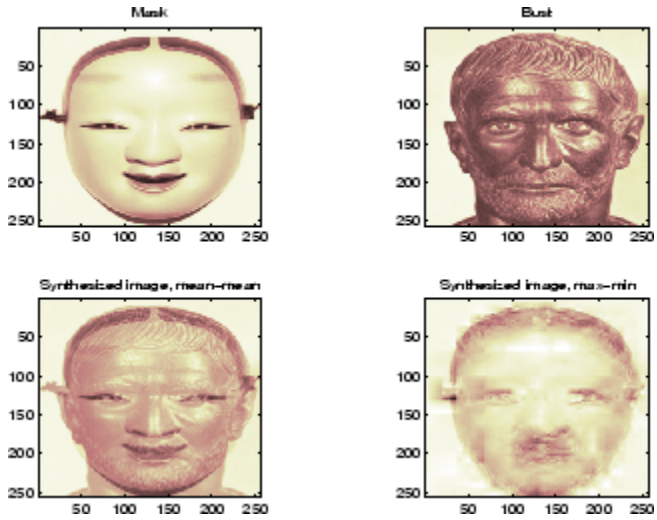
Examples

The following three examples examine the process of image fusion

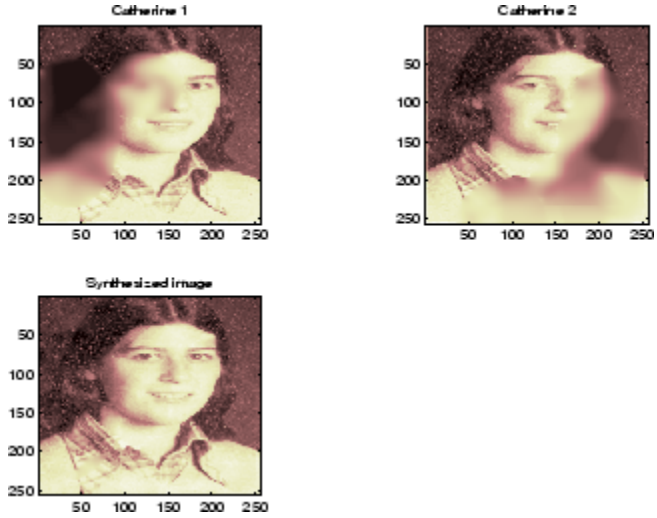
- The first example merges two different images leading to a new image
- The second example restores an image from two fuzzy versions of an original image.
- The third example shows how to make an image fusion using a user defined fusion method.

```
% Example 1: Fusion of two different images

% Load two original images: a mask and a bust
load mask; X1 = X;
load bust; X2 = X;

% Merge the two images from wavelet decompositions at level 5
% using db2 by taking two different fusion methods

% fusion by taking the mean for both approximations and details
XFUSmean = wfusimg(X1,X2,'db2',5,'mean','mean');

% fusion by taking the maximum for approximations and the
% minimum for the details
XFUSmaxmin = wfusimg(X1,X2,'db2',5,'max','min');

% Plot original and synthesized images
colormap(map);
subplot(221), image(X1), axis square, title('Mask')
subplot(222), image(X2), axis square, title('Bust')
subplot(223), image(XFUSmean), axis square,
```

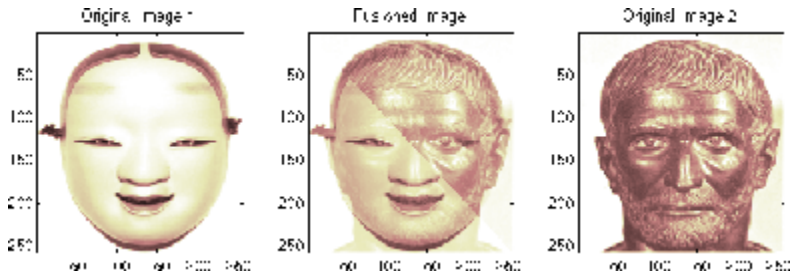
```
title('Synthesized image, mean-mean')
subplot(224), image(XFUSmaxmin), axis square,
title('Synthesized image, max-min')
```




```
% Example 2: Restoration by fusion of fuzzy images

% Load two fuzzy versions of an original image
load cathe_1; X1 = X;
load cathe_2; X2 = X;

% Merge the two images from wavelet decompositions at level 5
% using sym4 by taking the maximum of absolute value of the
% coefficients for both approximations and details
XFUS = wfusimg(X1,X2,'sym4',5,'max','max');

% Plot original and synthesized images
colormap(map);
subplot(221), image(X1), axis square,
title('Catherine 1')
subplot(222), image(X2), axis square,
title('Catherine 2')
subplot(223), image(XFUS), axis square,
title('Synthesized image')
```




```
% The synthesized image is a restored version of good
% quality of the common underlying original image.

% Example 3: Fusion using a user defined fusion method.
% This example calls a user fusion method defined by the
% file myfus_FUN.m which is listed below at the end of
% the example.

% load two images of the same size.
load mask; A = X;
load bust; B = X;

% Define the fusion method and call the fusion function
Fus_Method = struct('name','userDEF','param','myfus_FUN');
C = wfusmat(A,B,Fus_Method);

figure;
colormap(pink(220))
subplot(1,3,1), image(A), title('Original Image 1'), axis square
subplot(1,3,2), image(C), title('Fusioned Image'), axis square
subplot(1,3,3), image(B), title('Original Image 2'), axis square
```



```
%*******************************
% User defined fusion method.  *
%*******************************
 function C = myfus_FUN(A,B)

D = logical(triu(ones(size(A))));  t = 0.3;
C = A;
C(D)  = t*A(D)+(1-t)*B(D);
C(~D) = t*B(~D)+(1-t)*A(~D);
```

More About

Tips

X1 and X2 must be of same size (see `wextend` to resize images) and represent indexed images or truecolor images, which are `m`-by-`n` matrices or `m`-by-`n`-by-3 arrays, respectively.

For more information on image formats, see the `image` and `imfinfo` reference pages.

References

Zeeuw, P.M. (1998), "Wavelet and image fusion," CWI, Amsterdam, March 1998, `http://www.cwi.nl/~pauldz/`

Misiti, M.; Y. Misiti, G. Oppenheim, J.-M. Poggi (2003), "Les ondelettes et leurs applications," Hermes.

See Also

`wfusmat` | `wextend`

Introduced before R2006a

wfusmat

Fusion of two matrices or arrayz

Syntax

```
C = wfusmat(A,B,METHOD)
```

Description

`C = wfusmat(A,B,METHOD)` returns the fused matrix `C` obtained from the matrices `A` and `B` using the fusion method defined by `METHOD`.

The matrices `A` and `B` must be of the same size. The output matrix `C` is of the same size as `A` and `B`.

Available fusion methods are

- Simple, where `METHOD` is
  - `'max'` : `D = abs(A) ≥ abs(B) ; C = A(D) + B(~D)`
  - `'min'` : `D = abs(A) ≤ abs(B) ; C = A(D) + B(~D)`
  - `'mean'` : `C = (A+B) / 2 ; D = ones(size(A))`
  - `'rand'` : C = A(D) + B(~D); D is a Boolean random matrix
  - `'img1'` : C = A
  - `'img2'` : C = B
- Parameter-dependent, where METHOD is of the following form:

  ```
  METHOD = struct('name',nameMETH,'param',paramMETH)
  ```

  where `nameMETH` can be
  - `'linear'` : `C = A*paramMETH + B*(1-paramMETH)`,

    where `0 £ paramMETH ≤ 1`
  - `'UD_fusion'`: Up-down fusion, with paramMETH ≥ 0

```
x = linspace(0,1,size(A,1));
P = x.^paramMETH;
```

Then each row of C is computed with

```
C(i,:) = A(i,:)*(1-P(i)) + B(i,:)*P(i);
So C(1,:) = A(1,:) and C(end,:) = A(end,:)
```

- `'DU_fusion'`: Down-up fusion
- `'LR_fusion'`: Left-right fusion (columnwise fusion)
- `'RL_fusion'`: Right-left fusion (columnwise fusion)
- `'UserDEF'`: User-defined fusion, `paramMETH` is a string `'userFUNCTION'` containing a function name such that `C = userFUNCTION(A,B)`.

In addition, `[C,D] = wfusmat(A,B,METHOD)` returns the Boolean matrix `D` when defined, or an empty matrix otherwise.

Introduced before R2006a

wkeep

Keep part of vector or matrix

Syntax

```
Y = wkeep(X,L,OPT)
Y = wkeep(X,L,FIRST)
Y = wkeep(X,L)
Y = wkeep(X,L,'c')
Y = wkeep(X,S,[FIRSTR FIRSTC])
```

Description

`wkeep` is a general utility.

For a vector, `Y = wkeep(X,L,OPT)` extracts the vector `Y` from the vector `X`. The length of `Y` is `L`.

If `OPT` is equal to `'c'`, `'l'`, or `'r'`, `Y` is the central, left, or right part of `X`.

`Y = wkeep(X,L,FIRST)` returns the vector `X(FIRST:FIRST+L-1)`.

`Y = wkeep(X,L)` is equivalent to `Y = wkeep(X,L,'c')`.

For a matrix, `Y = wkeep(X,S)` extracts the central part of the matrix `X`. The size of `Y` is `S`.

`Y = wkeep(X,S,[FIRSTR FIRSTC])` extracts the submatrix of matrix `X`, of size `S` and starting from `X(FIRSTR,FIRSTC)`.

Examples

```
% For a vector.
x = 1:10;
y = wkeep(x,6,'c')
y =
```

```
    3     4     5     6     7     8

y = wkeep(x,6)
y =
    3     4     5     6     7     8

y = wkeep(x,7,'c')
y =
    2     3     4     5     6     7     8
y = wkeep(x,6,'l')
y =
    1     2     3     4     5     6

y = wkeep(x,6,'r')
y =
    5     6     7     8     9     10

% For a matrix.
x = magic(5)
x =
    17     24      1      8     15
    23      5      7     14     16
     4      6     13     20     22
    10     12     19     21      3
    11     18     25      2      9

y = wkeep(x,[3 2])
y =
    5     7
    6    13
   12    19
```

Introduced before R2006a

wmaxlev

Maximum wavelet decomposition level

Syntax

```
L = wmaxlev(S,'wname')
```

Description

`wmaxlev` is a one- or two-dimensional wavelet or wavelet packets oriented function.

`wmaxlev` can help you avoid unreasonable maximum level values. `L = wmaxlev(S,'wname')` returns the maximum level decomposition of signal or image of size `S` using the wavelet named in the string `'wname'` (see `wfilters` for more information).

`wmaxlev` gives the maximum allowed level decomposition, but in general, a smaller value is taken.

Usual values are 5 for the one-dimensional case, and 3 for the two-dimensional case.

Examples

```
% For a 1-D signal.
s = 2^10;
w = 'db1';

% Compute maximum level decomposition.
% The rule is the last level for which at least
% one coefficient is correct.
l = wmaxlev(s,w)

l =
    10

% Change wavelet.
w = 'db7';
```

```
% Compute maximum level decomposition.
l = wmaxlev(s,w)

l =
    6

% For a 2-D signal.
s = [2^9 2^7];
w = 'db1';

% Compute maximum level decomposition.
l = wmaxlev(s,w)

l =
    7

% which is the same as:
l = wmaxlev(min(s),w)

l =
    7

% Change wavelet.
w = 'db7';

% Compute maximum level decomposition.
l = wmaxlev(s,w)

l =
    3
```

See Also

wavedec | wpdec | wavedec2 | wpdec2

Introduced before R2006a

wmpalg

Matching pursuit

Syntax

```
YFIT = wmpalg(MPALG,Y,MPDICT)
[YFIT,R] = wmpalg(...)
[YFIT,R,COEFF] = wmpalg(...)
[YFIT,R,COEFF,IOPT] = wmpalg(...)
[YFIT,R,COEFF,IOPT,QUAL] = wmpalg(...)
[YFIT,R,COEFF,IOPT,QUAL,X] = wmpalg(...)
[YFIT,R,COEFF,IOPT,QUAL,X] = wmpalg(...,Name,Value)
```

Description

`YFIT = wmpalg(MPALG,Y,MPDICT)` returns an adaptive greedy approximation, `YFIT`, of the input signal, `Y`, in the dictionary, `MPDICT`. The adaptive greedy approximation uses the matching pursuit algorithm, `MPALG`. The dictionary, `MPDICT`, is typically an overcomplete set of vectors constructed using `wmpdictionary`.

`[YFIT,R] = wmpalg(...)` returns the residual, `R`, which is the difference vector between `Y` and `YFIT` at the termination of the matching pursuit.

`[YFIT,R,COEFF] = wmpalg(...)` returns the expansion coefficients, `COEFF`. The number of expansion coefficients depends on the number of iterations in the matching pursuit.

`[YFIT,R,COEFF,IOPT] = wmpalg(...)` returns the column indices of the retained atoms, `IOPT`. The length of `IOPT` equals the length of `COEFF` and is determined by the number of iterations in the matching pursuit.

`[YFIT,R,COEFF,IOPT,QUAL] = wmpalg(...)` returns the proportion of retained signal energy, `QUAL`, for each iteration of the matching pursuit. `QUAL` is the ratio of the ℓ^2 squared norm of the expansion coefficient vector, `COEFF`, to the ℓ^2 squared norm of the input signal, `Y`.

`[YFIT,R,COEFF,IOPT,QUAL,X] = wmpalg(...)` returns the normalized dictionary, `X`. `X` contains the unit vectors in the ℓ^2 norm corresponding to the columns of `MPDICT`.

`[YFIT,R,COEFF,IOPT,QUAL,X] = wmpalg(...,Name,Value)` returns an adaptive greedy approximation with additional options specified by one or more `Name,Value` pair arguments.

Input Arguments

`MPALG`

Matching pursuit algorithm as a string. Valid entries are:

- `'BMP'` — Basic matching pursuit
- `'OMP'` — Orthogonal matching pursuit
- `'WMP'` — Weak orthogonal matching pursuit

See “Matching Pursuit Algorithms”.

Default: `'BMP'`

`MPDICT`

Matching pursuit dictionary. `MPDICT` is a N-by-P matrix where N is equal to the length of the input signal, `Y`. You can construct `MPDICT` using `wmpdictionary`. In matching pursuit, `MPDICT` is commonly a frame, or overcomplete set of vectors. You may use the Name-Value pair `'lstcpt'` to specify a dictionary instead of using `MPDICT`. If you specify a value for `'lstcpt'`, `wmpalg` calls `wmpdictionary`.

`Y`

Signal for matching pursuit. `Y` is 1-D, real-valued row or column vector. The row dimension of `MPDICT` must match the length of `Y`.

Name-Value Pair Arguments

Specify optional comma-separated pairs of `Name,Value` arguments. `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside single quotes (`' '`). You can specify several name and value pair arguments in any order as `Name1,Value1,...,NameN,ValueN`.

`'itermax'`

Positive integer fixing the maximum number of iterations of the matching pursuit algorithm. If you do not specify a `'maxerr'` value, the number of expansion coefficients, `COEFF`, the number of dictionary vector indices, `IOPT`, and the length of the `QUAL` vector equal the value of `'itermax'`.

Default: 25

`'lstcpt'`

A cell array of cell arrays with valid subdictionaries. This name-value pair is only valid if you do not input a dictionary in `MPDICT`. Each cell array describes one subdictionary. Valid subdictionaries are:

- A valid Wavelet Toolbox orthogonal or biorthogonal wavelet family short name with the number of vanishing moments and an optional decomposition level and extension mode. For example, `{'sym4',5}` denotes the Daubechies least-asymmetric wavelet with 4 vanishing moments at level 5 and the default extension mode `'per'`. If you do not specify the optional number level and extension mode, the decomposition level defaults to 5 and the extension mode to `'per'`.
- A valid Wavelet Toolbox orthogonal or biorthogonal wavelet family short name preceded by `wp` with the number of vanishing moments and an optional decomposition level and extension mode. For example, `{'wpsym4',5}` denotes the Daubechies least-asymmetric wavelet packet with 4 vanishing moments at level 5. If you do not specify the optional number level and extension mode, the decomposition level defaults to 5 and the extension mode to `'per'`.
- `'dct'` Discrete cosine transform-II basis. The DCT-II orthonormal basis is:

$$\phi_k(n) = \begin{cases} \dfrac{1}{\sqrt{N}} & k = 0 \\ \sqrt{\dfrac{2}{N}} \cos\left(\frac{\pi}{N}\left(n+\frac{1}{2}\right)k\right) & k = 1,2,\ldots,N-1 \end{cases}$$

- `'sin'` Sine subdictionary. The sine subdictionary is:

$$\phi_k(t) = \sin(2\pi k t) \quad k = 1,2,\ldots\left\lceil \frac{N}{2} \right\rceil \quad 0 \le t \le 1$$

- `'cos'` Cosine subdictionary. The cosine subdictionary is

$$\phi_k(t) = \cos(2\pi kt) \quad k = 1,2,\ldots\left\lceil \frac{N}{2} \right\rceil \quad 0 \le t \le 1$$

- `'poly'` Polynomial subdictionary. The polynomial subdictionary is:

$$p_n(t) = t^{n-1} \quad n = 1,2,\ldots 20 \qquad 0 \le t \le 1$$

- `'RnIdent'` The shifted Kronecker delta subdictionary. The shifted Kronecker delta subdictionary is:

$$\phi_k(n) = \delta(n-k) \quad k = 0,1,\ldots N$$

If you use the `'lstcpt'` name-value pair to generate your dictionary, you can use the additional `'addbeg'` and `'addend'` name-value pairs to append and addend dictionary atoms. See `wmpdictionary` for details.

`'maxerr'`

Cell array containing the name of the norm and the maximum relative error in the norm expressed as a percentage. Valid norms are `'L1'`, `'L2'`, and `'Linf'`. The relative error expressed as a percentage is

$$100\frac{\|R\|}{\|Y\|}$$

where R is the residual at each iteration and Y is the input signal. For example, `{'L1',10}` sets maximum acceptable ratio of the L1 norms of the residual to the input signal to 0.10.

If you specify `'maxerr'`, the matching pursuit terminates when the first of the following conditions is satisfied:

- The number of iterations reaches the minimum of the length of the input signal, `Y`, or 500:
  `min(length(Y),500)`
- The relative error falls below the percentage you specify with the `'maxerr'` name-value pair.

`'stepplot'`

Number of iterations between successive plots. `'stepplot'` requires a positive integer. This name-value pair is only valid when `'typeplot'` is 2 or 3 (`'movie'` or `'stepwise'`).

`'typeplot'`

Type of plot to produce during the progression of matching pursuit. Valid entries for `'typeplot'` are: `0` or `'none'`, `1` or `'one'`, `2` or `'movie'`, `3` or `'stepwise'`. When `'typeplot'` is `'movie'` or `'stepwise'`, the plot updates based on the value of `'stepplot'`.

Default: `0` or `'none'`

`'wmpcfs'`

Optimality factor for weak orthogonal matching pursuit. The optimality factor is a real number in the interval (0,1]. This name-value pair is only valid when `MPALG` is `'WMP'`.

Default: `0.6`

Output Arguments

`YFIT`

Adaptive greedy approximation of the input signal, `Y`, in the dictionary

`R`

Residual after matching pursuit terminates

`COEFF`

Expansion coefficients in the dictionary. The selected dictionary atoms weighted by the expansion coefficients yield the approximated signal, `YFIT`.

`IOPT`

Column indices of the selected dictionary atoms. Using the column indices in `IOPT` with the expansion coefficients in `COEFF`, you can form the approximated signal, `YFIT`.

QUAL

Proportion of retained signal energy for each iteration in the matching pursuit. QUAL is a vector with each element equal to

$$\frac{||\alpha_k||_2^2}{||Y||_2^2}$$

where α_k is the vector of expansion coefficients after the k-th iteration.

X

The normalized matching pursuit dictionary. X is an N-by-P matrix where N is the length of the input signal, Y. The columns of X have unit norm.

Examples

Adaptive Approximation using Orthogonal Matching Pursuit

Approximate the cuspamax signal with the dictionary using orthogonal matching pursuit.

Use a dictionary consisting of sym4 wavelet packets and the DCT-II basis.

```
load cuspamax;
mpdict = wmpdictionary(length(cuspamax),'LstCpt',...
   {{'wpsym4',2},'dct'});
yfit = wmpalg('OMP',cuspamax,mpdict);
plot(cuspamax,'k'); hold on;
plot(yfit,'linewidth',2); legend('Original Signal',...
    'Matching Pursuit');
```

Return Residual, Expansion Coefficients, Selected Atoms, and Approximation Quality

Obtain the expansion coefficients in the dictionary, the column indices of the selected dictionary atoms, and the proportion of retained signal energy.

Create a dictionary consisting of sym4 wavelet packets and the DCT-II basis. Approximate the cuspamax signal with the dictionary using orthogonal matching pursuit.

```
load cuspamax;
```

```
mpdict = wmpdictionary(length(cuspamax),'LstCpt',...
    {{'wpsym4',2},'dct'});
[yfit,r,coeff,iopt,qual] = wmpalg('OMP',cuspamax,mpdict);
```

Specify the Maximum Number of Iterations

This example shows how to set the maximum number of iterations of the orthogonal matching pursuit to 50.

```
load cuspamax;
lstcpt = {{'wpsym4',1},{'wpsym4',2},'dct'};
mpdict = wmpdictionary(length(cuspamax),'LstCpt',lstcpt);
[yfit,r,coeff,iopt,qual] = wmpalg('OMP',cuspamax,mpdict,...
    'itermax',50);
```

Stepwise Plot of Weak Orthogonal Matching Pursuit

This example shows how to allow for a suboptimal choice in the update of the orthogonal matching pursuit.

Relax the requirement to be 0.8 times the optimal assignment. Plot the results stepwise and update the plot every 5 iterations.

```
load cuspamax;
lstcpt = {{'wpsym4',1},{'wpsym4',2},'dct'};
mpdict = wmpdictionary(length(cuspamax),'LstCpt',lstcpt);
[yfit,r,coeff,iopt,qual] = wmpalg('WMP',cuspamax,mpdict,...
    'wmpcfs',0.8,'typeplot','stepwise','stepplot',5);
```

Matching Pursuit of Electricity Consumption Data

Obtain a matching pursuit of electricity consumption measured every minute over a 24-hour period.

Load and plot data. The data shows electricity consumption sampled every minute over a 24-hour period. Because the data is centered, the actual usage values are not interpretable.

```
load elec35_nor;
y = signals(32,:);
plot(y); xlabel('Minutes'); ylabel('Usage');
set(gca,'xlim',[1 1440]);
```

Construct a dictionary for matching pursuit consisting of the Daubechies' extremal–phase wavelet with 2 vanishing moments at level 2, the Daubechies' least-asymmetric

wavelet with 4 vanishing moments at levels 1 and 4, the discrete cosine transform-II basis, and the sine basis.

```
dictionary = {{'db4',2},'dct','sin',{'sym4',1},{'sym4',4}};
[mpdict,nbvect] = wmpdictionary(length(y),'lstcpt',dictionary);
```

Implement orthogonal matching pursuit to obtain a signal approximation in the dictionary. Use 35 iterations. Plot the result.

```
[yfit,r,coef,iopt,qual] = wmpalg('OMP',y,mpdict,'itermax',35);
plot(y); hold on;
plot(yfit,'r'); xlabel('Minutes'); ylabel('Usage');
legend('Original Signal','OMP','Location','NorthEast');
set(gca,'xlim',[1 1440]);
```

Using the expansion coefficients in `coef` and the atom indices in `iopt`, construct the signal approximation, `yhat`, directly from the dictionary. Compare `yhat` with `yfit` returned by `wmpalg`.

```
[~,I] = sort(iopt);
X = mpdict(:,iopt(I));
yhat = X*coef(I);
max(abs(yfit-yhat))
```

- "Matching Pursuit"
- "Matching Pursuit Using 1–D Interactive Tool"

More About

- "Sparse Representation in Redundant Dictionaries"
- "Matching Pursuit Algorithms"

References

[1] Cai, T.T. and Wang,L. "Orthogonal Matching Pursuit for Sparse Signal Recovery with Noise". *IEEE® Transactions on Information Theory*, vol. 57, 7, 4680–4688, 2011.

[2] Donoho, D., Elad, M., and Temlyakov, V. "Stable Recovery of Sparse Overcomplete Representations in the Presence of Noise". *IEEE Transactions on Information Theory*. Vol. 52, 1, 6–18, 2004.

[3] Mallat, S. and Zhang, Z. “Matching Pursuits with Time-Frequency Dictionaries”. *IEEE Transactions on Signal Processing*, vol. 41, 12, 3397–3415, 1993

[4] bTropp, J.A. “Greed is good: Algorithmic results for sparse approximation”. *IEEE Transactions on Information Theory*, 50, pp. 2231–2242, 2004.

See Also

`wavemenu` | `wmpdictionary`

Introduced in R2012a

wmpdictionary

Dictionary for matching pursuit

Syntax

```
MPDICT = wmpdictionary(N)
[MPDICT,NBVECT] = wmpdictionary(N)
[MPDICT,NBVECT]= wmpdictionary(N,Name,Value)
[MPDICT,NBVECT,LST] = wmpdictionary(N,Name,Value)
[MPDICT,NBVECT,LST,LONGS] = wmpdictionary(N,Name,Value)
```

Description

`MPDICT = wmpdictionary(N)` returns the N-by-P dictionary, `MPDICT`, for the default subdictionaries `{{'sym4',5},{'wpsym4',5},'dct','sin'}`. The column dimension of `MPDICT` depends on `N`.

`[MPDICT,NBVECT] = wmpdictionary(N)` returns the row vector, `NBVECT`, which contains the number of vectors in each subdictionary. The order of the elements in `NBVECT` corresponds to the order of the subdictionaries and any prepended or appended subdictionaries. The sum of the elements in `NBVECT` is the column dimension of `MPDICT`.

`[MPDICT,NBVECT]= wmpdictionary(N,Name,Value)` returns the dictionary, `MPDICT`, using additional options specified by one or more `Name,Value` pair arguments.

`[MPDICT,NBVECT,LST] = wmpdictionary(N,Name,Value)` returns the cell array, `LST`, with descriptions of the subdictionaries.

`[MPDICT,NBVECT,LST,LONGS] = wmpdictionary(N,Name,Value)` returns the cell array, `LONGS`, containing the number of vectors in each subdictionary. `LONGS` is only useful for wavelet subdictionaries. In wavelet subdictionaries, the corresponding element in `LONGS` gives the number of scaling functions at the coarsest level and wavelet functions by level. See “Visualize Haar Wavelet Dictionary” on page 1-698 for an example using `LONGS`.

Input Arguments

`N`

A positive integer equal to the length of your input signal. The dictionary atoms are constructed to have `N` elements. `N` equals the row dimension of the dictionary, `MPDICT`.

Name-Value Pair Arguments

Specify optional comma-separated pairs of `Name,Value` arguments. `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside single quotes (`' '`). You can specify several name and value pair arguments in any order as `Name1,Value1,...,NameN,ValueN`.

`'addbeg'`

Prepended subdictionary. The prepended subdictionary is an N-by-M matrix where N is the length of the input signal. `wmpdictionary` does not check that the M column vectors of the prepended dictionary form a basis. If you do not specify a value for `lstcpt`, the subdictionary is prepended to the default dictionary. The column vectors in the prepended subdictionary do not have to be unit-norm.

`'addend'`

Appended subdictionary. The appended subdictionary is a N-by-M matrix where N is the length of the input signal. `wmpdictionary` does not check that the M column vectors of the prepended dictionary form a basis. If you do not specify a value for `lstcpt`, the subdictionary is appended to the default dictionary. The column vectors in the appended subdictionary do not have to be unit-norm.

`'lstcpt'`

A cell array of cell arrays with valid subdictionaries. Each cell array describes one subdictionary. Valid subdictionaries are:

- A valid Wavelet Toolbox orthogonal or biorthogonal wavelet family short name with the number of vanishing moments and an optional decomposition level and extension mode. For example, `{'sym4',5}` denotes the Daubechies least-asymmetric wavelet with 4 vanishing moments at level 5 and the default extension mode `'per'`. If you do

not specify the optional level and extension mode, the decomposition level defaults to 5 and the extension mode to `'per'`.

- A valid Wavelet Toolbox orthogonal or biorthogonal wavelet family short name preceded by `wp` with the number of vanishing moments and an optional decomposition level and extension mode. For example, `{'wpsym4',5}` denotes the Daubechies least-asymmetric wavelet packet with 4 vanishing moments at level 5. If you do not specify the optional level and extension mode, the decomposition level defaults to 5 and the extension mode to `'per'`.
- `'dct'` Discrete cosine transform-II basis. The DCT-II orthonormal basis is:

$$\phi_k(n) = \begin{cases} \frac{1}{\sqrt{N}} & k = 0 \\ \sqrt{\frac{2}{N}} \cos(\frac{\pi}{N}(n+\frac{1}{2})k) & k = 1,2,\ldots,N-1 \end{cases}$$

- `'sin'` Sine subdictionary. The sine subdictionary is

$$\phi_k(t) = \sin(2\pi kt) \quad k = 1,2,\ldots\left\lceil \frac{N}{2} \right\rceil \quad 0 \le t \le 1$$

where t is a linearly-spaced N-point vector.

- `'cos'` Cosine subdictionary. The cosine subdictionary is

$$\phi_k(t) = \cos(2\pi kt) \quad k = 1,2,\ldots\left\lceil \frac{N}{2} \right\rceil \quad 0 \le t \le 1$$

where t is a linearly-spaced N-point vector.

- `'poly'` Polynomial subdictionary. The polynomial subdictionary is:

$$p_n(t) = t^{n-1} \quad n = 1,2,\ldots 20 \qquad 0 \le t \le 1$$

where t is a linearly-spaced N-point vector.

- `'RnIdent'` The shifted Kronecker delta subdictionary. The shifted Kronecker delta subdictionary is:

$$\phi_k(n) = \delta(n-k) \quad k = 0,1,\ldots N$$

Default: `{{'sym4',5},{'wpsym4',5},'dct','sin'}`

Output Arguments

MPDICT

Matching pursuit dictionary. `MPDICT` is an N-by-P matrix with the row dimension, `N`, equal to the length of the input signal. The column dimension of the matrix depends on the size of the concatenated subdictionaries.

NBVECT

Number of vectors in subdictionaries. `NBVECT` is a row vector containing the number of elements in each subdictionary. The order of the elements in `NBVECT` corresponds to the order of the subdictionaries and any prepended or appended subdictionaries.

LST

Cell array describing the dictionary. `LST` is a 1-by-N cell array where N is the number of subdictionaries. Each element of the cell array contains a description of a subdictionary. If you specify a prepended or appended subdictionary, the first element of `LST` is `'AddBeg'` or `'AddEnd'`. If you specify a level for the wavelet or wavelet packet, the corresponding element of `LST` is a 1-by-2 cell array containing the wavelet or wavelet packet name in the first element and the level in the second element.

LONGS

Cell array containing the number of elements for each subdictionary. `LONGS` is useful only for wavelet subdictionaries. If you specify a wavelet subdictionary, the corresponding element of `LONGS` provides the number of scaling functions at the coarsest level and the number of wavelets at each level. See “Visualize Haar Wavelet Dictionary” on page 1-698 for an example using `LONGS`.

Examples

Default Dictionary

Create the default dictionary to represent a signal of length 100.

```
mpdict = wmpdictionary(100);
```

Discrete Cosine Transform and Kronecker Delta Dictionary

Create a DCT and shifted Kronecker delta dictionary to represent a signal of length 100.

```
mpdict = wmpdictionary(100,'lstcpt',{'dct','RnIdent'});
```

Haar Wavelet Packets and Discrete Cosine Transform Dictionary

Create a Haar wavelet packet (level 2) and DCT dictionary. Return the number of atoms in each subdictionary.

```
[mpdict,nbvect] = wmpdictionary(100,'lstcpt',{{'wphaar',2},'dct'});
```

Visualize Haar Wavelet Dictionary

Use the output argument, LONGS, to visualize a dictionary.

Create a Haar wavelet dictionary consisting of level-2 scaling functions and level-1 and level-2 wavelet functions.

```
[mpdict,~,~,longs] = wmpdictionary(100,'lstcpt',{{'haar',2}});
for nn = 1:size(mpdict,2)

    if (nn <= longs{1}(1))
        plot(mpdict(:,nn),'k','linewidth',2); grid on;
    xlabel('Translation');
        title('Haar Scaling Function - Level 2');
    elseif (nn>longs{1}(1) & nn<= longs{1}(1)+longs{1}(2))
         plot(mpdict(:,nn),'r','linewidth',2); grid on;
         xlabel('Translation');
        title('Haar Wavelet - Level 2');
    else
        title('Haar Wavelet - Level 1');
        plot(mpdict(:,nn),'b','linewidth',2); grid on;
        title('Haar Wavelet - Level 1');
        xlabel('Translation');
    end
        pause(0.2);
end
```

- “Matching Pursuit”
- “Matching Pursuit Using 1–D Interactive Tool”

More About

Matching Pursuit

Matching pursuit refers to a number of greedy or weak-greedy algorithms for computing an adaptive nonlinear expansion of a signal in a *dictionary*. In the majority of matching pursuit applications, a dictionary is an overcomplete set of vectors. The elements of the dictionary are referred to as *atoms* and are typically constructed to have certain time/frequency or time/scale properties. Matching pursuit takes the NP-hard problem of finding the best nonlinear expansion in a dictionary and implements it in an energy-perserving formulation that guarantees convergence. See "Matching Pursuit Algorithms" for more details.

- "Sparse Representation in Redundant Dictionaries"
- "Matching Pursuit Algorithms"

References

[1] Cai, T.T. and L. Wang "Orthogonal Matching Pursuit for Sparse Signal Recovery with Noise". *IEEE Transactions on Information Theory*, vol. 57, 7, 4680–4688, 2011.

[2] Donoho, D., M. Elad, and V. Temlyakov "Stable Recovery of Sparse Overcomplete Representations in the Presence of Noise". *IEEE Transactions on Information Theory*, 52,1, 6–18, 2004.

[3] Mallat, S. and Z. Zhang "Matching Pursuits with Time-Frequency Dictionaries". *IEEE Transactions on Signal Processing*, vol. 41, 12, 3397–3415, 1993

[4] Tropp, J.A. "Greed is good: Algorithmic results for sparse approximation". *IEEE Transactions on Information Theory*, 50, pp. 2231–2242, 2004.

See Also

`wavemenu` | `wmpalg`

Introduced in R2012a

wmspca

Multiscale Principal Component Analysis

Syntax

```
[X_SIM,QUAL,NPC,DEC_SIM,PCA_Params] = wmspca(X,LEVEL,WNAME,NPC)
[...] = wmspca(X,LEVEL,WNAME,'mode',EXTMODE,NPC)
[...] = wmspca(DEC,NPC)
[...] = wmspca(X,LEVEL,WNAME,'mode',EXTMODE,NPC)
```

Description

`[X_SIM,QUAL,NPC,DEC_SIM,PCA_Params] = wmspca(X,LEVEL,WNAME,NPC)` or `[...] = wmspca(X,LEVEL,WNAME,'mode',EXTMODE,NPC)` returns a simplified version `X_SIM` of the input matrix `X` obtained from the wavelet-based multiscale principal component analysis (PCA).

The input matrix `X` contains `P` signals of length `N` stored columnwise (`N > P`).

Wavelet Decomposition Parameters

The wavelet decomposition is performed using the decomposition level `LEVEL` and the wavelet `WNAME`.

`EXTMODE` is the extended mode for the DWT (See `dwtmode`).

If a decomposition `DEC` obtained using `mdwtdec` is available, you can use

`[...] = wmspca(DEC,NPC)` instead of

`[...] = wmspca(X,LEVEL,WNAME,'mode',EXTMODE,NPC)`.

Principal Components Parameter: NPC

If `NPC` is a vector, then it must be of length `LEVEL+2`. It contains the number of retained principal components for each PCA performed:

- `NPC(d)` is the number of retained noncentered principal components for details at level `d`, for 1 <= `d` <= `LEVEL`.
- `NPC(LEVEL+1)` is the number of retained non-centered principal components for approximations at level LEVEL.
- `NPC(LEVEL+2)` is the number of retained principal components for final PCA after wavelet reconstruction.

`NPC` must be such that 0 <= `NPC(d)` <= `P` for 1 <= `d` <= `LEVEL`+2.

If `NPC = 'kais'` (respectively, `'heur'`), then the number of retained principal components is selected automatically using Kaiser's rule (or the heuristic rule).

- Kaiser's rule keeps the components associated with eigenvalues greater the mean of all eigenvalues.
- The heuristic rule keeps the components associated with eigenvalues greater than 0.05 times the sum of all eigenvalues.

If `NPC = 'nodet'`, then the details are “killed” and all the approximations are retained.

Output Parameters

`X_SIM` is a simplified version of the matrix `X`.

`QUAL` is a vector of length `P` containing the quality of column reconstructions given by the relative mean square errors in percent.

`NPC` is the vector of selected numbers of retained principal components.

`DEC_SIM` is the wavelet decomposition of `X_SIM`

`PCA_Params` is a structure array of length `LEVEL+2` such that:

- `PCA_Params(d).pc` is a `P`-by-`P` matrix of principal components.

  The columns are stored in descending order of the variances.
- `PCA_Params(d).variances` is the principal component variances vector.
- `PCA_Params(d).npc = NPC`

Examples

Wavelet Principal Component Analysis of Noisy Multivariate Signal

Use wavelet multiscale principal component analysis to denoise a multivariate signal.

Load the dataset consisting of 4 signals of length 1024. Plot the original signals and the signals with additive noise.

```
load ex4mwden;
kp = 0;
for i = 1:4
    subplot(4,2,kp+1), plot(x_orig(:,i)); axis tight;
    title(['Original signal ',num2str(i)])
    subplot(4,2,kp+2), plot(x(:,i)); axis tight;
    title(['Noisy signal ',num2str(i)])
    kp = kp + 2;
end
```

Perform the first multiscale wavelet PCA using the Daubechies' least-asymmetric wavelet with 4 vanishing moments, `sym4`. Obtain the multiresolution decomposition down to level 5. Use the heuristic rule to decide how many principal components to retain.

```
level = 5;
wname = 'sym4';
npc = 'heur';
[x_sim, qual, npc] = wmspca(x,level,wname,npc);
```

Plot the result and examine the quality of the approximation.

```
qual
kp = 0;
for i = 1:4
    subplot(4,2,kp+1), plot(x(:,i)); axis tight;
    title(['Noisy signal ',num2str(i)])
    subplot(4,2,kp+2), plot(x_sim(:,i)); axis tight;
    title(['First PCA ',num2str(i)])
    kp = kp + 2;
end
```

The quality results are all close to 100%. The `npc` vector gives the number of principal components retained at each level.

Suppress the noise by removing the principal components at levels 1–3. Perform the multiscale PCA again.

```
npc(1:3) = zeros(1,3);
[x_sim, qual, npc] = wmspca(x,level,wname,npc);
```

Plot the result.

```
kp = 0;
for i = 1:4
    subplot(4,2,kp+1), plot(x(:,i)); axis tight;
    title(['Noisy signal ',num2str(i)])
    subplot(4,2,kp+2), plot(x_sim(:,i)); axis tight;
    title(['Second PCA ',num2str(i)])
    kp = kp + 2;
end
```

More About

Algorithms

The multiscale principal components generalizes the usual PCA of a multivariate signal seen as a matrix by performing simultaneously a PCA on the matrices of details of different levels. In addition, a PCA is performed also on the coarser approximation coefficients matrix in the wavelet domain as well as on the final reconstructed matrix. By selecting conveniently the numbers of retained principal components, interesting simplified signals can be reconstructed.

References

Aminghafari, M.; Cheze, N.; Poggi, J-M. (2006), "Multivariate de-noising using wavelets and principal component analysis," *Computational Statistics & Data Analysis*, 50, pp. 2381–2398.

Bakshi, B. (1998), "Multiscale PCA with application to MSPC monitoring," *AIChE J.*, 44, pp. 1596–1610.

See Also

`wmulden`

Introduced in R2006b

wmulden

Wavelet multivariate de-noising

Syntax

```
[X_DEN,NPC,NESTCOV,DEC_DEN,PCA_Params,DEN_Params] = ...
wmulden(X,LEVEL,WNAME,NPC_APP,NPC_FIN,TPTR,SORH)
[...] = wmulden(X,LEVEL,WNAME,'mode',EXTMODE,NPC_APP,...)
[...] = wmulden(DEC,NPC_APP)
[...] = wmulden(X,LEVEL,WNAME,'mode',EXTMODE,NPC_APP)
[DEC,PCA_Params] = wmulden('estimate',DEC,NPC_APP,NPC_FIN)
[X_DEN,NPC,DEC_DEN,PCA_Params] = wmulden('execute',DEC,PC_Params)
```

Description

`[X_DEN,NPC,NESTCOV,DEC_DEN,PCA_Params,DEN_Params] = ...`
`wmulden(X,LEVEL,WNAME,NPC_APP,NPC_FIN,TPTR,SORH)` or
`[...] = wmulden(X,LEVEL,WNAME,'mode',EXTMODE,NPC_APP,...)` returns a de-noised version `X_DEN` of the input matrix `X`. The strategy combines univariate wavelet de-noising in the basis where the estimated noise covariance matrix is diagonal with noncentered Principal Component Analysis (PCA) on approximations in the wavelet domain or with final PCA.

The input matrix `X` contains `P` signals of length N stored columnwise where `N > P`.

Wavelet Decomposition Parameters

The wavelet decomposition is performed using the decomposition level `LEVEL` and the wavelet `WNAME`.

`EXTMODE` is the extended mode for the DWT (See `dwtmode`).

If a decomposition `DEC` obtained using `mdwtdec` is available, you can use

`[...] = wmulden(DEC,NPC_APP)` instead of

```
[...] = wmulden(X,LEVEL,WNAME,'mode',EXTMODE,NPC_APP).
```

Principal Components Parameters: NPC_APP and NPC_FIN

The input selection methods NPC_APP and NPC_FIN define the way to select principal components for approximations at level LEVEL in the wavelet domain and for final PCA after wavelet reconstruction, respectively.

If NPC_APP (or NPC_FIN) is an integer, it contains the number of retained principal components for approximations at level LEVEL (or for final PCA after wavelet reconstruction).

NPC_XXX must be such that 0 <= NPC_XXX <= P

NPC_APP or NPC_FIN = 'kais' or 'heur' selects the number of retained principal components using Kaiser's rule or the heuristic rule automatically.

- Kaiser's rule keeps the components associated with eigenvalues greater than the mean of all eigenvalues.
- The heuristic rule keeps the components associated with eigenvalues greater than 0.05 times the sum of all eigenvalues.

NPC_APP or NPC_FIN = 'none' is equivalent to NPC_APP or NPC_FIN = P.

De-noising Parameters: TPTR and SORH

The default values for the de-noising parameters TPTR and SORH are:

TPTR = 'sqtwolog' and SORH = 's'

- Valid values for TPTR are

  ```
  'rigsure', 'heursure', 'sqtwolog', 'minimaxi',
  'penalhi', 'penalme', 'penallo'
  ```

- Valid values for SORH are:

  ```
  's' (soft) or 'h' (hard)
  ```

For additional information, see wden and wbmpen.

Output Parameters

`X_DEN` is a de-noised version of the input matrix `X`.

`NPC` is the vector of selected numbers of retained principal components.

`NESTCOV` is the estimated noise covariance matrix obtained using the minimum covariance determinant (MCD) estimator.

`DEC_DEN` is the wavelet decomposition of `X_DEN`.

`PCA_Params` is a structure such that:

```
PCA_Params.NEST = {pc_NEST,var_NEST,NESTCOV}
PCA_Params.APP  = {pc_APP,var_APP,npc_APP}
PCA_Params.FIN  = {pc_FIN,var_FIN,npc_FIN}
```

where:

- `pc_XXX` is a `P`-by-`P` matrix of principal components.

  The columns are stored in descending order of the variances.
- `var_XXX` is the principal component variances vector.
- `NESTCOV` is the covariance matrix estimate for detail at level 1.

`DEN_Params` is a structure such that:

- `DEN_Params.thrVAL` is a vector of length `LEVEL` which contains the threshold values for each level.
- `DEN_Params.thrMETH` is a string containing the name of the de-noising method (`TPTR`).
- `DEN_Params.thrTYPE` is a character variable containing the type of the thresholding (`SORH`).

Special Cases

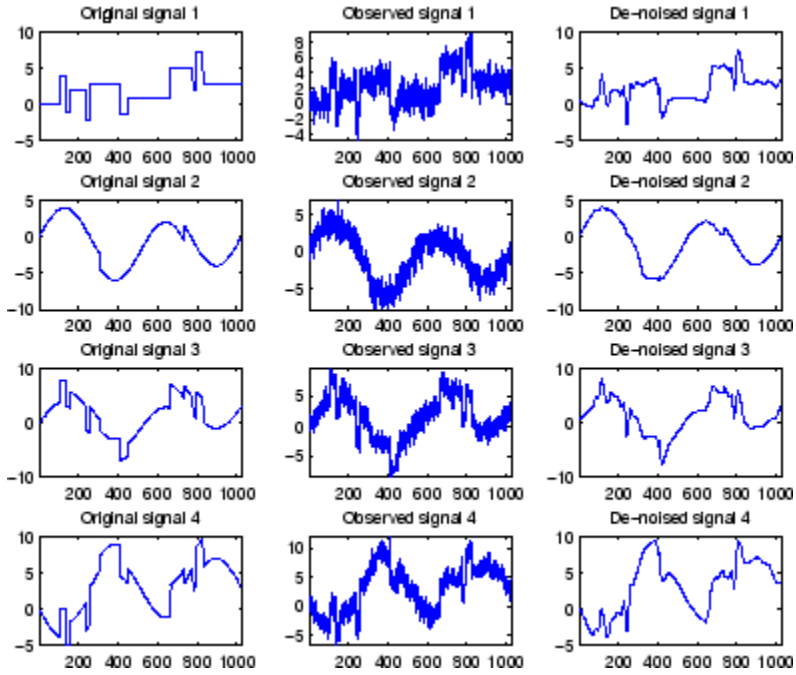
`[DEC,PCA_Params] = wmulden('estimate',DEC,NPC_APP,NPC_FIN)` returns the wavelet decomposition `DEC` and the Principal Components Estimates `PCA_Params`.

`[X_DEN,NPC,DEC_DEN,PCA_Params] = wmulden('execute',DEC,PC_Params)` uses the principal components estimates `PCA_Params` previously computed.

The input value DEC can be replaced by X, LEVEL, and WNAME.

Examples

```
%  Load a multivariate signal x together with
%  the original signals (x_orig) and true noise
%  covariance matrix (covar).

load ex4mwden

%  Set the de-noising method parameters.
level = 5;
wname = 'sym4';
tptr  = 'sqtwolog';
sorh  = 's';

% Set the PCA parameters to select the number of
% retained principal components automatically by
% Kaiser's rule.

npc_app = 'kais';
npc_fin = 'kais';

% Perform multivariate de-noising.
[x_den, npc, nestco] = wmulden(x, level, wname, npc_app, ...
                                    npc_fin, tptr, sorh);

% Display the original and de-noised signals.
kp = 0;
for i = 1:4
    subplot(4,3,kp+1), plot(x_orig(:,i));
    title(['Original signal ',num2str(i)])
    subplot(4,3,kp+2), plot(x(:,i));
    title(['Observed signal ',num2str(i)])
    subplot(4,3,kp+3), plot(x_den(:,i));
    title(['De-noised signal ',num2str(i)])
    kp = kp + 3;
end
```




```
% The results are good: the first function, which is
% irregular, is correctly recovered while the second
% function, more regular, is well de-noised.

% The second output argument gives the numbers
% of retained principal components for PCA for
% approximations and for final PCA.

npc

npc =

     2     2

% The third output argument contains the estimated
% noise covariance matrix using the MCD based
% on the matrix of finest details.

nestco
```

```
nestco =

    1.0784    0.8333    0.6878    0.8141
    0.8333    1.0025    0.5275    0.6814
    0.6878    0.5275    1.0501    0.7734
    0.8141    0.6814    0.7734    1.0967

% The estimation is satisfactory since the values are close
% to the true values given by covar.

covar

covar =

    1.0000    0.8000    0.6000    0.7000
    0.8000    1.0000    0.5000    0.6000
    0.6000    0.5000    1.0000    0.7000
    0.7000    0.6000    0.7000    1.0000
```

More About

Algorithms

The multivariate de-noising procedure is a generalization of the one-dimensional strategy. It combines univariate wavelet de-noising in the basis where the estimated noise covariance matrix is diagonal and non-centered Principal Component Analysis (PCA) on approximations in the wavelet domain or with final PCA.

The robust estimate of the noise covariance matrix given by the minimum covariance determinant estimator based on the matrix of finest details.

References

Aminghafari, M.; Cheze, N.; Poggi, J-M. (2006), "Multivariate de-noising using wavelets and principal component analysis," *Computational Statistics & Data Analysis*, 50, pp. 2381–2398.

Rousseeuw, P.; Van Driessen, K. (1999), "A fast algorithm for the minimum covariance determinant estimator," *Technometrics,* 41, pp. 212–223.

See Also

`wmspca`

Introduced in R2006b

wnoise

Noisy wavelet test data

Syntax

```
X = wnoise(FUN,N)
[X,XN] = wnoise(FUN,N,SQRT_SNR)
[X,XN] = wnoise(FUN,N,SQRT_SNR,INIT)
```

Description

`X = wnoise(FUN,N)` returns values of the test signal given by `FUN`, on a 2^N grid of [0,1].

`[X,XN] = wnoise(FUN,N,SQRT_SNR)` returns a test vector `X` as above, rescaled such that `std(X) = SQRT_SNR`. The returned vector `XN` contains the same test vector corrupted by additive Gaussian white noise $N(0,1)$. Then, `XN` has a signal-to-noise ratio of `SNR = (SQRT_SNR)`^{2}.

`[X,XN] = wnoise(FUN,N,SQRT_SNR,INIT)` returns previous vectors `X` and `XN`, but the generator seed is set to `INIT` value.

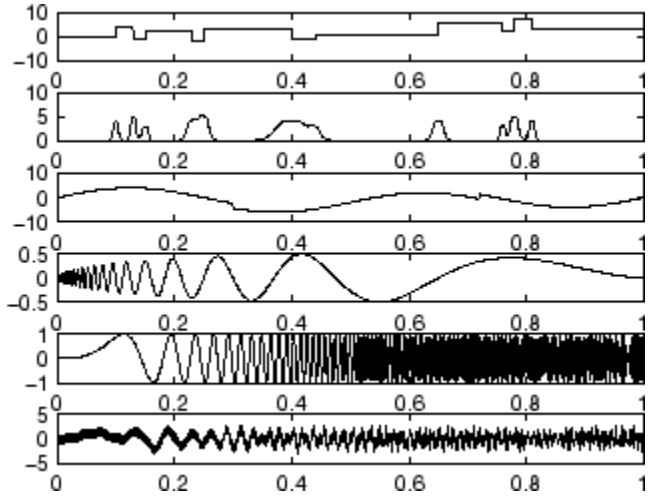
The six functions below are due to Donoho and Johnstone (See "References").

`FUN = 1`	or	`'blocks'`
`FUN = 2`	or	`'bumps'`
`FUN = 3`	or	`'heavy sine'`
`FUN = 4`	or	`'doppler'`
`FUN = 5`	or	`'quadchirp'`
`FUN = 6`	or	`'mishmash'`

Examples

```
% Generate 2^10 samples of 'Heavy sine' (item 3).
x = wnoise(3,10);
```

```
% Generate 2^10 samples of 'Doppler' (item 4) and of
% noisy 'Doppler' with a square root of signal-to-noise
% ratio equal to 7.
[x,noisyx] = wnoise(4,10,7);

% To introduce your own rand seed, a fourth
% argument is allowed:
init = 2055415866;
[x,noisyx] = wnoise(4,10,7,init);

% Plot all the test functions.
ind = linspace(0,1,2^10);
for i = 1:6
    x = wnoise(i,10);
    subplot(6,1,i), plot(ind,x)
end

% Editing some graphical properties,
% the following figure is generated.
```



References

Donoho, D.L.; I.M. Johnstone (1994), “Ideal spatial adaptation by wavelet shrinkage,” *Biometrika*, vol. 81, pp. 425–455.

Donoho, D.L.; I.M. Johnstone (1995), "Adapting to unknown smoothness via wavelet shrinkage via wavelet shrinkage," *JASA*, vol. 90, 432, pp. 1200–1224.

See Also

wden

Introduced before R2006a

wnoisest

Estimate noise of 1-D wavelet coefficients

Syntax

```
STDC = wnoisest(C,L,S)
STDC = wnoisest(C)
STDC = wnoisest(C)
```

Description

`STDC = wnoisest(C,L,S)` returns estimates of the detail coefficients' standard deviation for levels contained in the input vector `S`. `[C,L]` is the input wavelet decomposition structure (see `wavedec` for more information).

If `C` is a one dimensional cell array, `STDC = wnoisest(C)` returns a vector such that `STDC(k)` is an estimate of the standard deviation of `C{k}`.

If `C` is a numeric array, `STDC = wnoisest(C)` returns a vector such that `STDC(k)` is an estimate of the standard deviation of `C(k,:)`.

The estimator used is Median Absolute Deviation / 0.6745, well suited for zero mean Gaussian white noise in the de-noising one-dimensional model (see `thselect` for more information).

Examples

Estimate Noise Standard Deviation in The Presence of Outliers

Estimate of the noise standard deviation in an N(0,1) white Gaussian noise vector with outliers.

Create an N(0,1) noise vector with 10 randomly-placed outliers.

```
rng default;
x = randn(1000,1);
```

```
P = randperm(length(x));
indices = P(1:10);
x(indices(1:5)) = 10;
x(indices(6:end)) = -10;
```

Obtain the discrete wavelet transform down to level 2 using the Daubechies' extremal phase wavelet with 3 vanishing moments.

```
[c,l] = wavedec(x,2,'db3');
stdc = wnoisest(c,l,1:2)
```

In spite of the outliers, `wnoisest` provides a robust estimate of the standard deviation.

References

Donoho, D.L.; I.M. Johnstone (1994), "Ideal spatial adaptation by wavelet shrinkage," *Biometrika*, vol 81, pp. 425–455.

Donoho, D.L.; I.M. Johnstone (1995), "Adapting to unknown smoothness via wavelet shrinkage via wavelet shrinkage," *JASA*, vol 90, 432, pp. 1200–1224.

See Also

`thselect` | `wden` | `wavedec`

Introduced before R2006a

wp2wtree

Extract wavelet tree from wavelet packet tree

Syntax

```
T = wp2wtree(T)
```

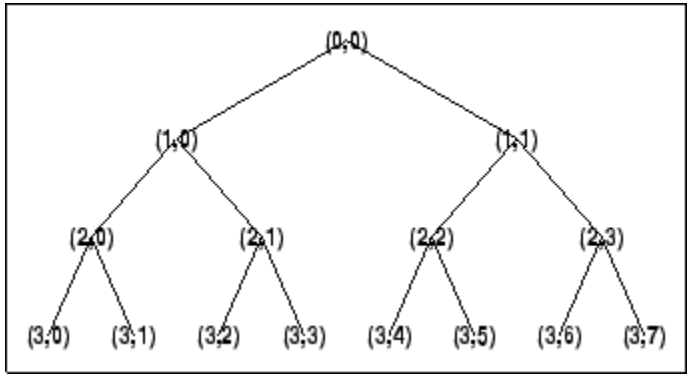
Description

`wp2wtree` is a one- or two-dimensional wavelet packet analysis function.

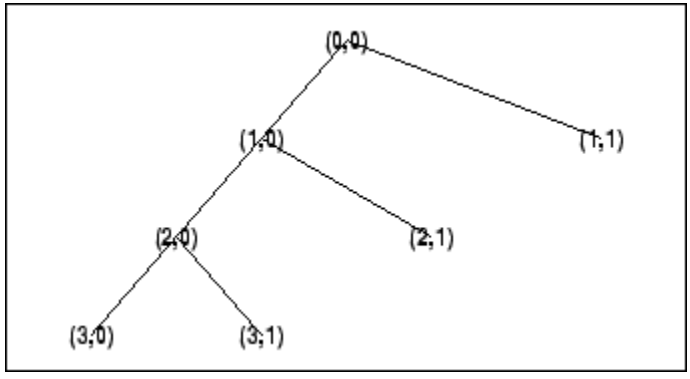
`T = wp2wtree(T)` computes the modified wavelet packet tree *T* corresponding to the wavelet decomposition tree.

Examples

```
% The current extension mode is zero-padding (see dwtmode).

% Load signal.
    load noisdopp; x = noisdopp;

% Decompose x at depth 3 with db1 wavelet packets
% using shannon entropy.
wpt = wpdec(x,3,'db1');

% Plot wavelet packet tree wpt.
plot(wpt)
```




```
% Compute wavelet tree.
wt = wp2wtree(wpt);

% Plot wavelet tree wt.
plot(wt)
```



See Also

wpdec | wpdec2

Introduced before R2006a

wpbmpen

Penalized threshold for wavelet packet de-noising

Syntax

```
THR = wpbmpen(T,SIGMA,ALPHA)
wpbmpen(T,SIGMA,ALPHA,ARG)
```

Description

`THR = wpbmpen(T,SIGMA,ALPHA)` returns a global threshold `THR` for de-noising. `THR` is obtained by a wavelet packet coefficients selection rule using a penalization method provided by Birge-Massart.

`T` is a wavelet packet tree corresponding to the wavelet packet decomposition of the signal or image to be de-noised.

`SIGMA` is the standard deviation of the zero mean Gaussian white noise in the de-noising model (see `wnoisest` for more information).

`ALPHA` is a tuning parameter for the penalty term. It must be a real number greater than 1. The sparsity of the wavelet packet representation of the de-noised signal or image grows with `ALPHA`. Typically `ALPHA` = 2.

`THR` minimizes the penalized criterion given by

let t^* be the minimizer of

```
crit(t) = -sum(c(k)^2,k≤t) + 2*SIGMA^2*t*(ALPHA + log(n/t))
```

where `c(k)` are the wavelet packet coefficients sorted in decreasing order of their absolute value and `n` is the number of coefficients, then THR=$|c(t^*)|$.

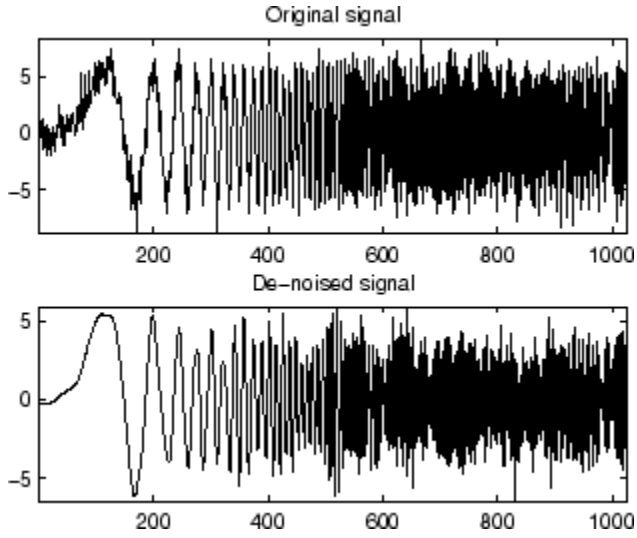
`wpbmpen(T,SIGMA,ALPHA,ARG)` computes the global threshold and, in addition, plots three curves:

- `2*SIGMA^2*t*(ALPHA + log(n/t))`

- sum(c(k)^2,k£t)
- crit(t)

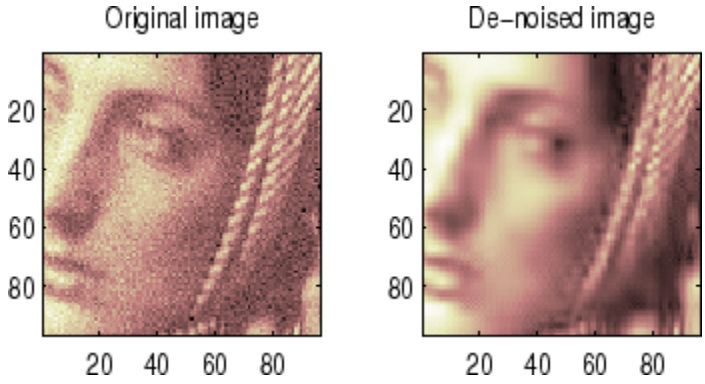
Examples

```
% Example 1: Signal de-noising.
% Load noisy chirp signal.
load noischir; x = noischir;

% Perform a wavelet packet decomposition of the signal
% at level 5 using sym6.
wname = 'sym6'; lev = 5;
tree = wpdec(x,lev,wname);

% Estimate the noise standard deviation from the
% detail coefficients at level 1,
% corresponding to the node index 2.
det1 = wpcoef(tree,2);
sigma = median(abs(det1))/0.6745;

% Use wpbmpen for selecting global threshold
% for signal de-noising, using the recommended parameter.
alpha = 2;
thr = wpbmpen(tree,sigma,alpha)

thr =

    4.5740


% Use wpdencmp for de-noising the signal using the above
% threshold with soft thresholding and keeping the
% approximation.
keepapp = 1;
xd = wpdencmp(tree,'s','nobest',thr,keepapp);

% Plot original and de-noised signals.
figure(1)
subplot(211), plot(x),
title('Original signal')
subplot(212), plot(xd)
title('De-noised signal')
```




```
% Example 2: Image de-noising.
% Load original image.
load noiswom;
nbc = size(map,1);

% Perform a wavelet packet decomposition of the image
% at level 3 using coif2.
wname = 'coif2'; lev = 3;
tree = wpdec2(X,lev,wname);

% Estimate the noise standard deviation from the
% detail coefficients at level 1.
det1 = [wpcoef(tree,2) wpcoef(tree,3) wpcoef(tree,4)];
sigma = median(abs(det1(:)))/0.6745;

% Use wpbmpen for selecting global threshold
% for image de-noising.
alpha = 1.1;
thr = wpbmpen(tree,sigma,alpha)

thr =

   38.5125
```

```
% Use wpdencmp for de-noising the image using the above
% thresholds with soft thresholding and keeping the
% approximation.
keepapp = 1;
xd = wpdencmp(tree,'s','nobest',thr,keepapp);

% Plot original and de-noised images.
figure(2)
colormap(pink(nbc));
subplot(221), image(wcodemat(X,nbc))
title('Original image')
subplot(222), image(wcodemat(xd,nbc))
title('De-noised image')
```



See Also

wbmpen | wden | wdencmp | wpdencmp

Introduced before R2006a

wpcoef

Wavelet packet coefficients

Syntax

```
X = wpcoef(T,N)
X = wpcoef(T)
```

Description

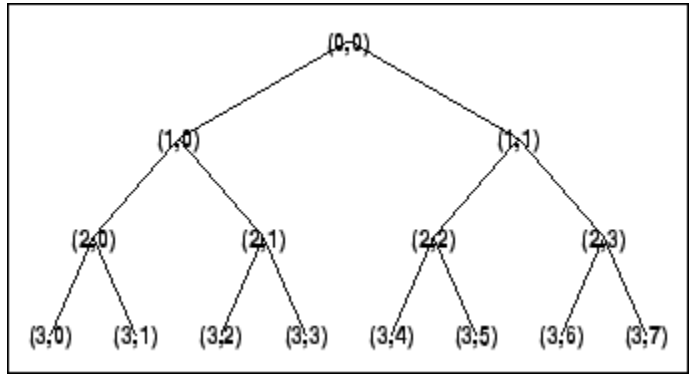
`wpcoef` is a one- or two-dimensional wavelet packet analysis function.

`X = wpcoef(T,N)` returns the coefficients associated with the node `N` of the wavelet packet tree *T*. If `N` doesn't exist, `X = [ ];`

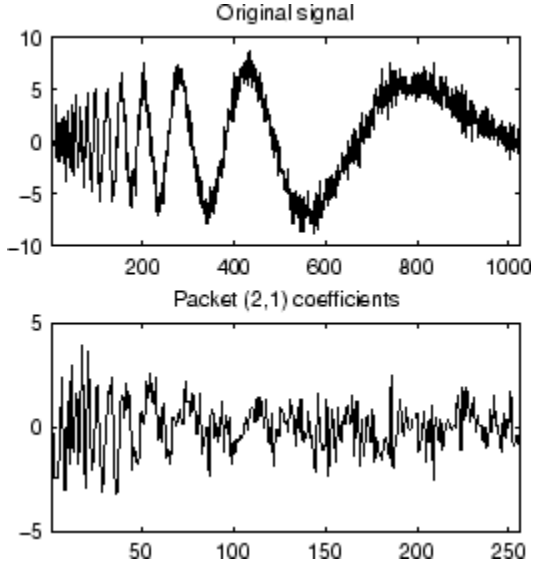
`X = wpcoef(T)` is equivalent to `X = wpcoef(T,0)`.

Examples

```
% The current extension mode is zero-padding (see dwtmode).

% Load signal.
load noisdopp; x = noisdopp;

figure(1); subplot(211);
plot(x); title('Original signal');

% Decompose x at depth 3 with db1 wavelet packets
% using Shannon entropy.
wpt = wpdec(x,3,'db1');

% Plot wavelet packet tree wpt.
plot(wpt)
```




```
% Read packet (2,1) coefficients.
cfs = wpcoef(wpt,[2 1]);

figure(1); subplot(212);
plot(cfs); title('Packet (2,1) coefficients');
```



More About

- “Reconstructing a Signal Approximation from a Node”

See Also

`wpcoef` | `wpdec` | `wpdec2` | `wprcoef`

Introduced before R2006a

wpcutree

Cut wavelet packet tree

Syntax

```
T = wpcutree(T,L)
T
[T,RN] = wpcutree(T,L)
```

Description

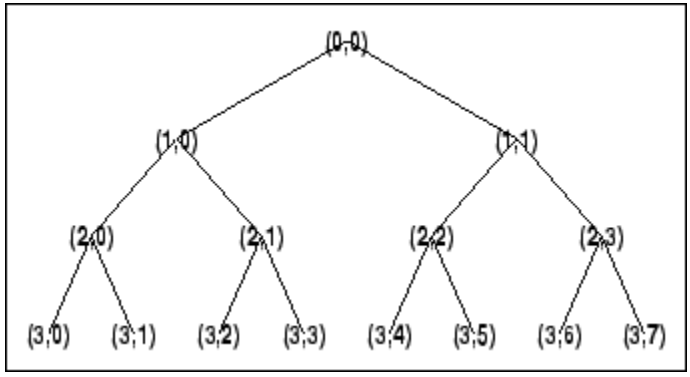
`wpcutree` is a one- or two-dimensional wavelet packet analysis function.

`T = wpcutree(T,L)` cuts the tree `T` at level `L`.

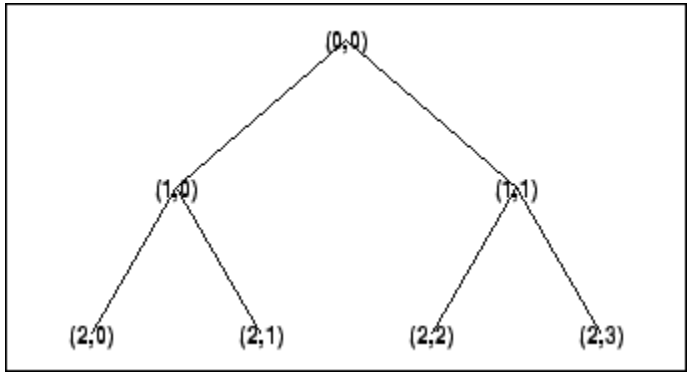
`[T,RN] = wpcutree(T,L)` returns the same arguments as above and, in addition, the vector `RN` contains the indices of the reconstructed nodes.

Examples

```
% The current extension mode is zero-padding (see dwtmode).

% Load signal.
load noisdopp; x = noisdopp;

% Decompose x at depth 3 with db1 wavelet packets
% using Shannon entropy.
wpt = wpdec(x,3,'db1');

% Plot wavelet packet tree wpt.
plot(wpt)
```




```
% Cut wavelet packet tree at level 2.
nwpt = wpcutree(wpt,2);

% Plot new wavelet packet tree nwpt.
plot(nwpt)
```



See Also

wpdec | wpdec2

Introduced before R2006a

wpdec

Wavelet packet decomposition 1-D

Syntax

```
T = wpdec(X,N,'wname',E,P)
T = wpdec(X,N,'wname')
T = wpdec(X,N,'wname','shannon')
T
```

Description

`wpdec` is a one-dimensional wavelet packet analysis function.

`T = wpdec(X,N,'wname',E,P)` returns a wavelet packet tree *T* corresponding to the wavelet packet decomposition of the vector `X` at level `N`, with a particular wavelet (`'wname'`, see `wfilters` for more information).

`T = wpdec(X,N,'wname')` is equivalent to `T = wpdec(X,N,'wname','shannon')`.

`E` is a string containing the type of entropy and *P* is an optional parameter depending on the value of `T` (see `wentropy` for more information).

Entropy Type Name (E)	Parameter (P)	Comments
`'shannon'`		`P` is not used.
`'log energy'`		`P` is not used.
`'threshold'`	$0 \le P$	`P` is the threshold.
`'sure'`	$0 \le P$	`P` is the threshold.
`'norm'`	$1 \le P$	`P` is the power.
`'user'`	`string`	`P` is a string containing the file name of your own entropy function, with a single input `X`.

Entropy Type Name (E)	Parameter (P)	Comments
`FunName`	`No constraints on P`	`FunName` is any other string except those used for the previous Entropy Type Names listed above. `FunName` contains the file name of your own entropy function, with `X` as input and `P` as additional parameter to your entropy function.

Note The `'user'` option is historical and still kept for compatibility, but it is obsoleted by the last option described in the table above. The `FunName` option do the same as the `'user'` option and in addition gives the possibility to pass a parameter to your own entropy function.

The wavelet packet method is a generalization of wavelet decomposition that offers a richer signal analysis. Wavelet packet atoms are waveforms indexed by three naturally interpreted parameters: position and scale as in wavelet decomposition, and frequency.

For a given orthogonal wavelet function, a library of wavelet packets bases is generated. Each of these bases offers a particular way of coding signals, preserving global energy and reconstructing exact features. The wavelet packets can then be used for numerous expansions of a given signal.

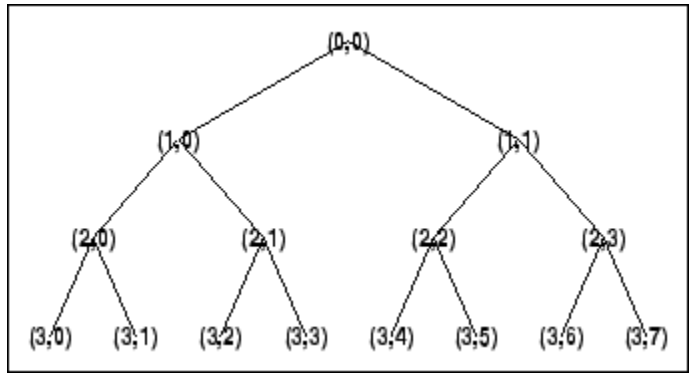
Simple and efficient algorithms exist for both wavelet packets decomposition and optimal decomposition selection. Adaptive filtering algorithms with direct applications in optimal signal coding and data compression can then be produced.

In the orthogonal wavelet decomposition procedure, the generic step splits the approximation coefficients into two parts. After splitting we obtain a vector of approximation coefficients and a vector of detail coefficients, both at a coarser scale. The information lost between two successive approximations is captured in the detail coefficients. The next step consists in splitting the new approximation coefficient vector; successive details are never re-analyzed.

In the corresponding wavelet packets situation, each detail coefficient vector is also decomposed into two parts using the same approach as in approximation vector splitting. This offers the richest analysis: the complete binary tree is produced in the one-dimensional case or a quaternary tree in the two-dimensional case.

Examples

```
% The current extension mode is zero-padding (see dwtmode).

% Load signal.
load noisdopp; x = noisdopp;

% Decompose x at depth 3 with db1 wavelet packets
% using Shannon entropy.
wpt = wpdec(x,3,'db1','shannon');

% The result is the wavelet packet tree wpt.

% Plot wavelet packet tree (binary tree, or tree of order 2).
plot(wpt)
```



More About

Algorithms

The algorithm used for the wavelet packets decomposition follows the same line as the wavelet decomposition process (see `dwt` and `wavedec` for more information).

References

Coifman, R.R.; M.V. Wickerhauser, (1992), “Entropy-based Algorithms for best basis selection,” *IEEE Trans. on Inf. Theory*, vol. 38, 2, pp. 713–718.

Meyer, Y. (1993), *Les ondelettes. Algorithmes et applications*, Colin Ed., Paris, 2nd edition. (English translation: *Wavelets: Algorithms and Applications*, SIAM).

Wickerhauser, M.V. (1991), "INRIA lectures on wavelet packet algorithms," *Proceedings ondelettes et paquets d'ondes*, 17–21 June, Rocquencourt, France, pp. 31–99.

Wickerhauser, M.V. (1994), *Adapted wavelet analysis from theory to software algorithms*, A.K. Peters.

See Also

`wavedec` | `waveinfo` | `wenergy` | `wpdec` | `wprec`

Introduced before R2006a

wpdec2

Wavelet packet decomposition 2-D

Syntax

```
T = wpdec2(X,N,'wname',E,P)
T = wpdec2(X,N,'wname')
T = wpdec2(X,N,wnam,'shannon')
```

Description

`wpdec2` is a two-dimensional wavelet packet analysis function.

`T = wpdec2(X,N,'wname',E,P)` returns a wavelet packet tree `T` corresponding to the wavelet packet decomposition of the matrix `X`, at level `N`, with a particular wavelet (`'wname'`, see `wfilters` for more information).

`T = wpdec2(X,N,'wname')` is equivalent to `T = wpdec2(X,N,wnam,'shannon')`.

`E` is a string containing the type of entropy and `P` is an optional parameter depending on the value of `T` (see `wentropy` for more information).

Entropy Type Name (E)	Parameter (P)	Comments
`'shannon'`		`P` is not used.
`'log energy'`		`P` is not used.
`'threshold'`	$0 \le P$	`P` is the threshold.
`'sure'`	$0 \le P$	`P` is the threshold.
`'norm'`	$1 \le P$	`P` is the power.
`'user'`	`string`	`P` is a string containing the file name of your own entropy function, with a single input `X`.
`STR`	`No constraints on P`	`STR` is any other string except those used for the previous Entropy Type Names listed above.

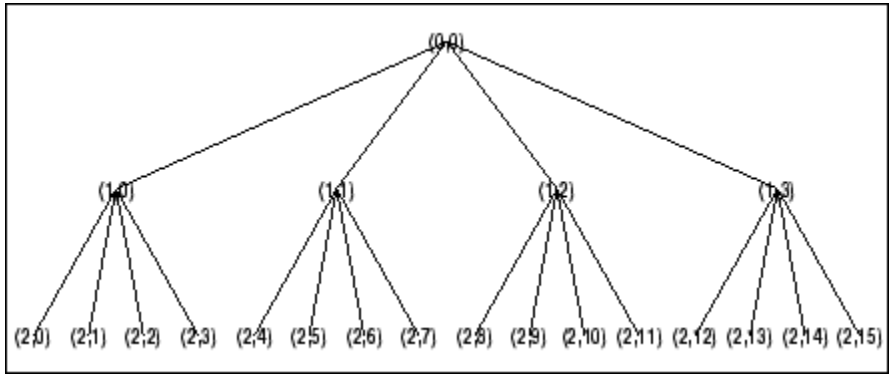
Entropy Type Name (E)	Parameter (P)	Comments
		STR contains the file name of your own entropy function, with X as input and P as additional parameter to your entropy function.

Note The `'user'` option is historical and still kept for compatibility, but it is obsoleted by the last option described in the preceding table. The `FunName` option does the same as the `'user'` option and in addition, allows you to pass a parameter to your own entropy function.

See `wpdec` for a more complete description of the wavelet packet decomposition.

Examples

```
% The current extension mode is zero-padding (see dwtmode).

% Load image.
load tire
% X contains the loaded image.

% For an image the decomposition is performed using:
t = wpdec2(X,2,'db1');
% The default entropy is shannon.

% Plot wavelet packet tree
% (quarternary tree, or tree of order 4).
plot(t)
```



More About

Tips

When X represents an indexed image, X is an `m`-by-`n` matrix. When X represents a truecolor image, it is an `m`-by-`n`-by-3 array, where each `m`-by-`n` matrix represents a red, green, or blue color plane concatenated along the third dimension.

For more information on image formats, see the `image` and `imfinfo` reference pages.

Algorithms

The algorithm used for the wavelet packets decomposition follows the same line as the wavelet decomposition process (see `dwt2` and `wavedec2` for more information).

References

Coifman, R.R.; M.V. Wickerhauser (1992), “Entropy-based algorithms for best basis selection,” *IEEE Trans. on Inf. Theory*, vol. 38, 2, pp. 713–718.

Meyer, Y. (1993), *Les ondelettes. Algorithmes et applications*, Colin Ed., Paris, 2nd edition. (English translation: *Wavelets: Algorithms and Applications*, SIAM).

Wickerhauser, M.V. (1991), “INRIA lectures on wavelet packet algorithms,” *Proceedings ondelettes et paquets d'ondes*, 17–21 June, Rocquencourt, France, pp. 31–99.

Wickerhauser, M.V. (1994), *Adapted wavelet analysis from theory to software Algorithms*, A.K. Peters.

See Also

`wavedec2` | `waveinfo` | `wenergy` | `wpdec` | `wprec2`

Introduced before R2006a

wpdencmp

De-noising or compression using wavelet packets

Syntax

```
[XD,TREED,PERFO,PERFL2] =
wpdencmp(X,SORH,N,'wname',CRIT,PAR,KEEPAPP)
[XD,TREED,PERFO,PERFL2] = wpdencmp(TREE,SORH,CRIT,PAR,KEEPAPP)
```

Description

`wpdencmp` is a one- or two-dimensional de-noising and compression oriented function.

`wpdencmp` performs a de-noising or compression process of a signal or an image, using wavelet packet. The ideas and the procedures for de-noising and compression using wavelet packet decomposition are the same as those used in the wavelets framework (see `wden` and `wdencmp` for more information).

`[XD,TREED,PERFO,PERFL2] = wpdencmp(X,SORH,N,'wname',CRIT,PAR,KEEPAPP)` returns a de-noised or compressed version `XD` of input signal `X` (one- or two-dimensional) obtained by wavelet packets coefficients thresholding.

The additional output argument `TREED` is the wavelet packet best tree decomposition (see `besttree` for more information) of `XD`. `PERFL2` and `PERFO` are L^2 energy recovery and compression scores in percentages.

`PERFL2 = 100 *` (vector-norm of WP-cfs of `XD` / vector-norm of WP-cfs of `X`^{2}.

If `X` is a one-dimensional signal and `'wname'` an orthogonal wavelet, `PERFL2` is reduced to

$$\frac{100\|XD\|^2}{\|X\|^2}$$

`SORH` equal to `'s'` or `'h'` is for soft or hard thresholding (see `wthresh` for more information).

Wavelet packet decomposition is performed at level `N` and `'wname'` is a string containing the wavelet name. Best decomposition is performed using entropy criterion defined by string `CRIT` and parameter `PAR` (see `wentropy` for more information). Threshold parameter is also `PAR`. If `KEEPAPP = 1`, approximation coefficients cannot be thresholded; otherwise, they can be.

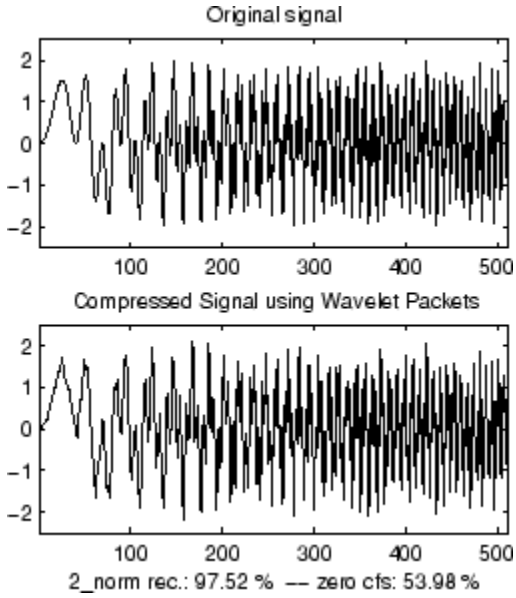
`[XD,TREED,PERFO,PERFL2] = wpdencmp(TREE,SORH,CRIT,PAR,KEEPAPP)` has the same output arguments, using the same options as above, but obtained directly from the input wavelet packet tree decomposition `TREE` (see `wpdec` for more information) of the signal to be de-noised or compressed.

In addition if `CRIT = 'nobest'` no optimization is done and the current decomposition is thresholded.

Examples

```
% The current extension mode is zero-padding (see dwtmode).

% Load original signal.
load sumlichr; x = sumlichr;

% Use wpdencmp for signal compression.
% Find default values (see ddencmp).
[thr,sorh,keepapp,crit] = ddencmp('cmp','wp',x)

thr =
    0.5193

sorh =
h

keepapp =
    1

crit =
    threshold

% De-noise signal using global thresholding with
% threshold best basis.
```

```
[xc,wpt,perf0,perfl2] = ...
wpdencmp(x,sorh,3,'db2',crit,thr,keepapp);

% Using some plotting commands,
% the following figure is generated.
```




```
% Load original image.
load sinsin

% Generate noisy image.
x = X/18 + randn(size(X));

% Use wpdencmp for image de-noising.
% Find default values (see ddencmp).
[thr,sorh,keepapp,crit] = ddencmp('den','wp',x)

thr =
    4.9685

sorh =
h
keepapp =
    1
```

```
crit =
sure
% De-noise image using global thresholding with
% SURE best basis.
xd = wpdencmp(x,sorh,3,'sym4',crit,thr,keepapp);

% Using some plotting commands,
% the following figure is generated.
```

```
% Generate heavy sine and a noisy version of it.
init = 1000;
[xref,x] = wnoise(5,11,7,init);

% Use wpdencmp for signal de-noising.
n = length(x);
thr = sqrt(2*log(n*log(n)/log(2)));
xwpd = wpdencmp(x,'s',4,'sym4','sure',thr,1);

% Compare with wavelet-based de-noising result.
xwd = wden(x,'rigrsure','s','one',4,'sym4');
```

References

Antoniadis, A.; G. Oppenheim, Eds. (1995), *Wavelets and statistics*, Lecture Notes in Statistics, 103, Springer Verlag.

Coifman, R.R.; M.V. Wickerhauser (1992), "Entropy-based algorithms for best basis selection," *IEEE Trans. on Inf. Theory*, vol. 38, 2, pp. 713–718.

DeVore, R.A.; B. Jawerth, B.J. Lucier (1992), "Image compression through wavelet transform coding," *IEEE Trans. on Inf. Theory*, vol. 38, No 2, pp. 719–746.

Donoho, D.L. (1993), "Progress in wavelet analysis and WVD: a ten minute tour," in Progress in wavelet analysis and applications, Y. Meyer, S. Roques, pp. 109–128. Frontières Ed.

Donoho, D.L.; I.M. Johnstone (1994), "Ideal spatial adaptation by wavelet shrinkage," *Biometrika*, vol. 81, pp. 425–455.

Donoho, D.L.; I.M. Johnstone, G. Kerkyacharian, D. Picard (1995), "Wavelet shrinkage: asymptopia," *Jour. Roy. Stat. Soc.*, series B, vol. 57 no. 2, pp. 301–369.

See Also

`besttree` | `ddencmp` | `wdencmp` | `wenergy` | `wpbmpen` | `wpdec` | `wpdec2` | `wthresh`

Introduced before R2006a

wpfun

Wavelet packet functions

Syntax

```
[WPWS,X] = wpfun('wname',NUM,PREC)
[WPWS,X] = wpfun('wname',NUM)
[WPWS,X] = wpfun('wname',NUM,7)
```

Description

`wpfun` is a wavelet packet analysis function.

`[WPWS,X] = wpfun('wname',NUM,PREC)` computes the wavelet packets for a wavelet `'wname'` (see `wfilters` for more information), on dyadic intervals of length $2^{-\mathrm{PREC}}$.

`PREC` must be a positive integer. Output matrix `WPWS` contains the W functions of index from 0 to `NUM`, stored row-wise as $[W_0; W_1; \ldots ; W_{\mathrm{NUM}}]$. Output vector `X` is the corresponding common `X`-grid vector.

`[WPWS,X] = wpfun('wname',NUM)` is equivalent to
`[WPWS,X] = wpfun('wname',NUM,7)`.

The computation scheme for wavelet packets generation is easy when using an orthogonal wavelet. We start with the two filters of length $2N$, denoted $h(n)$ and $g(n)$, corresponding to the wavelet.

Now by induction let us define the following sequence of functions ($W_n(x)$, $n = 0,1,2,\ldots$) by

$$W_{2n}(x) = \sqrt{2} \sum_{k=0,\ldots,2N-1} h(k) W_n(2x-k)$$

$$W_{2n+1}(x) = \sqrt{2} \sum_{k=0,\ldots,2N-1} g(k) W_n(2x-k)$$

where $W_0(x) = \phi(x)$ is the scaling function and $W_1(x) = \psi(x)$ is the wavelet function.

For example for the Haar wavelet we have

$$N = 1, h(0) = h(1) = \frac{1}{\sqrt{2}}$$

and

$$g(0) = -g(1) = \frac{1}{\sqrt{2}}$$

The equations become

$$W_{2n}(x) = W_n(2x) + W_n(2x-1)$$

and

$$(W_{2n+1}(x) = W_n(2x) - W_n(2x-1))$$

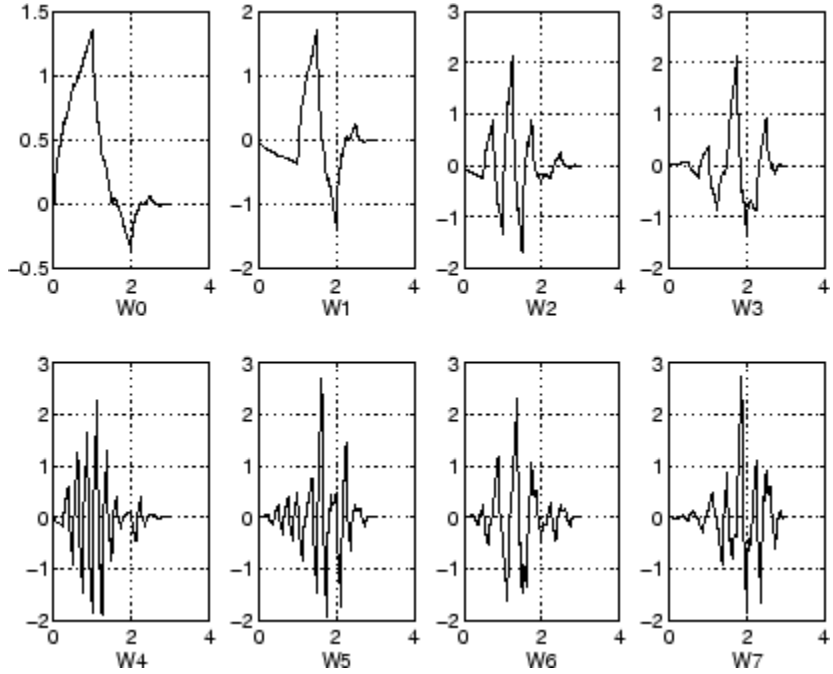
$W_0(x) = \phi(x)$ is the `haar` scaling function and $W_1(x) = \psi(x)$ is the `haar` wavelet, both supported in [0,1].

Then we can obtain W_{2n} by adding two 1/2-scaled versions of W_n with distinct supports [0,1/2] and [1/2,1], and obtain W_{2n+1} by subtracting the same versions of W_n.

Starting from more regular original wavelets, using a similar construction, we obtain smoothed versions of this system of *W*-functions, all with support in the interval [0, 2*N*-1].

Examples

```
% Compute the db2 Wn functions for n = 0 to 7, generating
% the db2 wavelet packets.
[wp,x] = wpfun('db2',7);

% Using some plotting commands,
% the following figure is generated.
```



References

Coifman, R.R.; M.V. Wickerhauser (1992), “Entropy-based Algorithms for best basis selection,” *IEEE Trans. on Inf. Theory*, vol. 38, 2, pp. 713–718.

Meyer, Y. (1993), *Les ondelettes. Algorithmes et applications*, Colin Ed., Paris, 2nd edition. (English translation: *Wavelets: Algorithms and applications*, SIAM).

Wickerhauser, M.V. (1991), “INRIA lectures on wavelet packet algorithms,” *Proceedings ondelettes et paquets d'ondes*, 17–21 June, Rocquencourt, France, pp. 31–99.

Wickerhauser, M.V. (1994), *Adapted wavelet analysis from theory to software algorithms*, A.K. Peters.

See Also

`wavefun` | `waveinfo`

Introduced before R2006a

wpjoin

Recompose wavelet packet

Syntax

```
T = wpjoin(T,N)
[T,X] = wpjoin(T,N)
T = wpjoin(T)
T = wpjoin(T,0)
[T,X] = wpjoin(T)
[T,X] = wpjoin(T,0)
```

Description

`wpjoin` is a one- or two-dimensional wavelet packet analysis function.

`wpjoin` updates the wavelet packet tree after the recomposition of a node.

The nodes are numbered from left to right and from top to bottom. The root index is 0.

`T = wpjoin(T,N)` returns the modified wavelet packet tree `T` corresponding to a recomposition of the node `N`.

`[T,X] = wpjoin(T,N)` also returns the coefficients of the node.

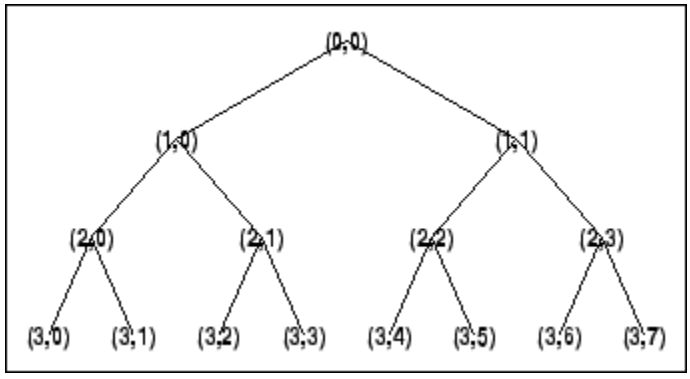
`T = wpjoin(T)` is equivalent to `T = wpjoin(T,0)`.

`[T,X] = wpjoin(T)` is equivalent to `[T,X] = wpjoin(T,0)`.

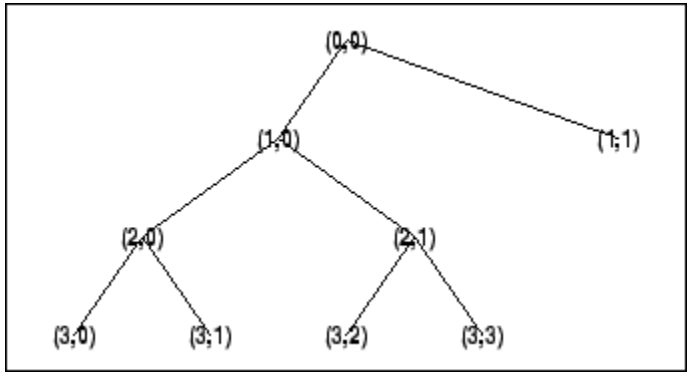
Examples

```
% The current extension mode is zero-padding (see dwtmode).

% Load signal.
load noisdopp; x = noisdopp;

% Decompose x at depth 3 with db1 wavelet packets.
```

```
wpt = wpdec(x,3,'db1');

% Plot wavelet packet tree wpt.
plot(wpt)
```




```
% Recompose packet (1,1) or 2
wpt = wpjoin(wpt,[1 1]);

% Plot wavelet packet tree wpt.
plot(wpt)
```



See Also

wpdec | wpdec2 | wpsplt

Introduced before R2006a

wprcoef

Reconstruct wavelet packet coefficients

Syntax

```
X = wprcoef(T,N)
X = wprcoef(T)
X = wprcoef(T,0)
```

Description

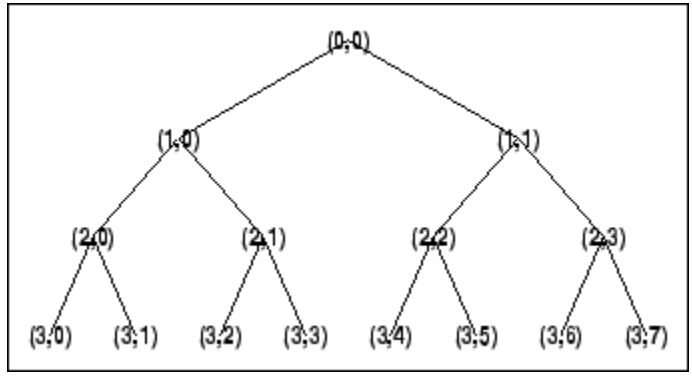
`wprcoef` is a one- or two-dimensional wavelet packet analysis function.

`X = wprcoef(T,N)` computes reconstructed coefficients of the node `N` of the wavelet packet tree *T*.

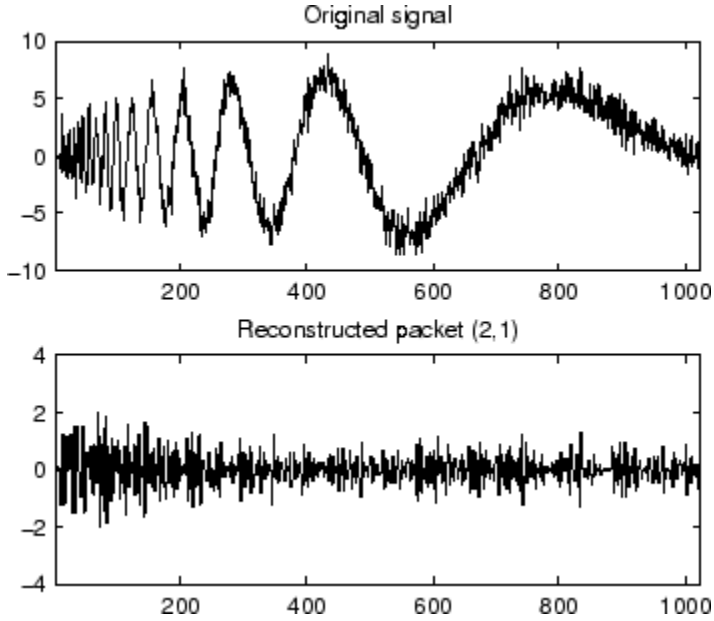
`X = wprcoef(T)` is equivalent to `X = wprcoef(T,0)`.

Examples

```
% The current extension mode is zero-padding (see dwtmode)

% Load signal.
load noisdopp; x = noisdopp;

figure(1); subplot(211);
plot(x); title('Original signal');

% Decompose x at depth 3 with db1 wavelet packets
% using Shannon entropy.
t = wpdec(x,3,'db1','shannon');

% Plot wavelet packet tree.
plot(t)
```




```
% Reconstruct packet (2,1).
rcfs = wprcoef(t,[2 1]);

figure(1); subplot(212);
plot(rcfs); title('Reconstructed packet (2,1)');
```



More About

- "Reconstructing a Signal Approximation from a Node"

See Also

`wpdec` | `wpdec2` | `wprec` | `wprec2`

Introduced before R2006a

wprec

Wavelet packet reconstruction 1-D

Syntax

```
X = wprec(T)
wprec(wpdec(X,'wname'))
```

Description

`wprec` is a one-dimensional wavelet packet analysis function.

`X = wprec(T)` returns the reconstructed vector `X` corresponding to a wavelet packet tree `T`.

`wprec` is the inverse function of `wpdec` in the sense that the abstract statement `wprec(wpdec(X,'wname'))` would give back `X`.

See Also

`wpdec` | `wpdec2` | `wpjoin` | `wprec2` | `wpsplt`

Introduced before R2006a

wprec2

Wavelet packet reconstruction 2-D

Syntax

```
X = wprec2(T)
wprec2(wpdec2(X,'wname'))
```

Description

`wprec2` is a two-dimensional wavelet packet analysis function.

`X = wprec2(T)` returns the reconstructed matrix `X` corresponding to a wavelet packet tree *`T`*.

`wprec2` is the inverse function of `wpdec2` in the sense that the abstract statement `wprec2(wpdec2(X,'wname'))` would give back `X`.

More About

Tips

If *`T`* is obtained from an indexed image analysis or a truecolor image analysis, `X` is an `m`-by-`n` matrix or an `m`-by-`n`-by-3 array, respectively.

For more information on image formats, see the `image` and `imfinfo` reference pages.

See Also

`wpdec` | `wpdec2` | `wprec` | `wpjoin` | `wpsplt`

Introduced before R2006a

wpspectrum

Wavelet packet spectrum

Syntax

```
[SPEC,TIMES,FREQ] = wpspectrum(WPT,Fs)
[...] = wpspectrum(WPT,Fs,'plot')
[...,TNFO] = wpspectrum(...)
```

Description

`[SPEC,TIMES,FREQ] = wpspectrum(WPT,Fs)` returns a matrix of wavelet packet spectrum estimates, `SPEC`, for the binary wavelet packet tree object, `WPT`. `Fs` is the sampling frequency in Hertz. `SPEC` is a 2^J-by-N matrix where J is the level of the wavelet packet transform and N is the length of the time series. `TIMES` is a 1-by-N vector of times and `FREQ` is a 1-by-2^J vector of frequencies.

`[...] = wpspectrum(WPT,Fs,'plot')` displays the wavelet packet spectrum.

`[...,TNFO] = wpspectrum(...)` returns the terminal nodes of the wavelet packet tree in frequency order.

Input Arguments

`WPT`

`WPT` is a binary wavelet packet tree of class `wptree`.

`Fs`

Sampling frequency in Hertz as a scalar of class double.

Default: 1

plot

The string `'plot'` displays the wavelet packet spectrum. Enter `'plot'` after `Fs` to produce a plot of the wavelet packet spectrum.

Output Arguments

SPEC

Wavelet packet spectrum. `SPEC` is a 2^J-by-N matrix where J is the level of the wavelet packet transform and N is the length of node 0 in the wavelet packet tree object.

The frequency spacing between the rows of `SPEC` is $Fs/2^{J+1}$.

TIMES

Time vector. `TIMES` is a vector of times in seconds equal in length to node 0 of the wavelet packet tree object. The time spacing between elements is $1/Fs$.

FREQ

Frequency vector. `FREQ` is a vector of frequencies of length 2^J where J is the level of the wavelet packet tree object. The frequency spacing in `FREQ` is $Fs/2^{J+1}$.

TNFO

Terminal nodes. `TNFO` is a vector of the terminal nodes of the wavelet packet tree object in frequency order.

Examples

Wavelet packet spectrum for signal consisting of two sinusoids with disjoint support:

```
fs = 500;
t = 0:1/fs:4;
y = sin(32*pi*t).*(t<2) + sin(128*pi*t).*(t>=2);
subplot(2,1,1);
plot(t,y);
axis tight
title('Analyzed Signal');
```

```
% Wavelet packet spectrum
level = 6;
wpt = wpdec(y,level,'sym6');
subplot(2,1,2);
[S,T,F] = wpspectrum(wpt,fs,'plot');
```

Wavelet packet spectrum of chirp:

```
fs = 1000;
t = 0:1/fs:2;
% create chirp signal
y = sin(256*pi*t.^2);

% Plot the analyzed signal
subplot(2,1,1);
plot(t,y);
axis tight
title('Analyzed Signal');

% Wavelet packet spectrum
level = 6;
wpt = wpdec(y,level,'sym8');
subplot(2,1,2);
[S,T,F] = wpspectrum(wpt,fs,'plot');
```

More About

Wavelet Packet Spectrum

The wavelet packet spectrum contains the absolute values of the coefficients from the frequency-ordered terminal nodes of the input binary wavelet packet tree. The terminal nodes provide the finest level of frequency resolution in the wavelet packet transform. If J denotes the level of the wavelet packet transform and Fs is the sampling frequency, the terminal nodes approximate bandpass filters of the form:

$$[\frac{nFs}{2^{J+1}},\frac{(n+1)Fs}{2^{J+1}}) \quad n=0,1,2,3,\ldots 2^{J}-1$$

At the terminal level of the wavelet packet tree, the transform divides the interval from 0 to the Nyquist frequency into bands of approximate width $Fs/2^{J+1}$.

Algorithms

`wpspectrum` computes the wavelet packet spectrum as follows:

- Extract the wavelet packet coefficients corresponding to the terminal nodes. Take the absolute value of the coefficients.
- Order the wavelet packet coefficients by frequency ordering.
- Determine the time extent on the original time axis corresponding to each wavelet packet coefficient. Repeat each wavelet packet coefficient to fill in the time gaps between neighboring wavelet packet coefficients and create a vector equal in length to node 0 of the wavelet packet tree object.

- "Wavelet Packet Spectrum"

References

Wickerhauser, M.V. *Lectures on Wavelet Packet Algorithms*, Technical Report, Washington University, Department of Mathematics, 1992.

See Also

`otnodes` | `wpdec`

Introduced in R2010b

wpsplt

Split (decompose) wavelet packet

Syntax

```
T = wpsplt(T,N)
[T,cA,cD] = wpsplt(T,N)
[T,cA,cH,cV,cD] = wpsplt(T,N)
```

Description

`wpsplt` is a one- or two-dimensional wavelet packet analysis function.

`wpsplt` updates the wavelet packet tree after the decomposition of a node.

`T = wpsplt(T,N)` returns the modified wavelet packet tree *T* corresponding to the decomposition of the node `N`.

For a one-dimensional decomposition,

`[T,cA,cD] = wpsplt(T,N)` with `cA` = approximation and `cD` = detail of node `N`.

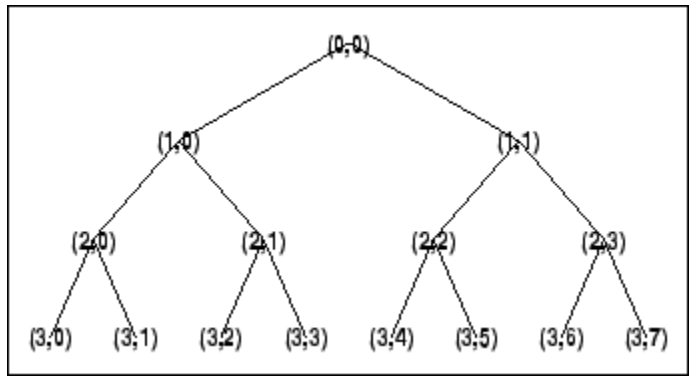
For a two-dimensional decomposition,

`[T,cA,cH,cV,cD] = wpsplt(T,N)` with `cA` = approximation and `cH,cV,c` = horizontal, vertical, and diagonal details of node `N`.

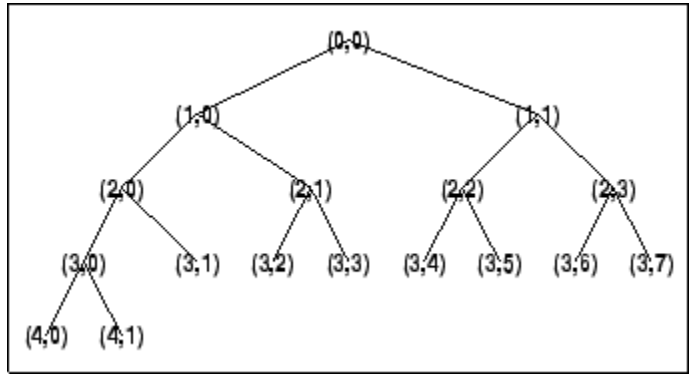
Examples

```
% The current extension mode is zero-padding (see dwtmode).

% Load signal.
load noisdopp;
x = noisdopp;

% Decompose x at depth 3 with db1 wavelet packets.
wpt = wpdec(x,3,'db1');
```

```
% Plot wavelet packet tree wpt.
plot(wpt)
```




```
% Decompose packet (3,0).
wpt = wpsplt(wpt,[3 0]);
% or equivalently wpsplt(wpt,7).

% Plot wavelet packet tree wpt.
plot(wpt)
```



See Also

wavedec | wpdec | wavedec2 | wpdec2 | wpjoin

Introduced before R2006a

wpthcoef

Wavelet packet coefficients thresholding

Syntax

```
NT = wpthcoef(T,KEEPAPP,SORH,THR)
```

Description

`wpthcoef` is a one- or two-dimensional de-noising and compression utility.

`NT = wpthcoef(T,KEEPAPP,SORH,THR)` returns a new wavelet packet tree `NT` obtained from the wavelet packet tree *`T`* by coefficients thresholding.

If `KEEPAPP` = 1, approximation coefficients are not thresholded; otherwise, they can be thresholded.

If `SORH` = `'s'`, soft thresholding is applied; if `SORH = 'h'`, hard thresholding is applied (see `wthresh` for more information).

`THR` is the threshold value.

See Also

`wpdec` | `wpdec2` | `wpdencmp` | `wthresh`

Introduced before R2006a

wptree

WPTREE constructor

Syntax

```
T = wptree(ORDER,DEPTH,X,WNAME,ENT_TYPE,PARAMETER)
T = wptree(ORDER,DEPTH,X,WNAME)
T = wptree(ORDER,DEPTH,X,WNAME,'shannon')
T = wptree(ORDER,DEPTH,X,WNAME,ENT_TYPE,ENT_PAR,USERDATA)
```

Description

`T = wptree(ORDER,DEPTH,X,WNAME,ENT_TYPE,PARAMETER)` returns a complete wavelet packet tree `T`.

`ORDER` is an integer representing the order of the tree (the number of "children" of each non terminal node). `ORDER` must be equal to 2 or 4.

If `ORDER = 2`, `T` is a WPTREE object corresponding to a wavelet packet decomposition of the vector (signal) `X`, at level `DEPTH` with a particular wavelet `WNAME`.

If `ORDER = 4`, `T` is a WPTREE object corresponding to a wavelet packet decomposition of the matrix (image) `X`, at level `DEPTH` with a particular wavelet `WNAME`.

`ENT_TYPE` is a string containing the entropy type and `ENT_PAR` is an optional parameter used for entropy computation (see `wentropy`, `wpdec`, or `wpdec2` for more information).

`T = wptree(ORDER,DEPTH,X,WNAME)` is equivalent to `T = wptree(ORDER,DEPTH,X,WNAME,'shannon')`

With `T = wptree(ORDER,DEPTH,X,WNAME,ENT_TYPE,ENT_PAR,USERDATA)` you may set a userdata field.

The function `wptree` returns a WPTREE object.

For more information on object fields, see the `get` function or type

```
help wptree/get
```

Class WPTREE (Parent class: DTREE)

Fields

'dtree'	DTREE parent object
'wavInfo'	Structure (wavelet information)
'entInfo'	Structure (entropy information)

The wavelet information structure, 'wavInfo', contains

'wavName'	Wavelet name
'Lo_D'	Low Decomposition filter
'Hi_D'	High Decomposition filter
'Lo_R'	Low Reconstruction filter
'Hi_R'	High Reconstruction filter

The entropy information structure, 'entInfo', contains

'entName'	Entropy name
'entPar'	Entropy parameter

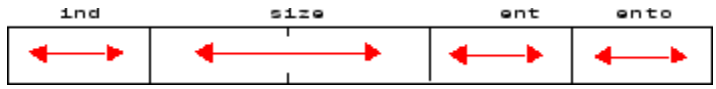
Fields from the DTREE parent object:

'allNI'	All nodes information

'allNI' is an array of size nbnode by 5, which contains

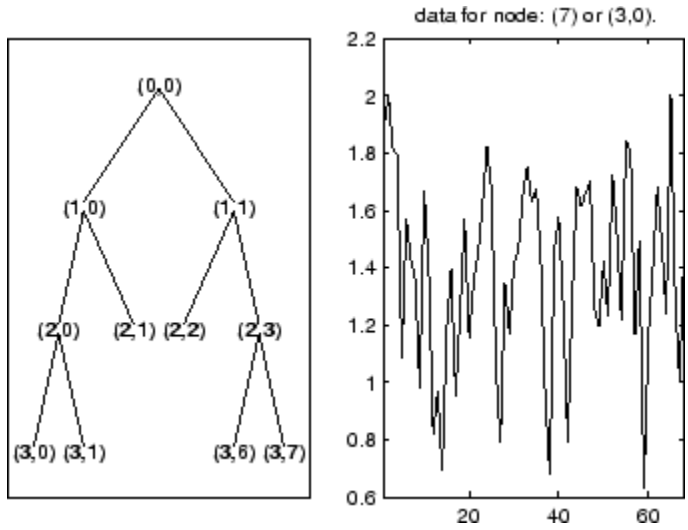
ind	Index
size	Size of data
ent	Entropy
ento	Optimal entropy

Each line is built based on the following scheme:



Examples

```
% Create a wavelet packet tree.
x = rand(1,512);
t = wptree(2,3,x,'db3');
t = wpjoin(t,[4;5]);

% Plot tree t4.
plot(t);

% Click the node (3,0), (see the plot function).
```



See Also

dtree | ntree

Introduced before R2006a

wpviewcf

Plot wavelet packets colored coefficients

Syntax

```
wpviewcf(T,CMODE)
wpviewcf(T,CMODE,NBCOL)
```

Description

`wpviewcf(T,CMODE)` plots the colored coefficients for the terminal nodes of the tree `T`.

`T` is a wavelet packet tree and `CMODE` is an integer, which represents the color mode. The color modes are listed in the table below.

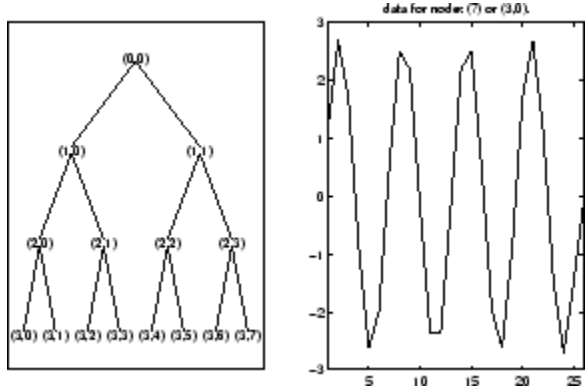
Color Mode	Description
1	Frequency order – Global coloration – Absolute values
2	Frequency order – By level – Absolute values
3	Frequency order – Global coloration – Values
4	Frequency order – By level coloration – Values
5	Natural order – Global coloration – Absolute values
6	Natural order – By level – Absolute values
7	Natural order – Global coloration – Values
8	Natural order – By level coloration – Values

`wpviewcf(T,CMODE,NBCOL)` uses `NBCOL` colors.

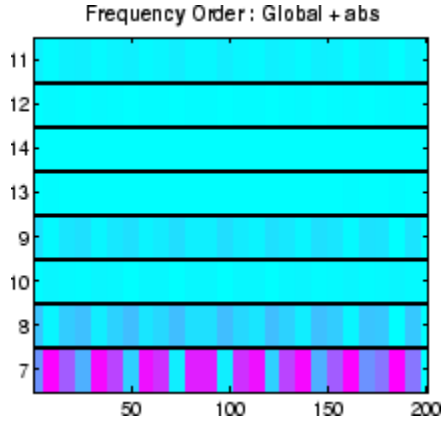
Examples

```
% Create a wavelet packet tree.
x = sin(8*pi*[0:0.005:1]);
t = wpdec(x,3,'db1');
```

```
% Plot tree t.
% Click the node (3,0), (see the plot function)
plot(t);
```




```
% Plot the colored wavelet packet coefficients.
wpviewcf(t,1);
```



See Also

wpdec

Introduced before R2006a

wrcoef

Reconstruct single branch from 1-D wavelet coefficients

Syntax

```
X = wrcoef('type',C,L,'wname',N)
X = wrcoef('type',C,L,Lo_R,Hi_R,N)
X = wrcoef('type',C,L,'wname')
X = wrcoef('type',C,L,Lo_R,Hi_R)
```

Description

`wrcoef` reconstructs the coefficients of a one-dimensional signal, given a wavelet decomposition structure (`C` and `L`) and either a specified wavelet (`'wname'`, see `wfilters` for more information) or specified reconstruction filters (`Lo_R` and `Hi_R`).

`X = wrcoef('type',C,L,'wname',N)` computes the vector of reconstructed coefficients, based on the wavelet decomposition structure `[C,L]` (see `wavedec` for more information), at level `N`. `'wname'` is a string containing the wavelet name.

Argument `'type'` determines whether approximation (`'type' = 'a'`) or detail (`'type' = 'd'`) coefficients are reconstructed. When `'type' = 'a'`, `N` is allowed to be 0; otherwise, a strictly positive number `N` is required. Level `N` must be an integer such that $N \leq$ `length(L)-2`.

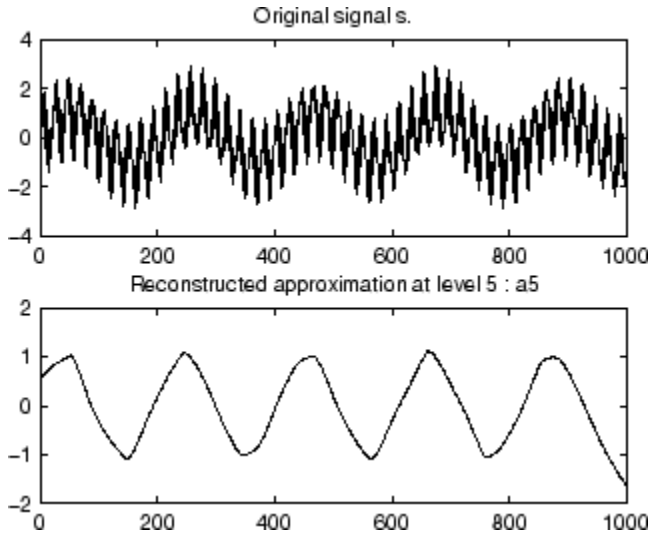
`X = wrcoef('type',C,L,Lo_R,Hi_R,N)` computes coefficients as above, given the reconstruction filters you specify.

`X = wrcoef('type',C,L,'wname')` and `X = wrcoef('type',C,L,Lo_R,Hi_R)` reconstruct coefficients of maximum level `N = length(L)-2`.

Examples

```
% The current extension mode is zero-padding (see dwtmode).
```

```
% Load a one-dimensional signal.
load sumsin; s = sumsin;

% Perform decomposition at level 5 of s using sym4.
[c,l] = wavedec(s,5,'sym4');

% Reconstruct approximation at level 5,
% from the wavelet decomposition structure [c,l].
a5 = wrcoef('a',c,l,'sym4',5);

% Using some plotting commands,
% the following figure is generated.
```



See Also

appcoef | detcoef | wavedec

Introduced before R2006a

wrcoef2

Reconstruct single branch from 2-D wavelet coefficients

Syntax

```
X = wrcoef2('type',C,S,'wname',N)
X = wrcoef2('type',C,S,Lo_R,Hi_R,N)
X = wrcoef2('type',C,S,'wname')
X = wrcoef2('type',C,S,Lo_R,Hi_R)
```

Description

`wrcoef2` is a two-dimensional wavelet analysis function. `wrcoef2` reconstructs the coefficients of an image.

`X = wrcoef2('type',C,S,'wname',N)` computes the matrix of reconstructed coefficients of level `N`, based on the wavelet decomposition structure `[C,S]` (see `wavedec2` for more information).

`'wname'` is a string containing the name of the wavelet (see `wfilters` for more information). If `'type' = 'a'`, approximation coefficients are reconstructed; otherwise if `'type' = 'h'` (`'v'` or `'d'`, respectively), horizontal (vertical or diagonal, respectively) detail coefficients are reconstructed.

Level `N` must be an integer such that $0 \le N \le$ `size(S,1)-2` if `'type' = 'a'` and such that $1 \le N \le$ `size(S,1)-2` if `'type' = 'h'`, `'v'`, or `'d'`.

Instead of giving the wavelet name, you can give the filters.

For `X = wrcoef2('type',C,S,Lo_R,Hi_R,N)`, `Lo_R` is the reconstruction low-pass filter and `Hi_R` is the reconstruction high-pass filter.

`X = wrcoef2('type',C,S,'wname')` or `X = wrcoef2('type',C,S,Lo_R,Hi_R)` reconstruct coefficients of maximum level `N = size(S,1)-2`.

Examples

```
% The current extension mode is zero-padding (see dwtmode).
```

```
% Load an image.
load woman;
% X contains the loaded image.

% Perform decomposition at level 2
% of X using sym5.
[c,s] = wavedec2(X,2,'sym5');

% Reconstruct approximations at
% levels 1 and 2, from the wavelet
% decomposition structure [c,s].
a1 = wrcoef2('a',c,s,'sym5',1);
a2 = wrcoef2('a',c,s,'sym5',2);

% Reconstruct details at level 2,
% from the wavelet decomposition
% structure [c,s].
% 'h' is for horizontal,
% 'v' is for vertical,
% 'd' is for diagonal.
hd2 = wrcoef2('h',c,s,'sym5',2);
vd2 = wrcoef2('v',c,s,'sym5',2);
dd2 = wrcoef2('d',c,s,'sym5',2);

% All these images are of same size sX.
sX = size(X)

sX =
 256 256

sa1 = size(a1)

sa1 =
 256 256

shd2 = size(hd2)

shd2 =
 256 256
```

More About

Tips

If `C` and `S` are obtained from an indexed image analysis (respectively a truecolor image analysis) then `X` is an `m`-by-`n` matrix (respectively an `m`-by-`n`-by-3 array).

For more information on image formats, see the reference pages of `image` and `imfinfo` functions.

See Also

`appcoef2` | `detcoef2` | `wavedec2`

Introduced before R2006a

wrev

Flip vector

Syntax

```
Y = wrev(X)
```

Description

`wrev` is a general utility.

`Y = wrev(X)` reverses the vector `X`.

Examples

```
v = [1 2 3];
wrev(v)
wrev(v')
```

See Also

`fliplr` | `flipud`

Introduced before R2006a

write

Write values in WPTREE fields

Syntax

```
T = write(T,'cfs',NODE,COEFS)
T = write(T,'cfs',N1,CFS1,'cfs',N2,CFS2, ...)
```

Description

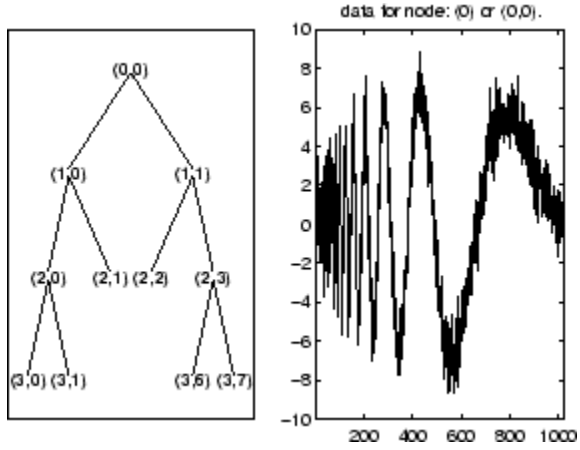
`T = write(T,'cfs',NODE,COEFS)` writes coefficients for the terminal node `NODE`.

`T = write(T,'cfs',N1,CFS1,'cfs',N2,CFS2, ...)` writes coefficients `CFS1, CFS2, ...` for the terminal nodes `N1, N2, ...`.

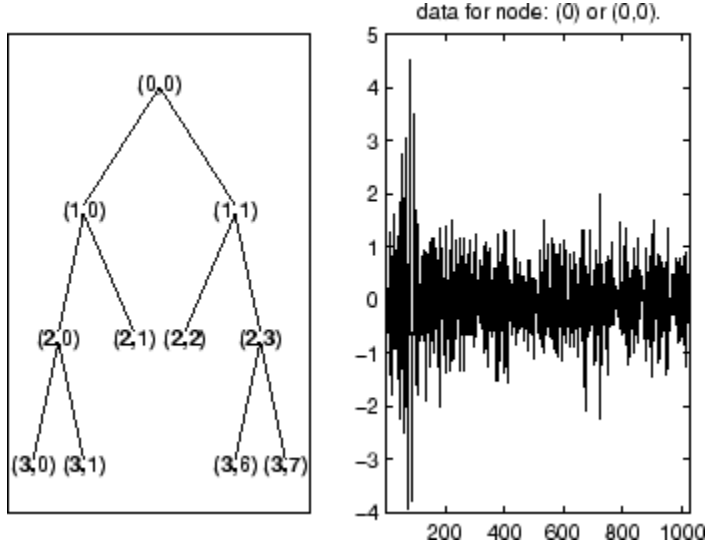
Caution The coefficients values must have the suitable size. You can use `S = read(T,'sizes',NODE)` or `S = read(T,'sizes',[N1;N2; ...])` in order to get those sizes.

Examples

```
% Create a wavelet packet tree.
load noisdopp; x = noisdopp;
t = wpdec(x,3,'db3');
t = wpjoin(t,[4;5]);

% Plot tree t and click the node (0,0) (see the plot function).
plot(t);
```




```
% Write values.
sNod = read(t,'sizes',[4,5,7]);
cfs4 = zeros(sNod(1,:));
cfs5 = zeros(sNod(2,:));
cfs7 = zeros(sNod(3,:));
t = write(t,'cfs',4,cfs4,'cfs',5,cfs5,'cfs',7,cfs7);

% Plot tree t and click the node (0,0) (see the plot function).
plot(t)
```



See Also

`disp` | `get` | `read` | `set`

Introduced before R2006a

wscalogram

Scalogram for continuous wavelet transform

Syntax

```
SC = wscalogram(TYPEPLOT,COEFS)
SC = wscalogram(TYPEPLOT,COEFS,'PropName1',PropVal1,...)
```

Description

`SC = wscalogram(TYPEPLOT,COEFS)` computes the scalogram `SC` which represents the percentage of energy for each coefficient. `COEFS` is the matrix of the continuous wavelet coefficients (see `cwt`).

The scalogram is obtained by computing:

```
S = abs(coefs.*coefs); SC = 100*S./sum(S(:))
```

When `TYPEPLOT` is equal to `'image'`, a scaled image of scalogram is displayed. When `TYPEPLOT` is equal to `'contour'`, a contour representation of scalogram is displayed. Otherwise, the scalogram is returned without plot representation.

`SC = wscalogram(TYPEPLOT,COEFS,'PropName1',PropVal1,...)` allows you to modify some properties. The valid choices for `PropName` are:

`'scales'`	Scales used for the CWT.
`'ydata'`	Signal used for the CWT.
`'xdata'`	*x* values corresponding to the signal values.
`'power'`	Positive real value. Default value is zero.

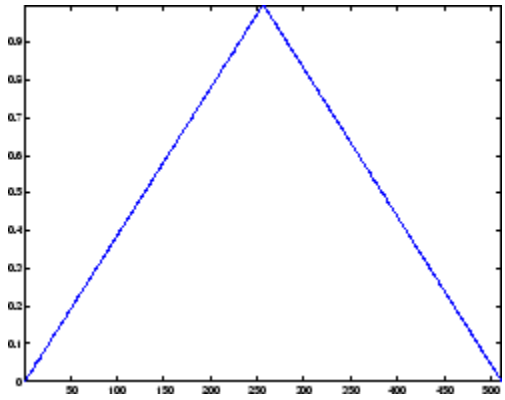
If `power > 0`, coefficients are first normalized

```
coefs(k,:) = coefs(k,:)/(scales(k)^power)
```

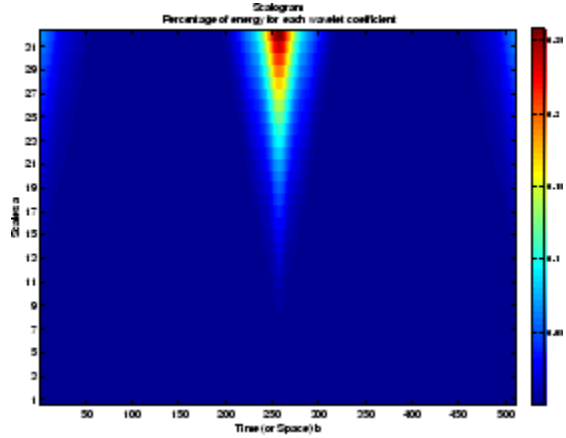
and then the scalogram is computed as explained above.

Examples

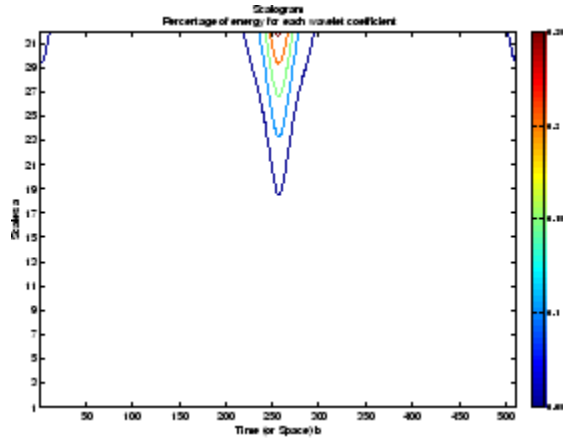
```
% Compute signal s
t = linspace(-1,1,512);
s = 1-abs(t);

% Plot signal s
figure;
plot(s), axis tight
```




```
% Compute coefficients COEFS using cwt
COEFS = cwt(s,1:32,'cgau4');

% Compute and plot the scalogram (image option)
figure;
SC = wscalogram('image',COEFS);
```




```
% Compute and plot the scalogram (contour option)
figure;
SC = wscalogram('contour',COEFS);
```



See Also

cwt

Introduced in R2008a

wsst

Wavelet synchrosqueezed transform

Syntax

```
sst = wsst(x)
[sst,f] = wsst(x)
[___] = wsst(x,fs)
[___] = wsst(x,ts)
[___] = wsst(___,wav)
wsst(___)
[___] = wsst(___,Name,Value)
```

Description

`sst = wsst(x)` returns the wavelet synchrosqueezed transform, `sst`, which you use to examine data in the time-frequency plane. The synchrosqueezed transform has reduced energy smearing when compared to the continuous wavelet transform. The input, `x`, must be a 1-D real-valued signal with at least four samples. `wsst` computes the synchrosqueezed transform using the analytic Morlet wavelet.

`[sst,f] = wsst(x)` returns a vector of frequencies, `f`, in cycles per sample. The frequencies correspond to the rows of `sst`.

`[___] = wsst(x,fs)` computes the sychrosqueezed transform using the specified sampling frequency, `fs`, in Hz, to compute the synchrosqueezed transform. If you specify an `f` output, `wsst` returns the frequencies in Hz. You can use any previous combination of output values.

`[___] = wsst(x,ts)` uses a `duration` `ts` with a positive, scalar input, as the sampling interval. The duration can be in years, days, hours, minutes, or seconds. If you specify `ts` and the `f` output, `wsst` returns the frequencies in `f` in cycles per unit time, where the time unit is derived from specified duration.

`[___] = wsst(___,wav)` uses the analytic wavelet specified by `wav` to compute the synchrosqueezed transform. Valid values are `'amor'` and `'bump'`, which specify the analytic Morlet and bump wavelet, respectively.

`wsst( ___ )` with no output arguments plots the synchrosqueezed transform as a function of time and frequency. If you do not specify a sampling frequency, `fs`, or interval, `ts`, the synchrosqueezed transform is plotted in cycles per sample. If you specify a sampling frequency, the synchrosqueezed transform is plotted in Hz. If you specify a sampling interval using a duration , the plot is in cycles per unit time. The time units are derived from the duration.

`[ ___ ] = wsst( ___ ,Name,Value)` returns the synchrosqueezed transform with additional options specified by one or more `Name,Value` pair arguments.

Examples

Synchrosqueezed Transform of Speech Signal

Obtain the wavelet synchrosqueezed transform of % a speech sample using default values.

```
load mtlb;
sst = wsst(mtlb);
```

Synchrosqueezed Transform and Reconstruction of Speech Signal

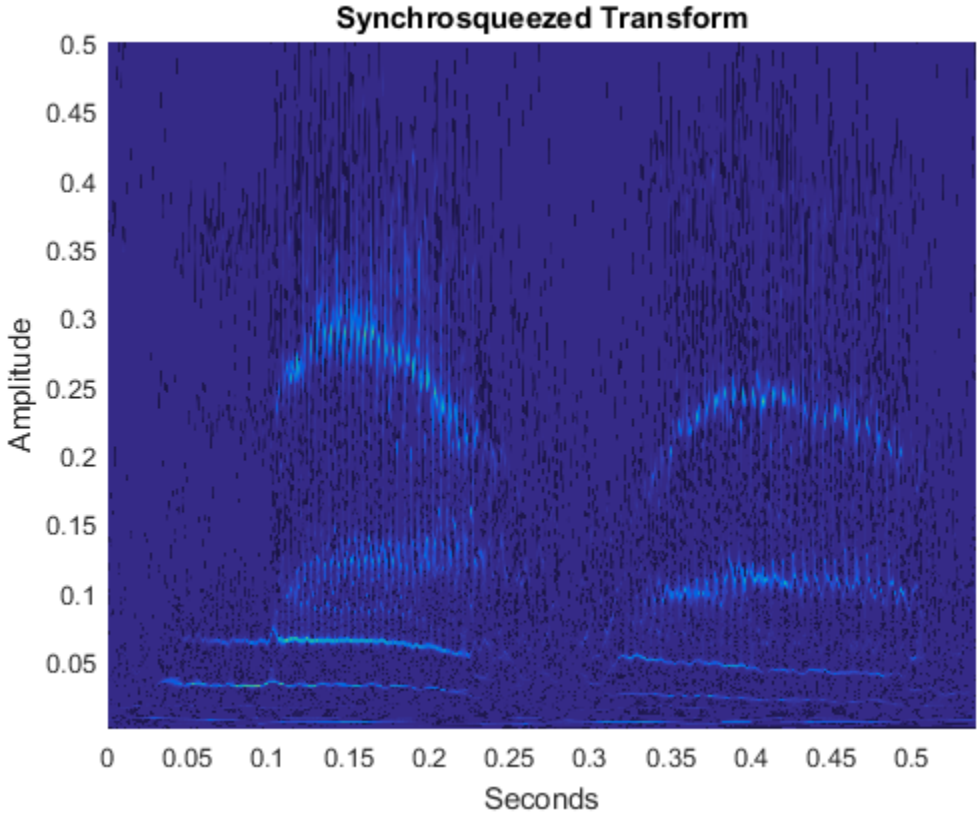
Obtain the wavelet synchrosqueezed transform of a speech signal and compare the original and reconstructed signals.

Load the speech signal and obtain its synchrosqueezed transform.

```
load mtlb;
soundsc(mtlb);
dt = 1/Fs;
t = 0:dt:numel(mtlb)*dt-dt;
[sst,f] = wsst(mtlb);
```

Plot the synchroqueezed transform.

```
pcolor(t,f,abs(sst));
shading interp
xlabel('Seconds'); ylabel('Amplitude');
title('Synchrosqueezed Transform');
```



Obtain the inverse synchroqueezed transform and play the reconstructed speech signal.

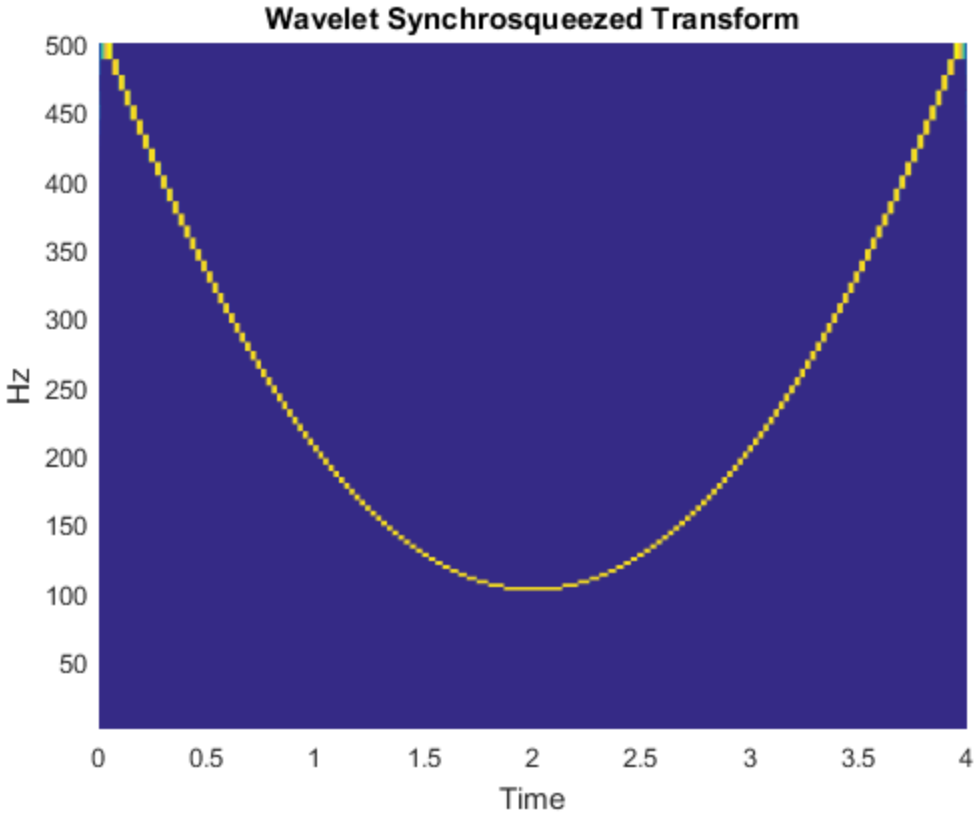
```
xrec = iwsst(sst);
soundsc(xrec);
```

Synchrosqueezed Transform of Quadratic Chirp

Obtain and plot the wavelet synchrosqueezed transform of a quadratic chirp. The chirp is sampled at 1000 Hz.

```
load quadchirp;
[sst,f] = wsst(quadchirp,1000);
hp = pcolor(tquad,f,abs(sst));
hp.EdgeColor = 'none';
```

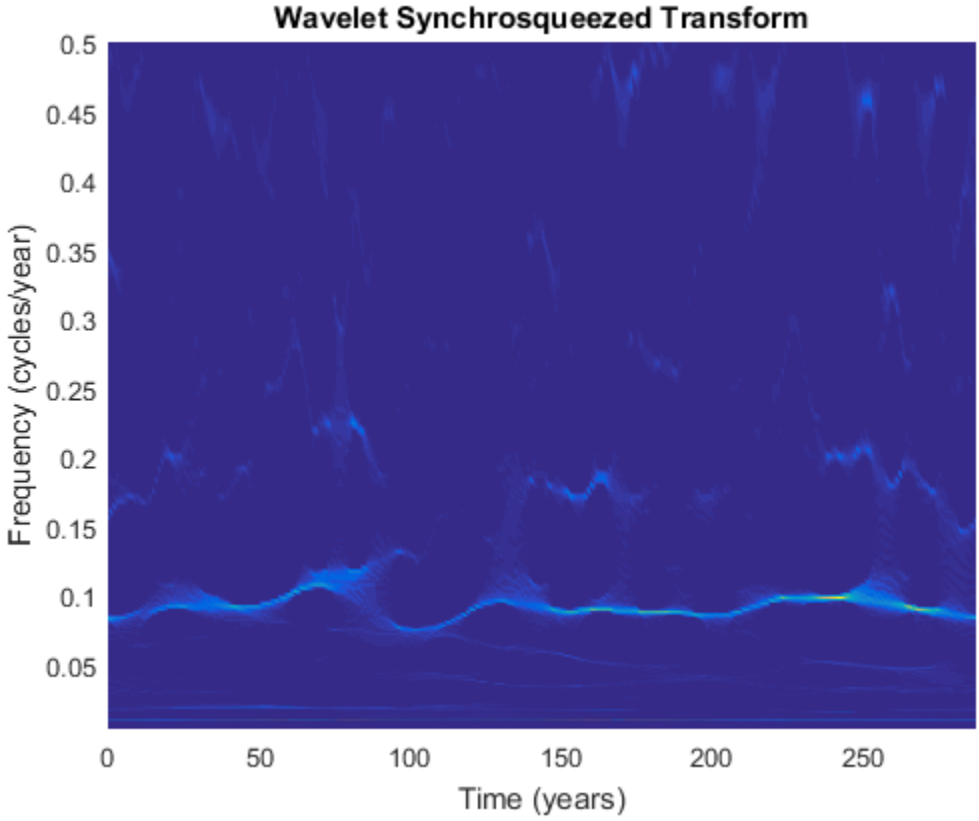
```
title('Wavelet Synchrosqueezed Transform');
xlabel('Time'); ylabel('Hz');
```



Synchrosqueezed Transform of Sunspot Data

This example shows how to obtain the wavelet synchrosqueezed transform of the sunspot data. Specify the sampling interval to be 1 year.

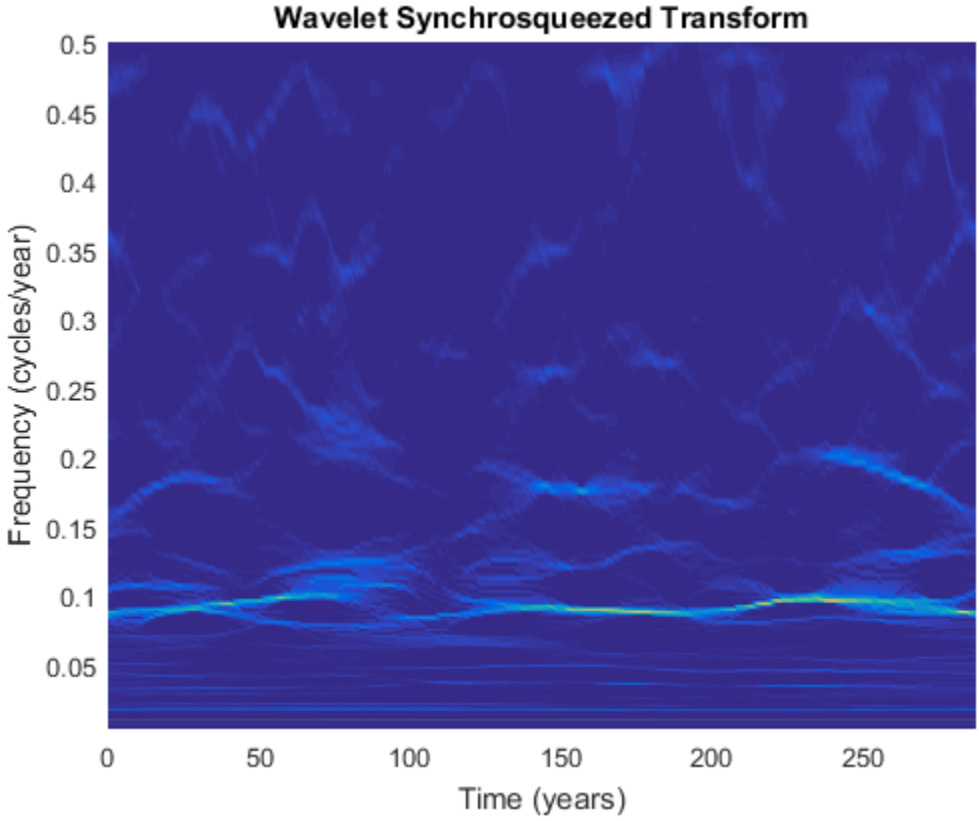
```
load sunspot.dat;
wsst(sunspot(:,2),years(1))
```



Synchrosqueezed Transform of Sunspot Data Using Bump Wavelet

Obtain and plot the wavelet synchrosqueezed transform of sunspot data using the bump wavelet. Specify the sampling interval to be 1 for one sample per year.

```
load sunspot.dat;
wsst(sunspot(:,2),years(1),'bump')
```



- "Time-Frequency Reassignment and Mode Extraction with Synchrosqueezing"

Input Arguments

`x` — Input signal

row or column vector of real values

Input signal, specified as a row or column vector. `x` must be a 1-D, real-valued signal with at least four samples.

`fs` — Sampling frequency

positive scalar

Sampling frequency, specified as a positive scalar.

`ts` — Sampling interval

duration with positive scalar input

Sampling interval, also known as the sampling period, specified as a `duration` with positive scalar input. Valid durations are `years`, `days`, `hours`, `seconds`, and `minutes`. You cannot use calendar durations (`caldays`, `calweeks`, `calmonths`, `calquarters`, or `calyears`). You cannot specify both `ts` and `fs`.

Example: `sst = wsst(x,hours(12))`

`wav` — Analytic wavelet

`'amor'` (default) | `'bump'`

Analytic wavelet used to compute the synchrosqueezed transform, specified as `'amor'` or `'bump'`. These strings specify the analytic Morlet wavelet and bump wavelet, respectively.

Name-Value Pair Arguments

Specify optional comma-separated pairs of `Name,Value` arguments. `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside single quotes (`' '`). You can specify several name and value pair arguments in any order as `Name1,Value1,...,NameN,ValueN`.

Example: `'VoicesPerOctave',26`

`'VoicesPerOctave'` — Number of voices per octave

32 (default) | integer from 10 to 48

Number of voices per octave to use in the synchrosqueezed transform, specified as the comma-separated pair consisting of `'VoicesPerOctave'` and an integer from 10 to 48. The product of the number of voices per octave and the number of octaves is the number of scales. The number of octaves depends on the size of the input `x` and is `floor(log2(numel(x)))-1`.

`'ExtendSignal'` — Extend input signal symmetrically

`false` (default) | `true`

Option to extend the input signal symmetrically, specified as the comma-separated pair consisting of `'ExtendSignal'` and either `false` or `true`. Extending the signal

symmetrically can mitigate boundary effects. If you specify `false`, then the signal is not extended. If you specify `true`, then the signal is extended.

Output Arguments

`sst` — Synchrosqueezed transform

matrix

Synchrosqueezed transform, returned as a matrix. By default, the synchrosqueezed transform uses `floor(log2(numel(x)))-1` octaves, 32 voices per octave, and the analytic Morlet wavelet. `sst` is an *Na*-by-*N* matrix where *Na* is the number of scales, and *N* is the number of samples in `x`. The default number of scales is `32*(floor(log2(numel(x)))-1)`.

`f` — Frequencies

vector

Frequencies of the synchrosqueezed transform, returned as a vector. The frequencies correspond to the rows of the `sst`. If you do not specify `fs` or `ts`, the frequencies are in cycles per sample. If you specify `fs`, the frequencies are in Hz. If you specify `ts`, the frequencies are in cycles per unit time. The length of the frequency vector is the same as the number of `sst` rows. If you specify `ts` as the sampling interval, `ts` is used to compute the scale-to-frequency conversion for `f`.

More About

- “Wavelet Synchrosqueezing”

References

[1] I. Daubechies, I., J. Lu, and H. T. Wu. "Synchrosqueezed Wavelet Transforms: an Empricial Mode Decomposition-like Tool", *Applied and Computational Harmonic Analysis*. Vol. 30(2), pp. 243–261.

[2] Thakur, G., E. Brevdo, N. S. Fučkar, and H. T. Wu. "The Synchrosqueezing algorithm for time-varying spectral analysis: robustness properties and new paleoclimate applications." *Signal Processing*. Vol. 93, pp. 1079–1094.

See Also

days | duration | hours | iwsst | minutes | seconds | wsstridge | years

Introduced in R2016a

wsstridge

Time-frequency ridges from wavelet synchrosqueezing

Syntax

```
fridge = wsstridge(sst)
[fridge,iridge] = wsstridge(sst)
[ ___ ] = wsstridge(sst,penalty)
[ ___ ] = wsstridge( ___ ,f)
[ ___ ]= wsstridge( ___ ,Name,Value)
```

Description

`fridge = wsstridge(sst)` extracts the maximum energy time-frequency ridge in cycles per sample from the wavelet synchrosqueezed transform, `sst`. The `sst` input is the output of `wsst`. Each ridge is a separate signal mode.

`[fridge,iridge] = wsstridge(sst)` returns in `iridge` the row indices of `sst`. The row indices are the maximum time-frequency ridge at each sample. Use `iridge` to reconstruct the signal mode along a time-frequency ridge using `iwsst`.

`[ ___ ] = wsstridge(sst,penalty)` multiplies the squared distance between frequency bins by the `penalty` value. You can include any of the ouitput arguments from previous syntaxes.

`[ ___ ] = wsstridge( ___ ,f)` returns the maximum energy time-frequency ridge in cycles per unit time based on the `f` input frequency vector. `f` is the frequency output of `wsst`. The `f` input and `fridge` output have the same units.

`[ ___ ]= wsstridge( ___ ,Name,Value)` returns the time-frequency ridge with additional options specified by one or more `Name,Value` pair arguments.

Examples

Extract Time-Frequency Ridge from Chirp Signal

Obtain the wavelet synchrosqueezed transform of a quadratic chirp and extract the maximum time-frequency ridge, in `fridge`, and the associated row indices, in `iridge`.

Load the chirp signal and obtain its synchrosqueezed transform.

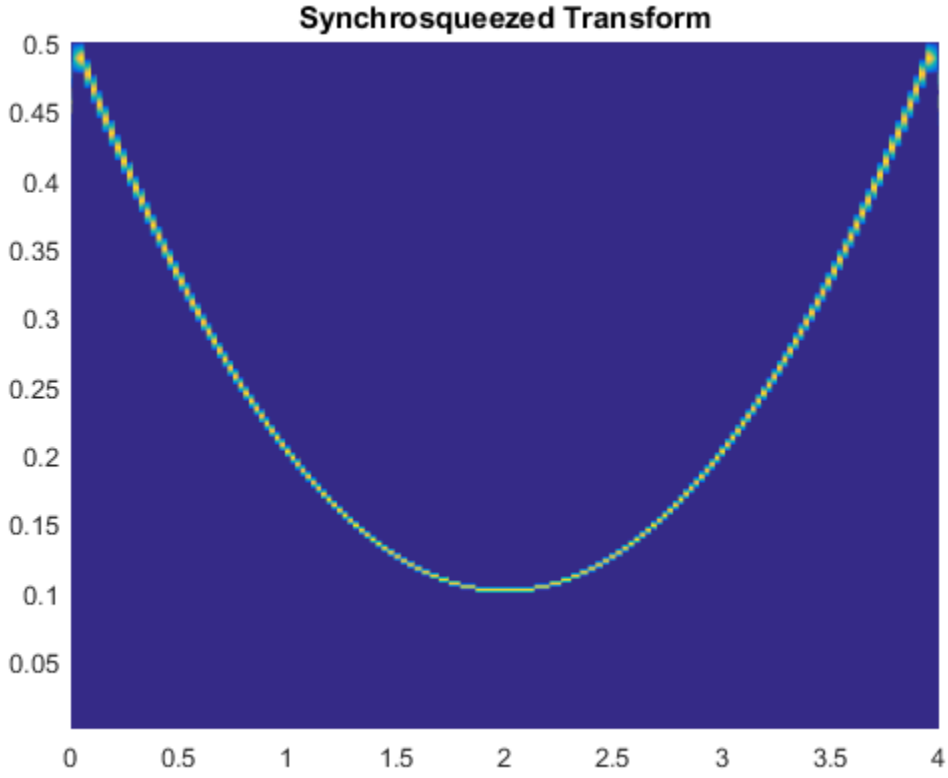
```
load quadchirp;
[sst,f] = wsst(quadchirp);
```

Extract the maximum time-frequency ridge.

```
[fridge,iridge] = wsstridge(sst);
```

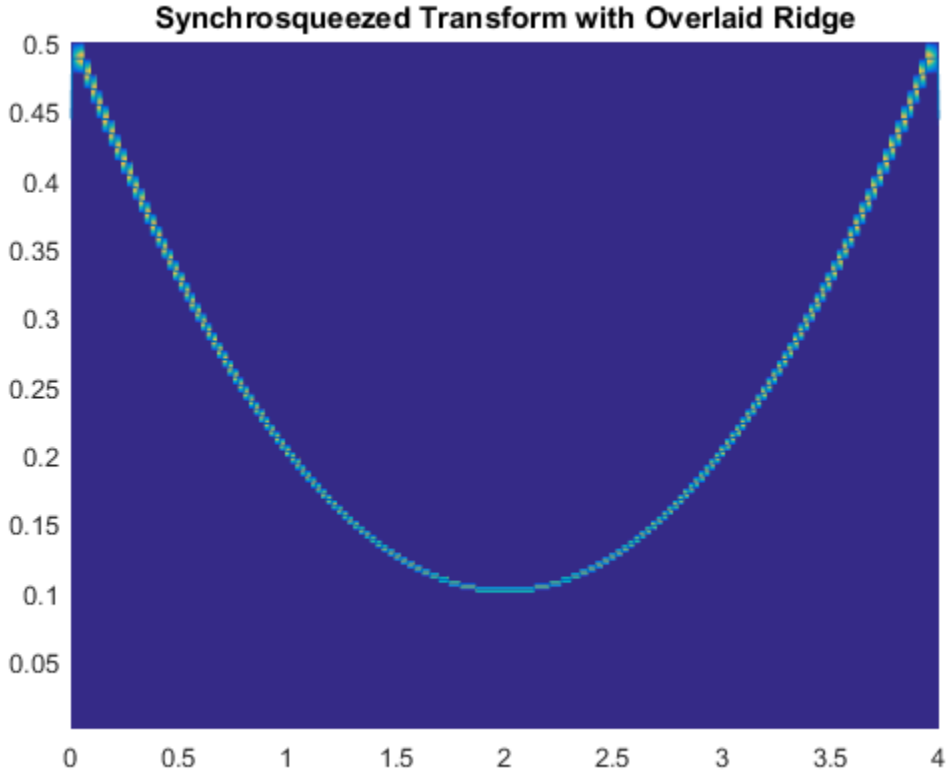
Plot the synchrosqueezed transform.

```
pcolor(tquad,f,abs(sst))
shading interp
title('Synchrosqueezed Transform')
```



Overlay the plot of the maximum energy frequency ridge.

```
hold on
plot(tquad,fridge)
title('Synchrosqueezed Transform with Overlaid Ridge')
```

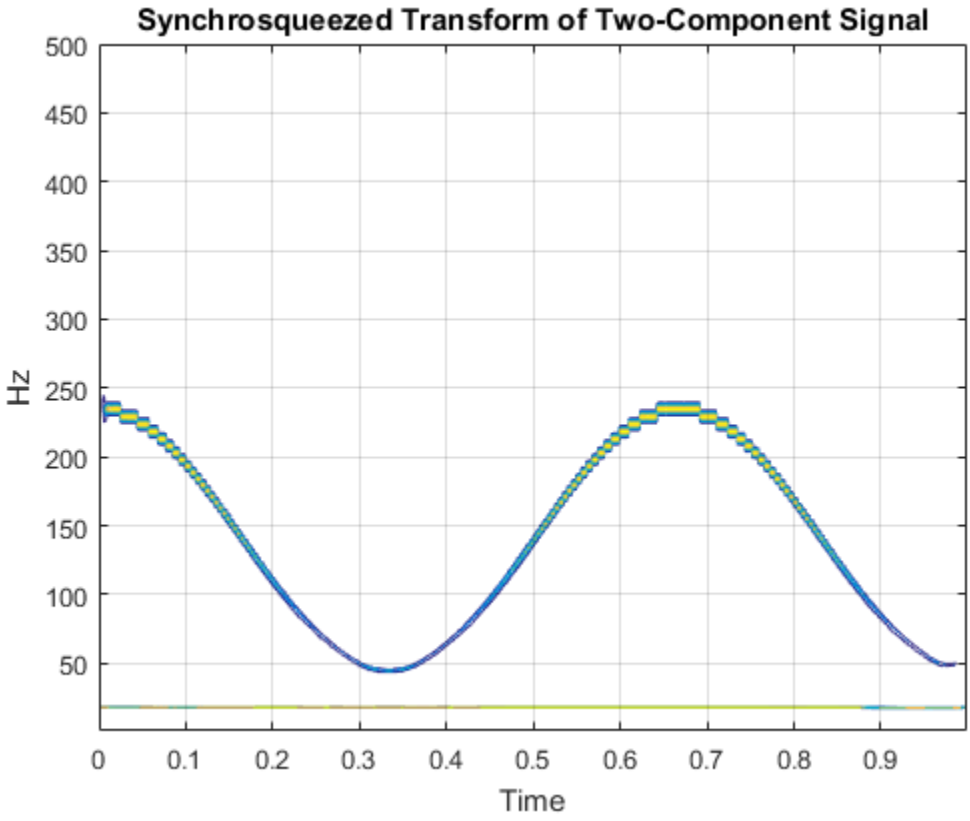


Extract Time-Frequency Ridge from Multicomponent Signal

Extract the two highest energy modes from a multicomponent signal.

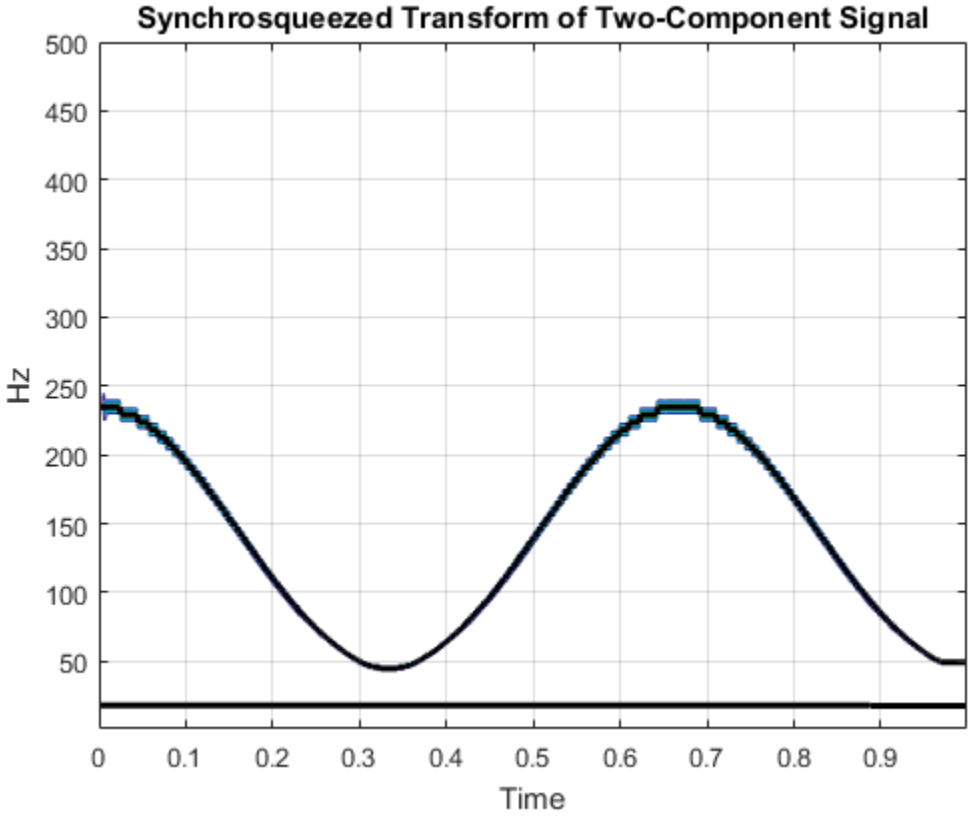
Obtain and plot the wavelet synchrosqueezed transform.

```
load multicompsig;
sig = sig1+sig2;
[sst,F] = wsst(sig,sampfreq);
contour(t,F,abs(sst));
xlabel('Time'); ylabel('Hz');
grid on;
title('Synchrosqueezed Transform of Two-Component Signal');
```



Using a penalty of 10, extract the two highest energy modes and plot the result.

```
[fridge,iridge] = wsstridge(sst,10,F,'NumRidges',2);
hold on;
plot(t,fridge,'k','linewidth',2);
```



- "Time-Frequency Reassignment and Mode Extraction with Synchrosqueezing"

Input Arguments

`sst` — Synchrosqueezed transform

matrix

Synchrosqueezed transform, specified as a matrix.

`penalty` — Frequency bins scaling penalty

0 (default) | nonnegative scalar

Frequency bins scaling penalty, specified as a nonnegative scalar. This input penalizes changes in frequency by multiplying the penalty value by the squared distance between frequency bins. Use a penalty term when you extract multiple ridges, or when you have a single modulated component in additive noise. The penalty term prevents jumps in frequency that occur when the region of highest energy in the time-frequency plane changes abruptly.

`f` — Synchrosqueezed transform frequencies

vector

Synchrosqueezed transform frequencies corresponding to the rows of the synchrosqueezed transform, which is the vector output of `wsst`. The number of elements in the frequency vector is equal to the number of rows in the `sst` input.

Name-Value Pair Arguments

Specify optional comma-separated pairs of `Name,Value` arguments. `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside single quotes (`' '`). You can specify several name and value pair arguments in any order as `Name1,Value1,...,NameN,ValueN`.

Example: `'NumRidges',3`

`'NumRidges'` — Number of highest energy time-frequency ridges

1 (default) | positive integer

Number of highest energy time-frequency ridges to extract, specified as the comma-separated pair consisting of `'NumRidges'` and a positive integer. If this integer is greater than 1, `wsstridge` iteratively determines the maximum energy time-frequency ridge by removing the previously computed ridges and the default or specified `'NumFrequencyBins'` on either side of each ridge bin.

`'NumFrequencyBins'` — Number of frequency bins to remove

4 (default) | positive integer

Number of frequency bins to remove from synchrosqueezed transform `sst` when extracting multiple ridges, specified as the comma-separated pair consisting of `'NumFrequencyBins'` and a positive integer. This integer must be less than or equal to `round(size(sst,1)/4)`. You can specify the nunber of frequency bins to remove only if you extract more than one ridge. After extracting the highest energy time-frequency ridge, `wsstridge` removes the `sst` values corresponding to the `iridge` indices at each

time step. The energy is removed along the time-frequency ridge extended on both sides of the `iridge` index by the specified number of frequency bins. If the index of the extended time-frequency ridge exceeds the number of frequency bins at any time step, `wsstridge` truncates the removal region at the first or last frequency bin. To specify `'NumFrequencyBins'`, you must specify `'NumRidges'`.

Output Arguments

`fridge` — Time-frequency ridge frequencies

vector or matrix

Time-frequency ridge frequencies, returned as a vector or matrix. The frequencies correspond to the time-frequency ridge at each time step. `fridge` is an *N*-by-`nr` matrix where *N* is the number of time samples (columns) in `sst` and `nr` is the number of ridges. The first column of the matrix contains the frequencies for the maximum energy time-frequency ridge in `sst`. Subsequent columns contain the frequencies for the time-frequency ridges in decreasing energy order. By default, `fridge` contains frequencies in cycles per sample.

`iridge` — Time-frequency ridge indices

vector or matrix

Time-frequency ridge row indices of `sst`, returned as a vector or matrix. The row indices in `iridge` correspond to the row index of the maximum time-frequency ridge for each `sst` column. `iridge` is an *N*-by-`nr` matrix where *N* is the number of time samples (columns) in `sst`, and `nr` is the number of ridges. The first column of the matrix contains the indices for the maximum energy time-frequency ridge in `sst`. Subsequent columns contain the indices for the time-frequency ridges in decreasing energy order.

More About

Algorithms

`wsstridge` uses a penalized forward-backward greedy algorithm to extract the maximum energy time-frequency ridges from the wavelet synchrosqueezed transform matrix. The algorithm finds the maximum time-frequency ridge by minimizing `-ln(E)` at each time point, where `E` is the absolute value of the synchrosqueezed transform. The is equivalent to maximizing the value of `E`. The algorithm optionally constrains jumps

in frequency with a penalty that is proportional to the square of the distance between frequency bins.

The following example illustrates the time-frequency ridge algorithm using a penalty that is 2 times the squared distance between frequency bins. This simple synchrosqueezed transform matrix has three frequency bins and three time steps. The second row represents a sine wave.

1. Obtain the complex matrix output, y, of the wavelet synchroqueezed transform using `wsst`. Compute the `(-ln(|y|))`. Suppose you have this `(-ln(|y|))` matrix output,

   ```
   1   4   4
   2   2   2
   5   5   4
   ```

2. Update the value for the (1,2) element.

   a. Leave the values the at the first time point unaltered. Begin the algorithm with the (1,2) element of the matrix, which presents the first frequency bin at the 2nd time point. Penalize the values in the first column based on their distance from the (1,2) element. Applying the penalty to the first column produces

   ```
   original value + penalty(distance squared)

   1 + 2(0*0) =  1
   2 + 2(1*1) =  4
   5 + 2(2*2)=  13

    1   4
    4   2
   13   5
   ```

   The minimum value of the first column is 1, which is in bin 1.

   b. Add the minimum value in column 1 to the current bin value, 4. The updated value for (1,2) becomes 5, which came from bin 1.

3. Update the values for the remaining elements in column 2.

   Recompute the original column 1 values with the penalty factor using the same process as in Step 2a. Obtain the remaining second column values using the same process as in Step 2b. Repeat Step 2 for the third column. The final matrix is

   $$1 \quad 5_{(1)} \quad 9_{(1)}$$

$$\begin{matrix} 2 & 4_{(2)} & 6_{(2)} \\ 5 & 7_{(2)} & 8_{(2)} \end{matrix}$$

The subscripts indicate the index of the bin in the previous column from which a value came.

4. Starting at the last column of the matrix, find the minimum value. Walk back in time through the matrix by going from the current bin to the origin of that bin at the previous time point. Keep track of the bin indices, which form the path composing the ridge. The algorithm smooths the transition by using the origin bin instead of the bin with the minimum value. For this example, the ridge indices are 2, 2, 2, which matches the energy path of the sine wave in row 2 of the matrix shown in Step 1.

5. If you are extracting multiple ridges, the algorithm removes the first ridge from the synchrosqueezed transform and repeats the process.

- "Wavelet Synchrosqueezing"

References

[1] I. Daubechies, I., J. Lu, and H. T. Wu. "Synchrosqueezed Wavelet Transforms: an Empricial Mode Decomposition-like Tool", *Applied and Computational Harmonic Analysis*. Vol. 30(2), pp. 243–261.

[2] Thakur, G., E. Brevdo, N. S. Fučkar, and H. T. Wu. "The Synchrosqueezing algorithm for time-varying spectral analysis: robustness properties and new paleoclimate applications." *Signal Processing*. Vol. 93, pp. 1079–1094.

See Also

`iwsst` | `wsst`

Introduced in R2016a

wtbo

WTBO constructor

Syntax

```
OBJ = wtbo
OBJ = wtbo(USERDATA)
```

Description

`OBJ = wtbo` returns a WTBO object. Any object in the Wavelet Toolbox software is parented by a WTBO object.

With `OBJ = wtbo(USERDATA)` you can set a userdata field.

Class WTBO (Parent class: none)

Fields

`wtboInfo`	Object information (not used in the current version of the toolbox)
`ud`	Userdata field

Introduced before R2006a

wtbxmngr

Wavelet Toolbox manager

Syntax

```
wtbxmngr(OPTION)
V = wtbxmngr('version')
```

Description

`wtbxmngr` or `wtbxmngr('version')` displays the current version of Wavelet Toolbox software.

`wtbxmngr(OPTION)` sets a toolbox option. Available options are

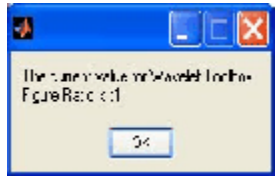
Option	Description
`'LargeFonts'`	Sets the size of future-created figures to use large fonts.
`'DefaultSize'`	Restores the default figure size for future- created figures.
`'FigRatio'`	Returns the current figure ratio value.
`'FigRatio',ratio`	Changes the size of future-created figures by multiplying the default size by the specified ratio, where ratio must be between 0.75 and 1.25.

`V = wtbxmngr('version')` saves the current version of the toolbox to variable `V`.

Examples

```
wtbxmngr('version')

**********************************
**  Wavelet Toolbox Version: V3.1  **
**********************************
```

```
wtbxmngr('FigRatio')      % Display the current figure ratio
wtbxmngr('FigRatio',1.25) % Set the figure ratio to 1.25
wtbxmngr('FigRatio')      % Display the current figure ratio
wtbxmngr('DefaultSize')   % Return to the default figure ratio
```



Introduced before R2006a

wthcoef

1-D wavelet coefficient thresholding

Syntax

```
NC = wthcoef('d',C,L,N,P)
NC = wthcoef('d',C,L,N)
NC = wthcoef('a',C,L)
NC = wthcoef('t',C,L,N,T,SORH)
```

Description

`wthcoef` thresholds wavelet coefficients for the denoising or compression of a 1-D signal.

`NC = wthcoef('d',C,L,N,P)` returns coefficients obtained from the wavelet decomposition structure `[C,L]` (see `wavedec` for more information), by rate compression defined in vectors `N` and `P`. `N` contains the detail levels to be compressed and `P` the corresponding percentages of lower coefficients to be set to zero. `N` and `P` must be of same length. Vector `N` must be such that $1 \leq$ `N(i)` $\leq$ `length(L)-2`.

`NC = wthcoef('d',C,L,N)` returns coefficients obtained from `[C,L]` by setting all the coefficients of detail levels defined in `N` to zero.

`NC = wthcoef('a',C,L)` returns coefficients obtained by setting approximation coefficients to zero.

`NC = wthcoef('t',C,L,N,T,SORH)` returns coefficients obtained from the wavelet decomposition structure `[C,L]` by soft (if `SORH ='s'`) or hard (if `SORH ='h'`) thresholding (see `wthresh` for more information) defined in vectors `N` and `T`. `N` contains the detail levels to be thresholded and `T` the corresponding thresholds. `N` and `T` must be of the same length.

`[NC,L]` is the modified wavelet decomposition structure.

See Also

`wavedec` | `wthresh`

Introduced before R2006a

wthcoef2

Wavelet coefficient thresholding 2-D

Syntax

```
NC = wthcoef2('type',C,S,N,T,SORH)
NC = wthcoef2('type',C,S,N)
NC = wthcoef2('a',C,S)
NC = wthcoef2('t',C,S,N,T,SORH)
```

Description

`wthcoef2` is a two-dimensional de-noising and compression oriented function.

For `'type' = 'h'` (`'v'` or `'d'`), `NC = wthcoef2('type',C,S,N,T,SORH)` returns the horizontal (vertical or diagonal, respectively) coefficients obtained from the wavelet decomposition structure `[C,S]` (see `wavedec2` for more information), by soft (if `SORH ='s'`) or hard (if `SORH ='h'`) thresholding defined in vectors `N` and `T`. `N` contains the detail levels to be thresholded and `T` the corresponding thresholds. `N` and `T` must be of the same length. The vector `N` must be such that `1 ≤ N(i) ≤ size(S,1)-2`.

For `'type' = 'h'` (`'v'` or `'d'`), `NC = wthcoef2('type',C,S,N)` returns the horizontal (vertical or diagonal, respectively) coefficients obtained from `[C,S]` by setting all the coefficients of detail levels defined in `N` to zero.

`NC = wthcoef2('a',C,S)` returns the coefficients obtained by setting approximation coefficients to zero.

`NC = wthcoef2('t',C,S,N,T,SORH)` returns the detail coefficients obtained from the wavelet decomposition structure `[C,S]` by soft (if `SORH ='s'`) or hard (if `SORH ='h'`) thresholding (see `wthresh` for more information) defined in vectors `N` and `T`. `N` contains the detail levels to be thresholded and `T` the corresponding thresholds which are applied in the three detail orientations. `N` and `T` must be of the same length.

`[NC,S]` is the modified wavelet decomposition structure.

See Also

`wavedec2` | `wthresh`

Introduced before R2006a

wthresh

Soft or hard thresholding

Syntax

```
Y = wthresh(X,SORH,T)
Y = wthresh(X,'s',T)
Y = wthresh(X,'h',T)
```

Description

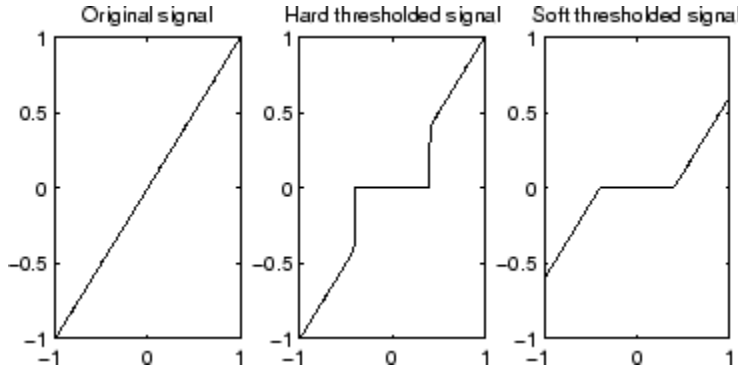
`Y = wthresh(X,SORH,T)` returns the soft (if `SORH = 's'`) or hard (if `SORH = 'h'`) thresholding of the input vector or matrix `X`. *T* is the threshold value.

`Y = wthresh(X,'s',T)` returns $Y = sign(X) \cdot (|X| - T)_+$, soft thresholding is wavelet shrinkage ($(x)_+ = 0$ if $x < 0$; $(x)_+ = x$, if $x \geq 0$).

`Y = wthresh(X,'h',T)` returns $Y = X.1_{(|X| > T)}$, hard thresholding is cruder.

Examples

```
% Generate signal and set threshold.
y = linspace(-1,1,100);
thr = 0.4;

% Perform hard thresholding.
ythard = wthresh(y,'h',thr);

% Perform soft thresholding.
ytsoft = wthresh(y,'s',thr);

% Using some plotting commands,
% the following figure is generated.
```



See Also

wden | wdencmp | wpdencmp

Introduced before R2006a

wthrmngr

Threshold settings manager

Syntax

```
THR = wthrmngr(OPTION,METHOD,VARARGIN)
```

Description

`THR = wthrmngr(OPTION,METHOD,VARARGIN)` returns a global threshold or level dependent thresholds depending on `OPTION`. The inputs, `VARARGIN`, depend on the `OPTION` and `METHOD` values.

This file returns the thresholds used throughout the Wavelet Toolbox software for de-noising and compression tools (command line files or GUI tools).

Valid options for the `METHOD` parameter are listed in the table below.

METHOD	Description
`'scarcehi'`	See `wdcbm` or `wdcbm2` when used with `'high'` predefined value of parameter `M`.
`'scarceme'`	See `wdcbm` or `wdcbm2` when used with `'medium'` predefined value of parameter `M`.
`'scarcelo'`	See `wdcbm` or `wdcbm2` when used with `'low'` predefined value of parameter `M`.
`'sqtwolog'`	See `'sqtwolog'` option in `thselect`, and see also `wden`.
`'sqtwologuwn'`	See `'sqtwolog'` option in `thselect`, and see also `wden` when used with `'sln'` option.
`'sqtwologswn'`	See `'sqtwolog'` option in `thselect`, and see also `wden` when used with `'mln'` option.
`'rigsure'`	See `'rigsure'` option in `thselect`, and see also `wden`.
`'heursure'`	See `'heursure'` option in `thselect`, and see also `wden`.
`'minimaxi'`	See `'minimaxi'` option in `thselect`, and see also `wden`.

METHOD	Description
'penalhi'	See wbmpen or wpbmpen when used with 'high' value of parameter ALPHA.
'penalme'	See wbmpen or wpbmpen when used with 'medium' value of parameter ALPHA.
'penallo'	See wbmpen or wpbmpen when used with 'low' value of parameter ALPHA.
'rem_n0'	This option returns a threshold close to 0. A typical THR value is median(abs(coefficients)).
'bal_sn'	This option returns a threshold such that the percentages of retained energy and number of zeros are the same.
'sqrtbal_sn'	This option returns a threshold equal to the square root of the value such that the percentages of retained energy and number of zeros are the same.

Discrete Wavelet 1-D Options

For 1–D wavelet transforms, the expansion coefficients are in the vector C and the lengths of the expansion coefficient vectors are stored in L.

Compression using a global threshold.

X is the signal to be compressed and [C,L] is the wavelet decomposition structure of the signal to be compressed.

```
THR = wthrmngr('dw1dcompGBL','rem_n0',X)
THR = wthrmngr('dw1dcompGBL','bal_sn',X)
```

Compression using level dependent thresholds.

X is the signal to be compressed and [C,L] is the wavelet decomposition structure of the signal to be compressed.

ALFA is a sparsity parameter (see wdcbm for more information).

```
THR = wthrmngr('dw1dcompLVL','scarcehi',C,L,ALFA)
      ALFA must be such that 2.5 < ALFA < 10
THR = wthrmngr('dw1dcompLVL','scarceme',C,L,ALFA)
      ALFA must be such that 1.5 < ALFA < 2.5
```

```
THR = wthrmngr('dw1dcompLVL','scarcelo',C,L,ALFA)
      ALFA must be such that 1 < ALFA < 2
```

De-noising using level dependent thresholds.

`[C,L]` is the wavelet decomposition structure of the signal to be de-noised, `SCAL` defines the multiplicative threshold rescaling (see `wden` for more information) and `ALFA` is a sparsity parameter (see `wbmpen` for more information).

```
THR = wthrmngr('dw1ddenoLVL','sqtwolog',C,L,SCAL)
THR = wthrmngr('dw1ddenoLVL','rigrsure',C,L,SCAL)
THR = wthrmngr('dw1ddenoLVL','heursure',C,L,SCAL)
THR = wthrmngr('dw1ddenoLVL','minimaxi',C,L,SCAL)
THR = wthrmngr('dw1ddenoLVL','penalhi',C,L,ALFA)
      ALFA must be such that 2.5 < ALFA < 10
THR = wthrmngr('dw1ddenoLVL','penalme',C,L,ALFA)
      ALFA must be such that 1.5 < ALFA < 2.5
THR = wthrmngr('dw1ddenoLVL','penallo',C,L,ALFA)
      ALFA must be such that 1 < ALFA < 2
```

Discrete Stationary Wavelet 1-D Options

De-noising using level dependent thresholds.

`SWTDEC` is the stationary wavelet decomposition structure of the signal to be de-noised, `SCAL` defines the multiplicative threshold rescaling (see `wden` for more information) and `ALFA` is a sparsity parameter (see `wbmpen` for more information).

```
THR = wthrmngr('sw1ddenoLVL',METHOD,SWTDEC,SCAL)
THR = wthrmngr('sw1ddenoLVL',METHOD,SWTDEC,ALFA)
```

The options for `METHOD` are the same as in the `'dw1ddenoLVL'` case.

Discrete Wavelet 2-D Options

For 2–D wavelet transforms, the expansion coefficients are in the vector `C` and the size of the coefficient matrices at each level is stored in `S`.

Compression using a global threshold.

`X` is the image to be compressed and `[C,S]` is the wavelet decomposition structure of the image to be compressed.

```
THR = wthrmngr('dw2dcompGBL','rem_n0',X)
THR = wthrmngr('dw2dcompGBL','bal_sn',C,S)
THR = wthrmngr('dw2dcompGBL','sqrtbal_sn',C,S)
```

Compression using level dependent thresholds.

X is the image to be compressed and [C,S] is the wavelet decomposition structure of the image to be compressed. ALFA is a sparsity parameter (see wdcbm2 for more information).

```
THR = wthrmngr('dw2dcompLVL','scarcehi',C,S,ALFA)
      ALFA must be such that 2.5 < ALFA < 10
THR = wthrmngr('dw2dcompLVL','scarceme',C,S,ALFA)
      ALFA must be such that 1.5 < ALFA < 2.5
THR = wthrmngr('dw2dcompLVL','scarcelo',C,S,ALFA)
      ALFA must be such that 1 < ALFA < 2
```

De-noising using level dependent thresholds.

[C,S] is the wavelet decomposition structure of the image to be de-noised, SCAL defines the multiplicative threshold rescaling (see wden for more information) and ALFA is a sparsity parameter (see wbmpen for more information).

```
THR = wthrmngr('dw2ddenoLVL','penalhi',C,S,ALFA)
      ALFA must be such that 2.5 < ALFA < 10
THR = wthrmngr('dw2ddenoLVL','penalme',C,S,ALFA)
      ALFA must be such that 1.5 < ALFA < 2.5
THR = wthrmngr('dw2ddenoLVL','penallo',C,S,ALFA)
      ALFA must be such that 1 < ALFA < 2
THR = wthrmngr('dw2ddenoLVL','sqtwolog',C,S,SCAL)
THR = wthrmngr('dw2ddenoLVL','sqrtbal_sn',C,S)
```

Discrete Stationary Wavelet 2-D Options

De-noising using level dependent thresholds.

SWTDEC is the stationary wavelet decomposition structure of the image to be de-noised, SCAL defines the multiplicative threshold rescaling (see wden for more information) and ALFA is a sparsity parameter (see wbmpen for more information).

```
THR = wthrmngr('sw2ddenoLVL',METHOD,SWTDEC,SCAL)
THR = wthrmngr('sw2ddenoLVL',METHOD,SWTDEC,ALFA)
```

The options for METHOD are the same as in the 'dw2ddenoLVL' case.

Discrete Wavelet Packet 1-D Options

Compression using a global threshold.

X is the signal to be compressed and WPT is the wavelet packet decomposition structure of the signal to be compressed.

```
THR = wthrmngr('wp1dcompGBL','bal_sn',WPT)
THR = wthrmngr('wp1dcompGBL','rem_n0',X)
```

De-noising using a global threshold.

WPT is the wavelet packet decomposition structure of the signal to be de-noised.

```
THR = wthrmngr('wp1ddenoGBL','sqtwologuwn',WPT)
THR = wthrmngr('wp1ddenoGBL','sqtwologswn',WPT)
THR = wthrmngr('wp1ddenoGBL','bal_sn',WPT)
THR = wthrmngr('wp1ddenoGBL','penalhi',WPT)
      see wbmpen with ALFA = 6.25
THR = wthrmngr('wp1ddenoGBL','penalme',WPT)
      see wbmpen with ALFA = 2
THR = wthrmngr('wp1ddenoGBL','penallo',WPT)
      see wbmpen with ALFA = 1.5
```

Discrete Wavelet Packet 2-D Options

Compression using a global threshold.

X is the image to be compressed and WPT is the wavelet packet decomposition structure of the image to be compressed.

```
THR = wthrmngr('wp2dcompGBL','bal_sn',WPT)
THR = wthrmngr('wp2dcompGBL','rem_n0',X)
THR = wthrmngr('wp2dcompGBL','sqrtbal_sn',WPT)
```

De-noising using a global threshold.

WPT is the wavelet packet decomposition structure of the image to be de-noised.

```
THR = wthrmngr('wp2ddenoGBL','sqtwologuwn',WPT)
THR = wthrmngr('wp2ddenoGBL','sqtwologswn',WPT)
THR = wthrmngr('wp2ddenoGBL','sqrtbal_sn',WPT)
```

```
THR = wthrmngr('wp2ddenoGBL','penalhi',WPT)
      see wbmpen with ALFA = 6.25
THR = wthrmngr('wp2ddenoGBL','penalme',WPT)
      see wbmpen with ALFA = 2
THR = wthrmngr('wp2ddenoGBL','penallo',WPT)
      see wbmpen with ALFA = 1.5
```

Examples

Level-Independent Threshold — Stationary Wavelet Transform

This example uses a level-independent threshold based on the finest-scale wavelet coefficients to implement hard thresholding with the stationary wavelet transform.

Load the noisy blocks signal. Obtain the stationary wavelet transform down to level 5 using the Haar wavelet.

```
load noisbloc;
L = 5;
swc = swt(noisbloc,L,'db1');
```

Make a copy of the wavelet transform coefficients. Determine the Donoho-Johnstone universal threshold based on the first-level detail coefficients. Using the `'sln'` option, `wthrmngr` returns a 1-by-L vector with every element equal to the same value. Take the mean of the vector to obtain a scalar threshold.

```
swcnew = swc;
ThreshSL = mean(wthrmngr('sw1ddenoLVL','sqtwolog',swc,'sln'));
```

Use the universal threshold to implement hard thresholding. The same threshold is applied to the wavelet coefficients at every level.

```
for jj = 1:L
swcnew(jj,:) = wthresh(swc(jj,:),'h',ThreshSL);
end
```

Invert the stationary wavelet transform on the thresholded coefficients, `swcnew`. Plot the original signal and the denoised signal for comparison.

```
noisbloc_denoised = iswt(swcnew,'db1');
plot(noisbloc); hold on;
```

```
plot(noisbloc_denoised,'r','linewidth',2);
```

Level-Dependent Threshold — Stationary Wavelet Transform

This example uses a level-dependent threshold derived from the wavelet coefficients at each scale to implement hard thresholding with the stationary wavelet transform.

Load the noisy blocks signal. Obtain the stationary wavelet transform down to level 5 using the Haar wavelet.

```
load noisbloc;
L = 5;
swc = swt(noisbloc,L,'db1');
```

Make a copy of the wavelet transform coefficients. Determine the Donoho-Johnstone universal threshold based on the detail coefficients for each scale. Using the `'mln'` option, `wthrmngr` returns a 1-by-L vector with each element of the vector equal to the universal threshold for the corresponding scale.

```
swcnew = swc;
ThreshML = wthrmngr('sw1ddenoLVL','sqtwolog',swc,'mln');
```

Use the universal thresholds to implement hard thresholding. The thresholds are applied in a scale-dependent manner.

```
for jj = 1:L
swcnew(jj,:) = wthresh(swc(jj,:),'h',ThreshML(jj));
end
```

Invert the stationary wavelet transform on the thresholded coefficients, `swcnew`. Plot the original signal and the denoised signal for comparison.

```
noisbloc_denoised = iswt(swcnew,'db1');
plot(noisbloc); hold on;
plot(noisbloc_denoised,'r','linewidth',2);
```

Image Compression— Birgé-Massart Thresholds

This example compresses an image using the Birgé-Massart strategy.

Load the image and add white Gaussian noise.

```
load sinsin
```

```
x = X+18*randn(size(X));
```

Obtain the 2-D discrete wavelet transform down to level 2 using the Daubechies' least-asymmetric wavelet with 8 vanishing moments. Obtain the compression thresholds using the Birgé-Massart strategy with `alpha` equal to 2.

```
[C,L] = wavedec2(x,2,'sym8');
alpha = 2;
THR = wthrmngr('dw2dcompLVL','scarcehi',C,L,alpha);
```

Compress the image and display the result.

```
 Xd = wdencmp('lvd',X,'sym8',2,THR,'s');
image(X); title('Original image');
figure;
image(x); title('Noisy image');
figure;
image(Xd); title('Denoised image');
```

Introduced before R2006a

wtreemgr

NTREE manager

Syntax

Description

`wtreemgr` is a tree management utility.

This function returns information on the tree *T* depending on the value of the `OPT` parameter.

Allowed values for `OPT` are listed in the table below.

`'allnodes'`	Tree nodes
`'isnode'`	True for existing node
`'istnode'`	True for terminal nodes
`'nodeasc'`	Node ascendants
`'nodedesc'`	Node descendants
`'nodepar'`	Node parent
`'ntnode'`	Number of terminal nodes
`'tnodes'`	Terminal nodes
`'leaves'`	Terminal nodes
`'noleaves'`	Not terminal nodes
`'order'`	Tree order
`'depth'`	Tree depth

See Also

`allnodes` | | `istnode` | `leaves` | `nodeasc` | `nodedesc` | `nodepar` | `noleaves` | `ntnode` | `tnodes` | `treedpth` | `treeord`

Introduced before R2006a

wvarchg

Find variance change points

Syntax

```
[PTS_OPT,KOPT,T_EST] = wvarchg(Y,K,D)
```

Description

`[PTS_OPT,KOPT,T_EST] = wvarchg(Y,K,D)` computes estimated variance change points for the signal `Y` for j change points, with j = 0, 1, 2, ... , `K`.

Integer `D` is the minimum delay between two change points.

Integer `KOPT` is the proposed number of change points ($0 \le$ `KOPT` $\le$ `K`). The vector `PTS_OPT` contains the corresponding change points.

For $1 \le$ `k` $\le$ `K`, `T_EST(k+1,1:k)` contains the `k` instants of the variance change points and then, if `KOPT > 0`, `PTS_OPT = T_EST(KOPT+1,1:KOPT)` else `PTS_OPT = []`.

`K` and `D` must be integers such that 1 < `K` << length(`Y`) and 1 $\le$ `D` << length(`Y`).

The signal `Y` should be zero mean.

`wvarchg(Y,K)` is equivalent to `wvarchg(Y,K,10)`.

`wvarchg(Y)` is equivalent to `wvarchg(Y,6,10)`.

Examples

Detect Variance Change Points

Add two variance change points to the blocks signal. Detect the variance change points using `wvarchg`.

Load the blocks signal. Add white noise with two variance change points located at index 180 and 600.

```
x = wnoise(1,10);
rng default;
bb = 1.5*randn(1,length(x));
cp1 = 180; cp2 = 600;
x = x + [bb(1:cp1),bb(cp1+1:cp2)/4,bb(cp2+1:end)];
```

Obtain the level-1 wavelet coefficients. Replace the top 2% of values with the mean value of the wavelet coefficients to remove all signal.

```
wname = 'db3'; lev = 1;
[c,l] = wavedec(x,lev,wname);
det = wrcoef('d',c,l,wname,1);
y = sort(abs(det));
v2p100 = y(fix(length(y)*0.98));
ind = find(abs(det)>v2p100);
det(ind) = mean(det);
```

Estimate the variance change points using the wavelet coefficients.

```
[pts_Opt,kopt,t_est] = wvarchg(det,5);
sprintf('The estimated change points are %d and %d\n',pts_Opt)
```

References

Lavielle, M. (1999), "Detection of multiple changes in a sequence of dependent variables," *Stoch. Proc. and their Applications*, 83, 2, pp. 79–102.

Introduced before R2006a